THE ELEMENTS OF ELECTRICITY

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PREFACE.

The following text book on electricity has been prepared for use of the Cadets of the United States Military Academy.

The course being required of all members of the third year class, explanations have been given in more detail than would be necessary were it elective. Recitations on the text proper are accompanied by the solution of numerous problems and class room instruction is supplemented by from eight to ten lectures and twenty laboratory periods.

WIRT ROBINSON.

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INTRODUCTORY.

CHAPTER 1.

UNITS.

1. Need of Units.—In the orderly study of any concrete science we early encounter the necessity for dealing with quantities. Quantities may be specified and an accurate conception of them conveyed to others only by stating how many times greater or less they are than some like quantity of which there is common knowledge. Those quantities employed as bases of comparison are called units.

2. Electrical Units to be Defined Later.—In beginning a study it might seem logical that we should first define the units to be used, but in electricity the number of units is perhaps greater than in any other one branch of science and a preliminary definition of them would from their mere number tend to confusion rather than to clearness; moreover, an accurate conception of some of them requires more or less knowledge of certain electrical principles and relations, therefore, it is found best to reserve these definitions until, in the development of the subject, the necessity for their use arises.

3. Fundamental Units.—There are, however, certain units of general application in all sciences and of these it is well to have from the beginning a definite conception. Such are the so-called "fundamental" units of length, mass and time and some others derived from these.

We may, in a sense, regard the unit of length alone as the fundamental unit for it is possible to define all the others more or less directly by reference to length. Thus, the unit of mass may be defined as the mass of water under certain conditions contained in a cube of certain dimensions, the unit of time in terms of the period of oscillation at a certain locality of a pendulum of a certain length, the unit of heat in terms of the linear expansion of mercury, etc.

The term "fundamental" is however applied to the units of length, mass and time because in addition to the simpler derived units of area, volume and weight, it is possible, as will be shown below, to express all such dynamical quantities as velocity, force, work, etc., in terms of these units. This does not mean that there is one universal fundamental unit of length or of mass or of time. The units are chosen arbitrarily, but once having been selected the system of derived units follows.

4. Standard of Length.—The desirability of having a single unvarying standard of length, one that could be reproduced should existing standards be destroyed, has long been evident. It has been proposed to take as such standard the length of the simple seconds pendulum at the sea level at some definite locality. This is about 39.14 inches.

The French government caused to be made most accurate measurements of several meridian arcs of the earth's surface whence was calculated the length of the meridian quadrant through Paris and one ten-millionth part of this quadrant (about 40 inches) was adopted as the measure of length and hence called the *meter*. A standard meter of platinum was made and is preserved in France. It is now known that an error was made in the determination of the length of the quadrant and that it is some 880 meters longer, so that practically the meter is the length of the platinum bar, the "*mètre des archives*" of France. Its length is 39.37+ inches.

5. Need of Multiples and Submultiples.—Although it would seem that there should be but one unit for any one kind of quantity, as a matter of fact this is not the case. The need of more than one arises mainly from the fact that the average human mind can not form a direct concrete conception of a quantity expressed by more than three figures. For example, should a person say that he had walked 63,360 inches we have no precise image of the actual distance, and even when expressed as 5280 feet we involuntarily translate into the next higher unit; but when he says that he has walked one mile we get a definite idea. In the other direction, to speak of an object as one 63,360th of a mile thick is almost meaningless but one inch conveys the exact impression. Therefore in practical affairs we require large units to measure large quantities and small units to measure small quantities.

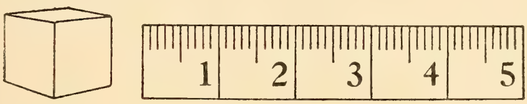
6. The Metric System.—It is not necessary to explain here the advantages of the metric or decimal system. The following table of English measures of length—

3 barleycorns make an inch
12 inches make a foot
3 feet make a yard
1760 yards make a mile

and the fact that besides these we have the line, the hand, the ell, the fathom, the rod, perch or pole, the chain, the furlong, the geographical mile, the nautical mile, the knot, the league, etc., between which in general no interrelation exists, are sufficient to show how illogical is our system. This is brought out all the more forcibly when we attempt to pass from one of these units to another or to make a calculation in which several are involved or to pass from linear dimensions to measures of capacity.

The metric system has by act of Congress been formally legalized for use in this country, but in spite of its advantages its introduction into every-day affairs has made but little progress and its employment is confined mainly to the sciences.

7. Unit of Length Selected by Electricians.—The meter is subdivided into ten parts, decimeters, a unit but little used, and these are again subdivided into ten parts, centimeters. This last



cubic cm. centimeter scale
Fig. 1.

unit, a length only very little less than four-tenths of an inch, is adopted by electricians as their fundamental unit of length. The selection of the centimeter rather than the meter was largely influenced by the fact that the cubic centimeter of water weighs one gram and consequently to determine the specific gravity of a substance it is only necessary to obtain the weight of a cubic centimeter of it in grams.

8. The Unit of Mass.—Mass and weight should not be confused. The mass of a body is the quantity of matter which it contains and is invariable but its weight varies as it changes its position with respect to the earth's center of gravity. Neverthe-

less, the masses of similar bodies under like conditions are proportional to their weights and practically we compare masses by comparing their weights.

In the metric system the unit of mass is the mass of a cubic decimeter of distilled water at its maximum density, 4° C. The weight of this, the kilogram (about 2.2 pounds), is the French industrial unit of weight and is perpetuated in a platinum standard.

The kilogram being inconveniently large for their purposes, electricians and other physicists have taken as their fundamental unit the gram, the mass of a cubic centimeter of distilled water at 4° C. Our five-cent nickel coin weighs about 5.26 grams.

9. The Unit of Time.—The unit of time used by electricians is the mean solar second. As the earth's orbit is not circular but elliptical, the velocity of the earth varies at various points and the apparent solar day, or the interval of time between two successive passages of the sun across the meridian, also varies. The average throughout the year of these apparent solar days is taken as the mean solar day and this is considered as composed of 24 hours of 60 minutes of 60 seconds, or as divided into 86,400 mean solar seconds.

10. The C. G. S. System.—The centimeter, the gram and the second were recommended as fundamental units by a committee of the British Association in 1873 and were formally adopted by the International Congress of Electricians in Paris in September, 1881. From these are obtained the various derived units and the system is therefore usually referred to as the "C. G. S. system."

11. Absolute Units.—Derived units are of two classes, absolute and practical. The term absolute, first used in this connection by Gauss, is applied to those units which are derived from the fundamental units of the system, depend upon them absolutely and exclusively and are independent of the force of gravity or of any instrument or apparatus or of any arbitrary weight or size of any arbitrary material. Many of the absolute units are inconveniently small, others are inconveniently large, and this gives rise to the practical units which more nearly fulfill the requirements of the practical electrician.

Area.—The absolute unit of area is the square centimeter.

Volume.—The absolute unit of volume is the cubic centimeter.

Velocity.—The absolute unit of velocity is the velocity of a body which moves at the rate of one centimeter per second. The practical unit in the metric system is one meter per second and in the English system one foot per second.

Acceleration.—Acceleration is the rate of change of velocity and the absolute unit is the acceleration of a body which changes its velocity one centimeter per second.

Force.—Force is measured by the acceleration which it imparts to a given mass. The absolute unit, the *dyne,* is that force which acting for one second upon a mass of one gram causes its velocity to change one centimeter per second. If the mass starts from rest, at the end of the first second it will have acquired a velocity of one centimeter per second; if the mass be moving its velocity will be accelerated or retarded one centimeter per second. The dyne is a very small force. The weight of one gram corresponds to 981 dynes, that of our five-cent piece to about 5160 and the head of the average pin to about 15. The practical unit in the English system is the pound, which is nearly 445,000 dynes.

Work.—Work is the expenditure of energy in overcoming a resistance over a path. The absolute unit of work, the *erg,* is the work performed in pushing or pulling against a force of one dyne over a path of one centimeter. The erg is a very small unit. The English practical unit, the foot-pound, or the work performed in lifting a weight of one pound for one foot against the force of gravity, is in round numbers 13,560,000 ergs.

Energy.—Energy is the capacity of a body to do work and hence is measured by the work which it can do, therefore, the absolute unit of energy is also the erg.

Heat.—The absolute unit of heat, the *small calorie,* is the amount of heat required to raise the temperature of one gram of water from 0° to 1° on the Centigrade scale. According to the latest determination of the mechanical equivalent of heat it requires an expenditure of 1402 foot-pounds to raise one pound of water from 0° to 1° C. The small calorie is therefore equivalent to 42,000,000 ergs.

In the C. G. S. system the practical units are some power of ten times the absolute units and these practical units have been named after distinguished electricians.

CHAPTER 2.

ELECTRICITY.

12. Origin of the Name.—Among the stones esteemed precious by the ancients was amber to which the Greeks applied the name "*elektron.*" This substance, which is now known to be a fossil resin, is found in various localities but especially along the shores of the Baltic where it is cast up on the beaches after storms. It was prized on account of its golden yellow color and luster and also because of certain talismanic properties attributed to it. It is quite soft and easily fashioned into beads which can be given a high polish by rubbing with a dry, woolen cloth. The workmen engaged in preparing these soon noticed that upon rubbing a piece it acquired a property which it had not before possessed, that is, it attracted to itself light substances such as particles of lint and dust, bits of straw, feathers, etc. This property quickly died away but could be revived by renewed rubbing. These observations are recorded by writers of 2500 years ago who, as was usual in such cases, fell back upon the supernatural for an explanation and ascribed to the substance certain mystical qualities.

For over two thousand years such remained the state of knowledge. During the reign of Queen Elizabeth a certain Doctor Gilbert, an Englishman, carried out a very remarkable series of experiments and observations upon the then vaguely known properties of magnets, and as allied to magnets investigated other bodies in which powers of attraction could be produced. He discovered that this property was by no means confined to amber and in Chapter II, Book Second, of his work, *De Magnete, Magneticisque Corporibus* (On the Magnet and Magnetic Bodies), published in 1600 he enumerates a list of substances, mainly vitreous or crystalline and resinous or resinoid, which possess it. He mentions among others the diamond, sapphire, opal, varieties of rock crystal, glass, fluor spar, rock salt, mica, sealing wax, resin, jet, sulphur, etc., and to all these bodies in which, like amber or *elektron*, the power of attraction could be produced by rubbing he applied the term "*electrics.*" From this it was an easy transi-

tion to the word "*electricity*" applied both to the study or science and to the agent itself.

13. Electricity.—At the very outset we are compelled to admit that we do not know what electricity is. It is not matter since it is devoid of physical dimensions and weight; yet in its production, transmission and manifestation it must always be associated with matter. Mechanical or chemical energy applied to matter at one point may be used to produce electricity which may be transmitted to some other point and there used to reproduce energy of either kind. Its great value in the industrial world consists in this very ability to transfer energy instantly to great distances and to deliver it with minimum loss.

Fortunately for our purposes a theory is not essential, for although our knowledge of the agent, electricity, is restricted to the various phenomena which it produces, the laws under which it operates are definite and well known and under any given set of conditions we are able to predict what the electrical outcome will be. The study of electricity which we are about to take up is therefore but an orderly and logical presentation of these phenomena and of the laws which govern them.

14. Divisions of the Subject.—Like any other science electricity can not be studied as a whole but must be separated into subdivisions, more or less artificial, and these subdivisions taken in such order and detail as may appear most suited to the development of the subject while at the same time avoiding undue repetition or presentation of facts involving anticipation of principles not yet explained.

It is customary to consider electricity under four heads corresponding to the four conditions under which its effects are usually observed.

1st, Electricity may exist as a motionless charge upon bodies. If a wooden ball at the end of a stiff wire be dipped under water and then withdrawn it will be covered with a film of moisture and this is very roughly analogous to the charge of electricity which may be imparted to a metal ball supported upon a glass stem. This is termed stationary or *static electricity.*

2nd, With a suitable path to direct it, electricity may flow in a constant stream. This is *current electricity.*

3rd, Associated with certain metals, mainly iron, its oxides and steel, there are met manifestations, termed magnetic, which take the form of forces traversing the metal, emerging at one end, following a curved path and re-entering at the other end. An electric current is surrounded by similar whirling forces; electricity may be made to produce magnetic effects and conversely from magnetic forces electricity may be produced. A third division is therefore *magnetism.*

4th, Finally, typically in the case of wireless telegraphy, the electricity is not in the form of a charge nor of a current but is transmitted through space by means of intermittent oscillations or *waves.*

From a practical standpoint, the least important of the above is the static electricity but it is now to be considered because of its historical interest, its development being chronologically the first and associated with the names of many noted scientists, among whom our Franklin played a prominent part. It also enables us to present in a simple manner certain useful principles and conceptions and thus serves as a stepping-stone to what follows.

PART I.

STATIC ELECTRICITY.

CHAPTER 3.

ELECTRIC ATTRACTION AND REPULSION.

15. Electric Attraction.—If on a dry day a rod of glass or of resin or of some resinoid substance such as amber, sealing wax, vulcanized rubber, sulphur, celluloid, etc., be rubbed with a piece

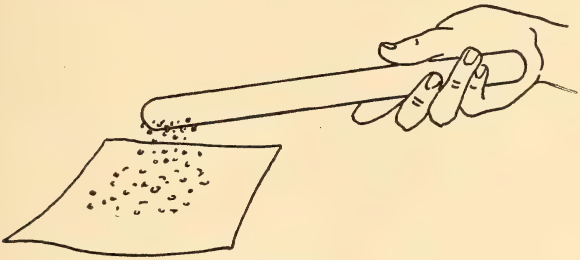


Fig. 2.

of fur or woolen cloth (wool is fur) and then held immediately above small particles of light substances such as bits of tissue paper, feathers, straw or chaff, the particles will leap up and cling to the rod. In the case of a glass rod the effect is more pronounced if it be rubbed with silk instead of with fur. The rod is said to be *electrified* and the state persists for some time in dry weather but disappears quickly if it be damp. The electrification is instantly lost if the rod be rubbed over its entire surface with the hand, or if it be dipped into water or passed quickly through a flame.

If an excited or electrified rod be held above a small block of wood no appreciable effect will be produced, but if the block be cut up into fine shavings they will be readily attracted. Although the block is attracted the electric force is too feeble to move it as a whole but easily moves the light pieces. In experimenting with electric attraction, on account of this feebleness it is customary to

use balls of pith, a substance which combines bulk with extreme lightness.

16. Electrified Bodies Attract Non-Electrified.—An electrified body attracts all non-electrified, including the metals, liquids, etc. Gilbert, who made this discovery, excepted only such bodies as are "afire or flaming or the thinnest air" and devised a piece of apparatus, a *versorium* (rotating needle, revolving pointer), by which this may be shown. Light needles of various substances were made and like compass needles balanced free to turn upon a pivot. If these be approached by an electrified body they will turn towards it. If an electrified piece of amber be held above a spherical globule of water the globule will assume a conical shape as if reaching up to the amber, so also the dense smoke from a recently extinguished candle will be attracted.

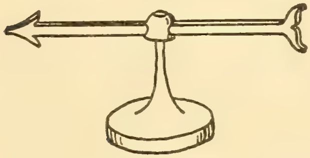


Fig. 3.

17. Electrified Bodies are Attracted by Non-Electrified.—The attraction between an electrified body and a non-electrified is mutual. This follows necessarily from a fundamental principle of mechanics but may easily be shown by suspending by a fine thread an electrified rod so as to turn horizontally like Gilbert's versorium. If a non-electrified body be held near, the rod will be attracted and turn towards it.

18. Electric Charge.—If two rods of sealing wax be rubbed with a woolen cloth they each become electrified. If they be rubbed one against the other no effect is produced. Finally, if one be electrified by rubbing and then the second be touched by the first, the second will be found to be slightly electrified. In other words, the electrified rod communicates a portion of its electrification to the non-electrified. The electrification upon a body is spoken of as a *charge*; an electrified body is said to be *charged*; and when the electrification is withdrawn it is said to be *discharged*.

19. Conductors and Non-Conductors.—In 1729 Stephen Gray, experimenting with electric attraction, used, instead of a glass rod, a tube into the open ends of which he had stuck corks to keep out the dust. Upon rubbing the glass tube he was surprised to find that the corks which had not been rubbed had nevertheless

acquired the property of attraction as if the charge generated upon the glass had spread upon them. To test this further he inserted in the corks long wands of wood or metal terminating in balls and found that when the glass was rubbed the balls attracted light objects. In place of the wands he next tried cords and wires by which he suspended a ball from a glass tube held in a balcony above and found that the ball became electrified as soon as the tube was rubbed. Wishing to continue this experiment at a greater distance than could be obtained from his balcony he was obliged to stretch his cord horizontally, and to keep it clear of the ground he hung it up at intervals by bits of thread attached to nails in a post. Under these conditions he was unable to electrify the ball and he surmised correctly that the charge had escaped by way of the suspending threads. A friend who was assisting him suggested that they use a smaller thread which would give a smaller path by which the charge could escape and a spool of silk being at hand it was tried with the result that he was able to electrify the ball at greater and greater distances up to as far as 765 feet. Finally, the silk thread breaking under the strain, he tried a fine wire, even smaller than the silk, but was unable to electrify the ball and now perceived for the first time that the escape of the charge depended not upon the size of the suspensions but upon the material of which they were made. As a result of a continuation of these experiments he was enabled to arrange all bodies in two classes, one which transmitted electricity to a distance and which he called *conductors*, the other preventing this transmission

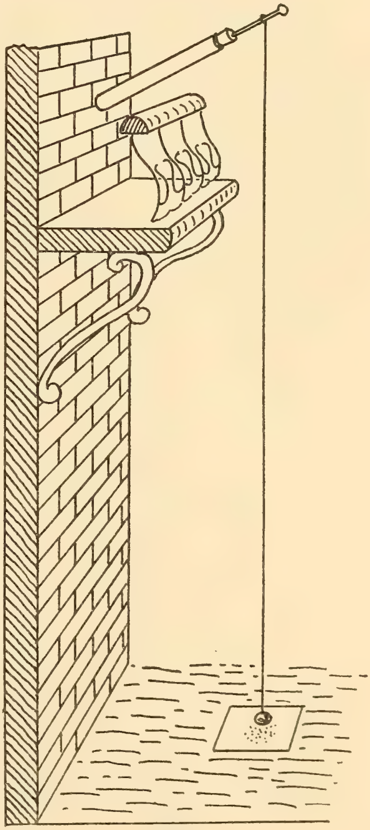


Fig. 4.

and called *non-conductors* or *insulators*. In the light of modern investigation we now know that there is no strict dividing line between the two and that there is no such thing as a perfect conductor nor a perfect insulator but that all bodies offer resistance to the passage of electricity, those that offer but little being termed conductors, those that offer a great deal being termed non-conductors. Joubert concisely defines good conductors as those bodies which when electrified at one point are immediately found to be electrified all over.

20. Table of Conductors and Non-Conductors.—In the following list the commoner conductors, partial conductors and non-conductors are arranged in order of their conductivity beginning with silver, the best conductor, and ending with air, the poorest conductor (or best non-conductor). This arrangement is not rigorously exact since relative conductivity may vary with change of temperature and other circumstances:

Good Conductors:	Non-Conductors:
Silver	Slate
Copper	Oils
Aluminum	Porcelain
Brass	Leather
Platinum	Paper
Iron	Wool
Lead	Silk
Mercury	Resin
Fair Conductors:	Rubber
Compact carbon	Shellac
Acid solutions	Vulcanized rubber
Salt solutions	Mica
Living plants	Paraffine
Damp earth	Glass
Partial Conductors:	Air
Water	
Animal bodies	
Flame	
Cotton	
Woods	
Marble	

The foregoing explains why an electrified body is discharged when rubbed with the hand or dipped into water or passed through a flame, also why, as Gilbert discovered, damp weather is unfavorable for the production of electrification.

21. All Bodies Susceptible of Electrification.—In contradistinction to his *electrics* Gilbert designated as *non-electrics* those bodies in which he was unable to produce electrical attraction by friction. Among these he enumerates various flints and agates, marble, bone, ivory, the metals, the lodestone, the human body, etc. We now know that he was in error in supposing that they could not be electrified. Examination of the table above will show that his electrics are all non-conductors and his non-electrics are all conductors. When he attempted to electrify a piece of metal the charge upon it was instantly conducted away. If the metal be attached to a glass handle it is readily electrified. If a person stand upon a glass-legged stool or upon a cake of resin or be suspended by silk cords and then be touched by an electrified glass rod or stroked by a piece of fur he will be strongly electrified, small light particles will fly to him as to the electrified amber and if a second person attempt to touch him, just when the distance between the outstretched hand and the electrified person becomes very small a faint snapping noise will be heard and both persons will perceive a slight pricking sensation. In the dark it will be seen that this noise accompanies a spark. All bodies if properly insulated so that the charge upon them can not escape may be electrified.

22. Electric Repulsion.—Reverting to the first experiment in electric attraction (Par. 15), if the electrified rod with the particles adhering to it be observed for a brief interval, the particles will be seen to leap or dart away from the rod as if shot away by a repelling force. This repulsion does not take place until *after* the particles have been in contact with the electrified rod. To exhibit this better, use is made of the so-called *electric pendulum*, a pith ball suspended by a fine silk thread (Fig. 5). If the ball be approached by an electrified rod it will fly to the rod and after a short contact will be repelled.

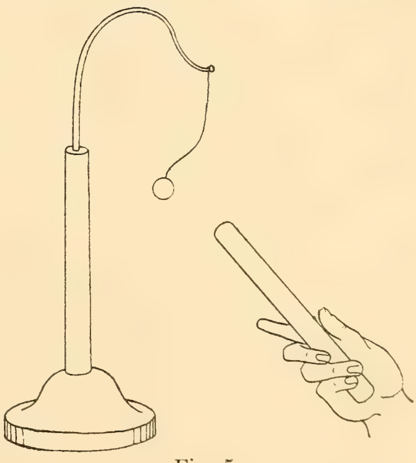


Fig. 5.

If the rod be moved in pursuit the ball will continue to move away avoiding the rod. The ball is now charged, as may be shown by its being attracted by any non-electrified body held near it; the repulsion must therefore be due to the charge which it acquired by its contact with the rod.

23. Two Kinds of Electrification.—If the pith ball of an electric pendulum be approached by a stick of sealing wax which has been rubbed with fur, it will first be attracted and after contact will be repelled. Similarly, if it be approached by a glass rod which has been rubbed with silk, it will be attracted until contact is made and thereafter repelled. But the strange part is that the ball repelled by the electrified sealing wax is attracted by the electrified glass and the ball repelled by the glass is attracted by the sealing wax. The electrification produced upon the glass must therefore be different from that produced upon the sealing wax. Dufay, who in 1733 made this discovery, designated these by the terms *vitreous* and *resinous*, vitreous being that produced by rubbing glass with silk and resinous that by rubbing sealing wax with fur. It has since been discovered that the kind of electricity produced does not depend entirely upon the material of the body rubbed but also upon that of the rubber and moreover varies in a surprising manner with the polish, the temperature and even the color of the body rubbed. Glass rubbed by silk is vitreously electrified but if it be rubbed by fur it is resinously electrified. It is possible to arrange a list of substances so that any one body in it is vitreously electrified when rubbed by any other below it on the list. The following is such a list:—Fur, glass, flannel, feathers, silk, paper, wax, metals, vulcanized rubber, celluloid.

In view of the above it is better, for reasons given in Par. 27, to follow Franklin and employ the terms *positive* and *negative*, the vitreous being positive, the resinous negative.

24. Like Charges Repel, Unlike Attract.—If two pith balls suspended side by side by separate silk threads (Fig. 6) be approached by an electrified rod of glass or of sealing wax they will both be attracted to the rod and, as soon as they have touched it, will be repelled, but not only this, they will repel each other and no longer hang side by side but will diverge and stand apart. If two separate pendulums be used and the pith ball of one be charged

from a glass rod, the other from a rod of sealing wax, the balls will attract each other. We therefore see that bodies charged with like electricity repel each other; those charged with unlike electricity attract each other.

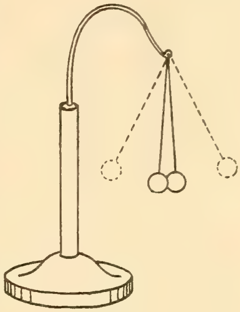


Fig. 6.

25. Electroscopes.—Instruments for determining (a) whether a body is charged or not and (b) the nature of the charge are called electroscopes. The simplest form of an electroscope is Gilbert's versorium described in Par. 16. The electric pendulum is frequently used as an electroscope. If the pith ball after being touched by the hand is attracted by the body being investigated, the body is charged. After we have in this way ascertained that the body is charged we next determine the nature of the charge by charging the pith ball, say positively, or from a glass rod which has been rubbed by silk, after which when held near the body it will be repelled if the latter be charged positively and attracted if it be charged negatively.

The large insulated metal conductors, used with certain electrical machines to be described later, often have attached to them as charge indicators small electric pendulums which differ from the one already described in that the support is a brass rod and that the pith ball is suspended by a linen or cotton thread or by a very slender metal filament instead of by a silk thread. When the conductor is charged the pith ball becomes charged through the suspending thread and is repelled and stands out at an angle from the vertical brass support. Since the greater the charge the greater the repulsion and the greater the angle at which the pith ball stands, this instrument indicates roughly the relative amount of the charge.

The gold-leaf electroscope, a much more sensitive form, is described further on (Par. 34).

26. Simultaneous Production of Equal Amounts of Both Kinds of Electricity.—In producing electricity by friction the body rubbed acquires a certain kind of charge and the rubber acquires the other kind; thus in rubbing a glass rod with silk the rod is charged with vitreous or positive electricity and the silk can be shown to have a resinous or negative charge. Furthermore, as

may be shown in several ways, the amounts of the two kinds are exactly equal. If two substances are rubbed together and acquire opposite charges and their charges be imparted successively to a third body the third body will not be electrified. If a disc of glass and one covered with silk, both being mounted on glass handles, be rubbed together they will each separately attract pith balls but when placed together will have no effect, the charge on the one exactly counterbalancing or neutralizing that on the other.

27. Theories of Electricity.—Two theories were advanced to account for the above phenomena. The first is *Symmer's Two Fluid Theory.* According to this there exist in all bodies two electrical fluids of opposite kinds but in exact balance, thus neutralizing each other. When a body is excited by friction this balance is disturbed and one of the fluids is drawn off upon the rubber leaving the remaining fluid unbalanced and in excess. In this country the theory most generally accepted is *Franklin's Single Fluid Theory.* In brief this is to the effect that all bodies in their natural state are charged with a certain quantity of electricity, in each body this quantity being of definite amount. When two bodies are rubbed together one parts with a portion of its electricity which is appropriated by the other. The latter then has more than its normal share and is positively electrified; the former has less and is negatively electrified. It is proper to state here that although we do not know what electricity is, we do know that it is not a fluid yet we retain the term for convenience, and that although we speak of bodies being positively or negatively electrified we really do not know which has the greater charge and the terms are used purely in a conventional sense, just as in analytical geometry distances to the right of the vertical axis are by convention considered positive and those to the left negative. Finally, no satisfactory explanation is given why bodies should acquire unlike charges by friction. The amount of electrification is not proportional to the amount of mechanical energy spent in friction, since it is immaterial whether the friction be of the ordinary kind or be rolling, but it is proportional to the amount spent in pulling apart two bodies held together by the mutual attraction due to their opposite electrical states.

CHAPTER 4.

ELECTROSTATIC INDUCTION.

28. Electrification by Influence.—In Fig. 7, *A* represents a metallic ball attached to a stand by a glass stem and *B* a metallic cylinder similarly mounted and carrying on its under side a series of pairs of pith balls hanging from linen threads. So far as electrical results are concerned, it is immaterial whether the ball and cylinder be solid or hollow. They may even be of wood covered with tin-foil or gilded but are usually made of thin brass.

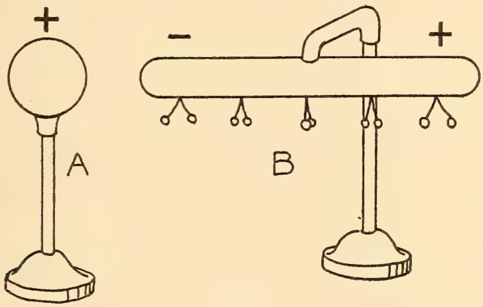


Fig. 7.

If now the ball *A*, while at some distance from *B*, be given a charge, say positive, and then be moved up towards *B*, the pith balls beneath *B* will be observed to diverge indicating that *B* is charged. Since *A* has not touched *B* and since the same effect is produced when a sheet of glass is interposed between *A* and *B* and, finally, since it can be shown that the charge upon *A* is undiminished, the charge upon *B* could not have been communicated from *A* but must have been *induced* or produced by the *influence* of *A*'s charge. This phenomenon may be called "*induction*" but, as will be seen later, there is a more important and different kind of induction and it is better to use the term "*influence.*" If *A* be withdrawn, the charge upon *B* disappears.

29. Distribution of the Induced Charge.—If we return to the preceding experiment and examine *B* while it is under the in-

fluence of A, it will be noticed that the pairs of pith balls do not diverge to the same extent, those at the ends standing far apart but the divergence decreasing towards the center and the pair at the center not diverging at all. This indicates that the charge has accumulated at the ends of B and that the center is not charged. Examination with an electroscope will show that the charges at the ends of B are of different kinds, that nearest A (in the case assumed) being negative, that farthest away being positive; in other words, the positive charge on A has induced on B and drawn as near to itself as possible a negative charge and repelled as far as possible a positive charge.

In Par. 24 it is stated that bodies charged with like electricity repel each other and those charged with unlike attract. The above experiment seems to indicate that it is not the charged *bodies* that attract or repel each other but the *charges* themselves; however, as we can not obtain a charge separate from a material body the matter is not susceptible of absolutely convincing proof.

30. Electric Attraction and Repulsion Explained.—The foregoing affords an explanation of the phenomena of attraction and repulsion already described. When an electrified rod is presented to a pith ball, a like charge is induced on the far side of the ball and an opposite charge on the near side. The like charge is repelled, the opposite attracted and the opposite being the nearer, the force of attraction is greater than that of repulsion and the ball moves bodily to the rod. Upon contact with the rod the opposite charge on the ball is neutralized by a portion of the charge on the rod, leaving the ball with the same kind of charge as that remaining on the rod and consequently the ball is repelled.

31. Amount of Induced Charge.—A given charge always induces on surrounding objects an exactly equal opposite charge. If a small charged sphere be placed at the center of a hollow conducting sphere there will be induced upon the inner surface of the latter an exactly equal opposite charge, and this no matter what the size of the outer sphere or the thickness or the nature of the intervening non-conductor. If the charged sphere be displaced from the center so as to be nearer one side of the cavity than the other, the amount of the induced charge is unaltered

but the greater portion will accumulate upon the side of the cavity nearest the sphere. A charged body inside of a room induces upon the ceiling, walls, floor and surrounding objects opposite charges which in the aggregate exactly equal the central charge and which accumulate most upon those objects nearest to it. Finally, if the charged body be at a distance from others, as for example in an open field, the induced charge will still be the same but will be spread over the surface of the ground, the greater portion being immediately beneath the body. If while in this position a conducting body be brought up close to it, practically the entire induced charge will be found upon the second body and the portion upon the earth becomes so small that it may be neglected. In ordinary laboratory experiments where the charged body is a foot or more from the table beneath and is supported by an insulated stem and the conductor upon which the charge is induced is brought up to a distance of an inch or so from the first, the induced charge upon the table and more distant objects becomes less and less and gathers more and more upon the conductor. Under such conditions we may say that the amount of the induced charge upon the conductor depends upon—

(a) The amount of the primary or inducing charge;

(b) The distance between the primary and the induced charge;

(c) The nature of the medium between them.

The greater the primary charge, the greater its influence and the greater the induced charge.

The nearer the primary charge to the conductor, the greater the induced charge.

With a constant primary charge at a constant distance from the conductor, the amount of the induced charge is found to vary with the nature of the separating medium, that is, whether it be air or oil or glass or sulphur or mica or other non-conductor and this variation is not in proportion to the value of the substance as a non-conductor but to an inherent property of the substance termed by Faraday its *dielectric capacity* (see Par. 90).

The maximum charge that could ever be induced is one at the far end of the conductor equal and similar to the primary charge and one at the near end equal and opposite. As the distance between the primary and the opposite induced charge diminishes a point is reached where the attraction between them becomes

great enough to break down the resistance of the remaining thickness of the medium intervening, a spark leaps across, the primary charge and the opposite induced charge neutralize each other, the original charged body is found to be discharged and the conductor is left charged with the similar charge which at first was repelled to its far end.

32. Separation of the Induced Charges.—If we repeat the preceding experiment with the charged ball *A* and a divided con-

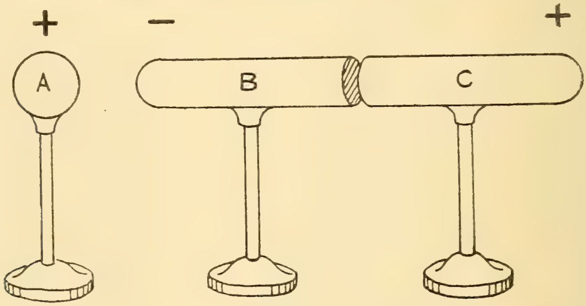


Fig. 8.

ductor consisting of two parts *B* and *C*, Fig. 8, which may be placed in close contact, the induced positive charge will be repelled into the far end *C* and the negative charge drawn into the near end *B*. While under the influence of *A*, *C* may be removed first and then *B* and each will be found to be charged, *C* positively and *B* negatively. If the two parts while distant from *A* be again joined together their electrification vanishes. This is an additional proof of the fact stated in Par. 26 of the simultaneous production of equal amounts of both kinds of electricity.

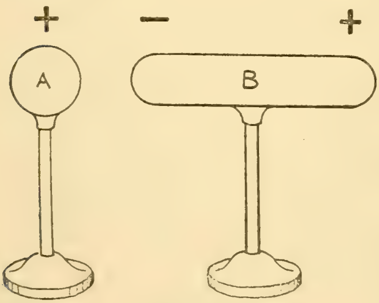


Fig. 9.

33. Free and Bound Charges.—Let us again consider the case of the charged insulated ball and the insulated conductor as shown in Fig. 9. The positive charge on *A* has induced and attracted to the near end of *B* a negative charge which is held securely by their mutual attraction. The hand may be placed on *B*, a wire may be attached to the near

end of B, still the negative charge refuses to budge and the only way by which it can be made to shift its position is by connecting it to some conductor which will allow it to approach A nearer than it is now. Such a charge, that is an induced charge held by a primary charge of the opposite kind, is said to be *"bound."* On the other hand, the positive charge on the far end of B is being repelled by A and will take advantage of any path whatsoever which will enable it to withdraw more remote from A. Thus if the hand be placed upon the near end or upon any other point of B the positive charge immediately escapes through the body and finally to the earth, even though in doing so it must in a part of its pathway draw nearer to A. Such a charge, in contradistinction to the bound charge, is said to be *"free"* and this name is also applied to any charge upon an insulated conductor not under the influence of some other charge. Since the free charge always escapes when the conductor is touched and the bound charge remains, the following rule is given: *If while under the influence of a charged body a conductor be touched, it acquires a charge of the opposite sign.*

We are now in a position to understand the operation of two pieces of apparatus, the gold-leaf electroscope and the electrophorus.

34. The Gold-Leaf Electroscope.—This is a very sensitive piece of apparatus for detecting the presence of electric charges and determining their character. The simplest form consists of a glass jar (Fig. 10) closed by a stopper of insulating material through which passes a brass rod which terminates above in a metal knob or disc and below is bent like the letter L. Fastened to the horizontal arm of the L so as to hang face to face in contact and vertically are two small ribbon-like strips of gold-leaf. This is used because on account of its extreme thinness it is lighter than any other material of equal strength and adds the advantage of being an excellent conductor. The glass jar serves as an insulating support and protects the leaves from currents of air which would cause them to flutter. When a charged body, such as an electrified rod of glass or of sealing wax, even though the charge be very small, is brought within a foot or so of the apparatus the hanging leaves will diverge. The explanation is that the charged body induces and attracts an unlike charge into the knob of the apparatus and repels a charge of similar kind to its own as far as possible,

that is, into the gold leaves; these having like charges repel each other and stand apart.

To determine the nature of a charge, the electroscope is given a preliminary charge of a known kind. This causes the leaves to

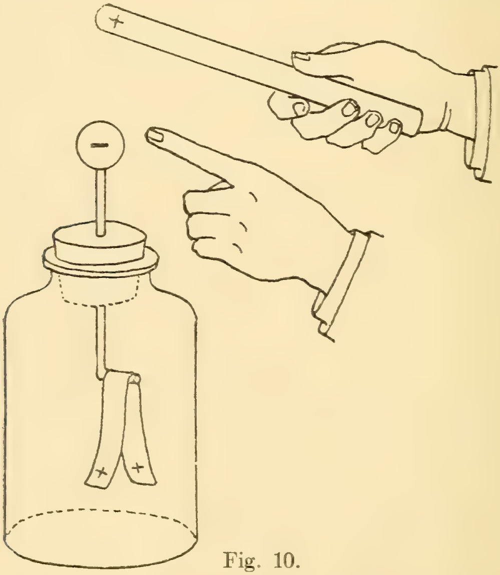


Fig. 10.

diverge. If now it be approached by a charge of the same kind the leaves will diverge more while if the charge be of opposite kind they will droop together.

By taking advantage of the principle given in the preceding paragraph we may with a single charged body impart to the electroscope a charge of either kind desired. Thus with a positively charged glass rod we may touch the knob and impart a slight positive charge (we really neutralize the induced negative charge in the knob and leave the induced positive charge). To charge it negatively we hold the glass rod near the knob (Fig. 10). This induces a bound negative charge and a free positive charge. If now the knob be touched by the remaining hand the free charge will be removed as will be indicated by the leaves instantly falling together. Now withdraw the hand and finally remove the rod. The bound negative charge, which had been attracted into the knob, will surge back and distribute itself as will be shown by the leaves again diverging.

35. The Electrophorus.—Volta's invention, the electrophorus, Fig. 11, an instrument for producing static charges by influence, consists of two parts. The first, analogous to the charged rod used in the experiments described in the preceding paragraph, is a flat cake of some resinoid body, resin, sealing wax, sulphur, vulcanized rubber or celluloid, mounted in a shallow metal dish. The second is a circular disc of metal, or of wood covered with tin-foil, at the back of which is a glass handle. To use the instrument, the cake dry and free from dust is rubbed with a warm, dry, woolen cloth or piece of fur. It thus acquires a negative charge. The metal disc is then placed upon the cake. It is in mathematical contact with the cake in only a few points and the cake being a non-conductor only the minute portions of the charge at these points of contact flow into the disc. Therefore the disc is a conductor separated from a charged body, the cake, by a layer of air as thin as a sheet of paper and consequently a bound positive charge is induced upon its lower face and a free negative charge upon its upper face. While in this condition it is touched by the finger, the free charge escapes and, in accordance with the rule in Par. 33, it is left with a positive charge. It may then be lifted by the glass handle and its charge being no longer bound can be used to give a spark, to charge other bodies, etc. As practically none of the primary charge on the cake is removed, this process could be repeated an indefinite number of times without the necessity of recharging the cake but, as a matter of fact, the primary charge gradually weakens due to leakage into the air.

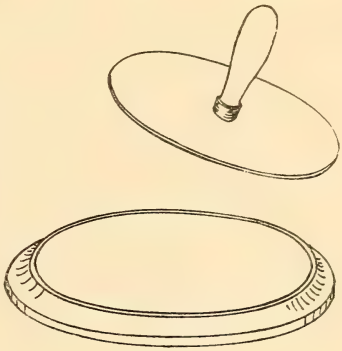


Fig. 11.

In the production of electricity, energy must always be expended. It requires more force to pull the disc away from the charged cake than it does from the cake before it is charged; the extra energy thus expended accounts for the production of the charge.

Machines have been invented by which this operation of bringing up the conductor, touching it and then withdrawing it is performed automatically and the movement of these machines, being one of rotation, the production of the charge is almost continuous.

CHAPTER 5.

DISTRIBUTION OF CHARGE.

36. Charge on a Non-Conductor.—An electric charge imparted to a body is differently distributed according to whether the body is a conductor or a non-conductor. In the case of a non-conductor the charge clings to the spot where it was generated or placed. If a stick of sealing wax be rubbed, only the part which has been rubbed will be found to be charged. If a cake of non-conducting material be touched by a charged body, only the spots actually touched will be charged. If such a cake be charged over its entire surface and then be touched by the finger or by a conductor, only the spots actually touched will be discharged. Lichtenberg devised a means by which the above may be shown to the eye. A charged body is moved like a pencil over a dry sheet of glass or of resin and a pattern is traced. Finely powdered red lead and sulphur mixed together are then sifted over this pattern through a piece of muslin. In the mixing and sifting the red lead becomes positively electrified, the sulphur negatively, and if the original charge be positive, the sulphur will be attracted, the red lead repelled and there will be produced a yellow pattern on a red back ground. In performing this experiment it will be noticed that the sulphur does not follow absolutely the mathematical lines originally traced but spreads slightly in mossy or frost-like patterns. Charges while not flowing over a non-conductor still have a tendency to creep or spread and the fern-like forms are due to minute particles of dust which lead the charge now in one direction, now in another.

37. Charge on a Conductor.—On the other hand, a charge imparted to any point of a conductor spreads immediately over the entire body and if a charged conductor be touched at any point so as to afford a path to the earth it is immediately discharged. It is possible with the apparatus described in the next paragraph to remove a portion of the charge. As soon as this portion is removed the remaining charge redistributes itself.

38. The Charge Confined to the Surface.—With size, shape and other conditions constant it is found that the same charge may be imparted to a conductor whether it be solid or hollow or even made of non-conducting material covered with tin-foil or gilded. The inevitable conclusion is that the charge resides upon the surface of a conductor. This is shown directly by the following experiments. A hollow metallic sphere (Fig. 12) with an opening in its top and mounted upon a glass support is given a charge. In order to take a sample portion of a charge for investigation, Coulomb devised a piece of apparatus which he called a *proof plane*. This is a little circular disc of metal or gilded paper fastened to the end of a small glass rod. If the disc be touched to a charged body it receives a portion of the charge and may then be removed, and the charge tested by an electroscope or otherwise. If the charged sphere be touched by a proof plane it will part with a portion of its charge. If, however, the proof plane be inserted through the opening in the sphere and the inside of the sphere be touched, the plane will show no sign of any charge.

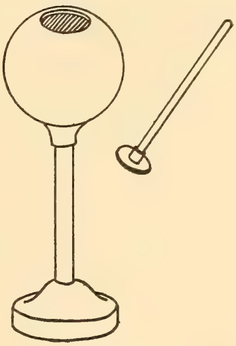


Fig. 12.

The above fact may be even more conclusively shown as follows: A small metal ball suspended by a silk thread is brought into contact with the outside of the charged hollow sphere. While touching the sphere it is practically a portion of the latter's outer surface and it receives a charge. The charged ball is then lowered through the opening until it touches the inside of the sphere. At that instant when it forms a part of the latter's inner surface it is discharged, the charge passing through to the outside of the sphere.

Faraday showed the same thing with a cylinder of wire gauze instead of the sphere.

39. Biot's Experiment.—Another demonstration of the surface distribution of the charge is given by Biot's experiment. In Fig. 13, *A* is an insulated metallic sphere and *B* and *C* are glass-handled metallic hemispheres slightly larger than the sphere. If the sphere be charged and then the hemispheres placed so as to completely cover it but not to touch it the charge will still remain on the sphere. If the covers be allowed to touch the sphere the charge will im-

mediately pass to the hemispheres which when separated will be found to be charged and the sphere discharged. The reason for this is given later (Par. 68).

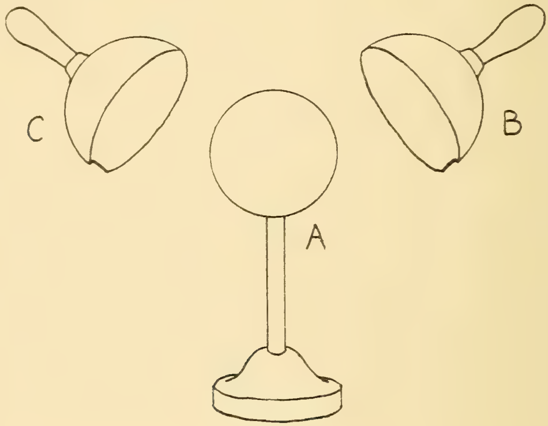


Fig. 13.

As an exception to the foregoing general statement there is one set of conditions under which it is possible to have a charge on the interior of a conductor. If through the opening of the sphere shown in Fig. 12 there be inserted a charged insulated body, there will be induced upon the inside of the surrounding sphere a charge of opposite kind, the charge of like kind being repelled to the exterior.

Finally, it must be remembered that we are now discussing static charges, for, as will be shown later, current electricity penetrates throughout the conductor.

40. Distribution of Charge.—Although, as was stated in Par. 37 above, a charge imparted to a conductor spreads over it immediately, the distribution is not uniform but more of the charge

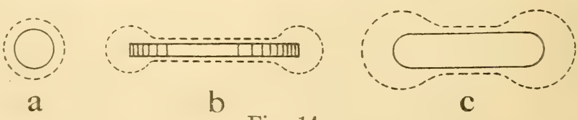


Fig. 14.

will be found about the edges and angles than upon the flatter surfaces. In fact, there is only one body, the sphere, upon which the distribution is uniform and this is so only when the sphere is so remote from other charged bodies that the effects of induction

are not felt. This uniform distribution may be represented graphically as in (a) in Fig. 14 by drawing about the circle representing the sphere a concentric dotted circle as if the charge were a material of the thickness represented by the distance between the full and the dotted circles.

On a metallic disc (b) the charge is heaped up around the edges but uniformly distributed over the flat surfaces. Advantage is taken of this in a piece of apparatus, the attracted disc electrometer (Par. 101).

If the conductor be a cylinder with rounded ends (c), such as is used with many electrical machines, the amount of charge at the ends is much greater than upon the cylindrical portion.

41. Surface Density.—This material conception of the charge is not confined to graphic representation but in our calculations we may and do treat it as if it were a substance the component particles of which repel each other and combine in a resultant action upon other charges. Thus we speak of it as spread with a certain density over the surface of a conductor or as being denser at certain points than at others. This surface density is measured by the amount of electrification or number of units of electricity per unit area. What these units are is explained later (Par. 56). An isolated sphere is the only body over which the distribution is uniform and the surface density is determined by dividing the total charge by the area of the sphere.

On neither conductors nor on non-conductors may a charge be accumulated indefinitely, but when in air the surface density at any point reaches about 20 units per square centimeter a discharge will occur either along the surface of the body or through the body or through the surrounding medium.

42. Effect of Points.—Coulomb found that in an ellipsoid of revolution the surface density at the extremities of the axes were to each other as the lengths of the respective axes. In a spindle-shaped ellipsoid where the axis of revolution is much longer than the minor axis the density at the pointed end is very much greater than that on the equatorial surface, and this disproportion increases as the ellipsoid becomes more and more pointed until finally the particles of air adjacent to the point become charged. Having like charges, these particles repel each other and are repelled from the point. They therefore move off, giving way for

others which likewise become charged and move off, thus producing a continuous electric wind and rapidly discharging the body. In consequence of the foregoing, all points, sharp corners and angles, unless they be designedly used, are carefully avoided in electrical apparatus.

43. Franklin's Experiment.—To illustrate the effect of points Franklin devised the following experiment. From the ceiling there is suspended by a silk thread a pith ball as large as a marble and upon the floor immediately beneath is placed a glass jar upon whose mouth is balanced a metal ball (Fig. 15). The thread is of

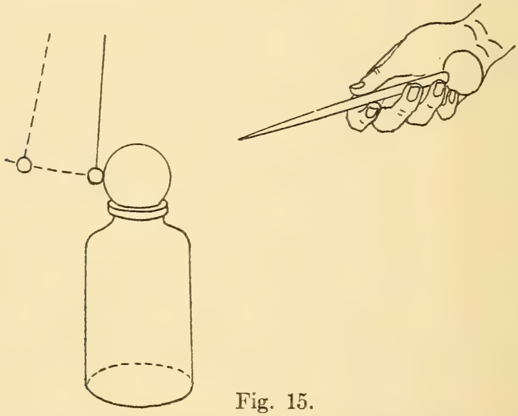


Fig. 15.

such length that the pith ball hangs against the side of the metal ball. A charge is communicated to the metal ball and the pith ball is at once repelled and hangs at a distance of four or five inches. If now a sharp-pointed wire or a needle held in the hand be brought up to within six or eight inches of the metal ball, its charge is instantly lost as will be shown by the pith ball falling against it at once. In the dark a faint light, like that of a firefly, will be seen around the point of the needle. Franklin stated that the needle drew the electric fire from the ball. A more accurate explanation is that the charge upon the ball induced up through the body of the experimenter and out to the needle an opposite charge which escaped from the point, passed over to the ball and neutralized its charge. This experiment is noteworthy as it suggested to Franklin the invention of the lightning-rod.

44. Other Experiments with Points.—The existence of the electric wind referred to above can be shown in several ways. If

a point attached to a charged conductor be held near the face the wind can be distinctly felt. If such a point be held close to the flame of a candle the flame will be blown to one side or perhaps even extinguished.

As the charged particles of air are repelled from the point, the point must experience an equal repulsion in the opposite direction. This is illustrated by the *electric whirl* shown in Fig. 16. It consists of a light metal hub with a set of pointed wire spokes, the ends all being bent at right angles and all pointing in the same direction, clockwise or counter-clockwise. The hub is placed upon a pointed pivot so as to turn freely like a compass needle. The pivot is connected to an electric machine and when a continuous charge is supplied the whirl rotates in the opposite direction to that in which the ends of the wires point.

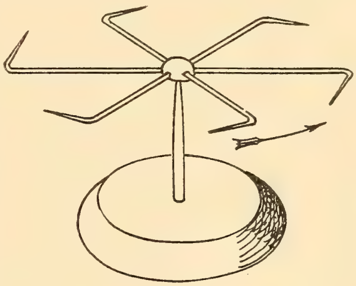


Fig. 16.

There is a final point in connection with this electric wind which is to be noted. Just as the spray from an atomizer moistens the surface against which it is directed, so the electrified particles of air striking the surface of a non-conductor impart a charge to this surface. This property is utilized in the operation of certain electric machines described in the next chapter.

45. Division of Charge.—If a charged body be brought into contact with one not charged, both being insulated, the charge is divided between the two in proportion to their electric capacities, a property to be defined later (Par. 79). If both bodies be charged they may be considered to make common stock of their charges and to redistribute the total as stated above. This is true as well for charges of opposite kinds; enough of the greater charge is consumed to neutralize the lesser and the remainder, whether positive or negative, is distributed between the two bodies. Spheres of equal size have equal capacities, therefore, if an insulated charged sphere be touched by an equal uncharged one, likewise insulated, the original charge will be divided into halves. This enables us to get two similar and equal charges and, as will shortly be shown, is of very great importance in the determination of the laws of electrical attraction and of repulsion and in the measurement of electrical charges.

CHAPTER 6.

ELECTRICAL MACHINES.

46. Kinds of Machines.—In the preceding chapters we have seen how electric charges may be produced first by friction of dissimilar substances and second by influence, as typically in the case of the electrophorus. Based upon these two principles there have been constructed two distinct classes of machines designated respectively as frictional and influence machines. These substitute for the intermittent motion of friction and for the alternate lowering and raising of the disc of the electrophorus a motion of rotation by which wasteful expenditure of energy is avoided, the production of the charge becomes continuous, and a much greater charge can be obtained than by the other means. Many kinds have been constructed and though they are of interest the limits of time and space restrict us to a brief description and explanation of a typical form of each.

47. Frictional Machines.—Frictional machines comprise three parts, the material which is rubbed, the rubber and the body, called the *prime conductor,* upon which the charge is accumulated.

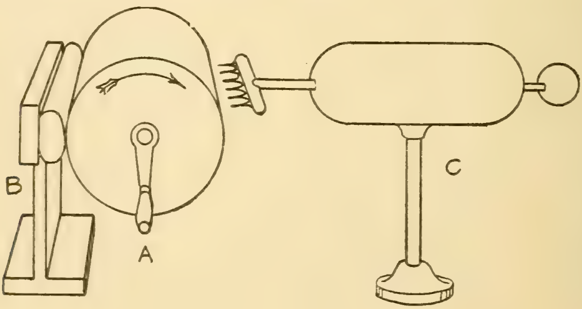


Fig. 17.

The earliest form, invented by Von Guericke, consisted of a globe of sulphur cast upon a wooden axis by which it was rotated. As the globe revolved it was pressed by the bare hand and the charge was gathered by a light chain which dangled against the globe

and hung from the prime conductor, an iron bar suspended by silk chords. Many changes and improvements were made by subsequent inventors. The operation of the modern machine is best explained from the form shown in Fig. 17, the *cylinder machine.*

48. Cylinder Machine.—This consists of a glass cylinder A rotating on a horizontal axis, a hair-stuffed pad B pressing against one side of the cylinder and the prime conductor C placed on the other side and insulated upon a glass support. This conductor is of hollow brass, of the shape shown, from one end of which projects a T-shaped rod carrying on its outer side a row of needle-like spikes. The quantity of electricity produced depends upon the extent of the two surfaces in contact and also upon the material of which these consist. The farther these are apart in the list of substances in Par. 23, the greater the electrical effect produced by rubbing them together. The material of the cylinder, glass, being near the top of the list, the rubber should be some substance near the bottom. The metals come near the bottom but their rigidity interferes with their use as rubbers. However, certain metals dissolve readily in mercury producing a more or less pasty amalgam which alone or mixed with grease may be smeared upon the rubber. Zinc, tin and the sulphide of tin are used in these amalgams.

The operation of the machine is as follows: The cylinder is rotated in a clockwise direction, the glass becomes positively electrified, the rubber negatively. As the positive charge on the surface of the glass comes around opposite the prime conductor, the points are said to collect it or take it off, but actually a negative charge is induced on the near end of this conductor, a positive charge on the far end, the negative charge escapes from the needle points in an electric wind, strikes against the cylinder and neutralizes the positive charge on its surface (Par. 43) and the conductor acquires an increasing positive charge.

If the rubber is insulated, a negative charge may be drawn from it but it is generally connected to the ground by means of a light chain or otherwise.

More modern forms use rotating glass plates instead of the cylinder but the principle of their operation is the same. It will be noted that although designated frictional machines, influence as well as friction is involved in the production of the

charge. They are very sensitive to hygroscopic moisture and frequently fail to work on account of atmospheric conditions, for which reason they are now superseded by the influence machines.

49. Toepler's Influence Machine.—This machine, as shown in its simplest form in Fig. 18, consists of two plates of glass mounted

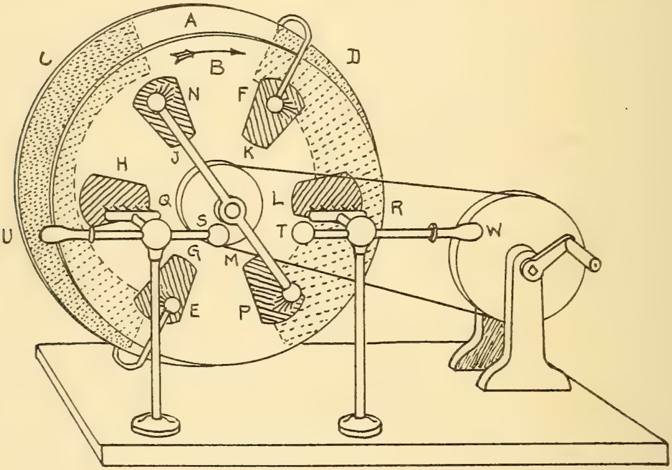


Fig. 18.

a short distance apart upon a common horizontal axis about which one may be rotated, the other one being fastened rigidly to the frame of the apparatus. The rotating plate is circular in form. Fig. 19 (in which for clearness the relative proportions and positions of the parts have been greatly distorted) represents an edgewise view of the glass plates, the eye of the observer being supposed to travel around the circumference while being continually directed towards the axis of the machine. The letters on these two figures correspond. *A* represents the fixed plate and *B* the moving one, the direction of motion being indicated by the arrow. On the outer surface of *A* and diametrically opposite to each other are the two *field plates* *C* and *D*. These are sheets of tin-foil glued to the glass, their thickness being greatly exaggerated in Fig. 19. Extending from each of these field plates there is a conductor which passes around the outer edge of the two glass plates to the *appropriating brushes* *E* and *F* on the outer side

of the revolving plate. These brushes are of fine brass wire like a paint brush and sweep along the face of the plate *B* as it revolves. On the outer surface of *B* there are glued six *carriers, G, H, J, K, L, M,* likewise of tin-foil. Outside of these and opposite the farther edge of the field plates are the *neutralizing brushes, N* and *P,* connected to each other by a conductor. Midway between the appropriating and the neutralizing brushes are the two *combs, Q* and *R,* which connect to the two *discharging knobs, S* and *T.* These knobs are on the ends of rods which by means of the glass handles *U* and *W* may be slid in or out thus adjusting the distance between the knobs. The operation of the machine is as follows: From an excited glass rod *Z* a small initial charge is imparted to the plate *C.* This induces a negative charge on the inner side of the carrier *H* and a positive charge on the outer side. As the plate *B* rotates *H* moves to the position *J* where it is touched by the neutralizing brush *N* which allows its free positive charge to escape, as shown by the small arrow, and leaves it with a negative charge. Upon reaching the position *K* the greater part of this negative charge, being no longer bound, is drawn off by the appropriating brush *F* and conveyed to the field plate *D.* When the carrier reaches the position *L* the negative charge on *D* induces a positive charge on the inner surface and a negative charge on the outer. In the position *M* the carrier is touched by the neutralizing brush *P* and the free negative charge is neutral-

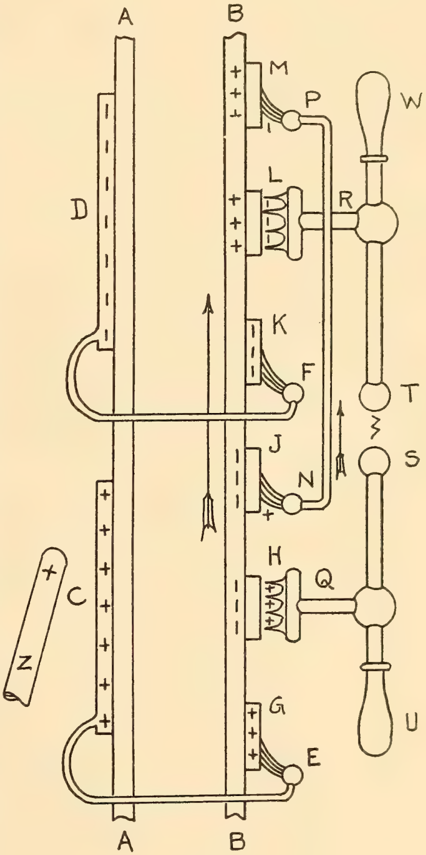


Fig. 19.

ized by the positive charge coming from N, M being left with a positive charge. The carrier next reaches G, is touched by the appropriating brush E and gives up the greater part of its charge to the field plate C. C now has a greater positive charge than in the beginning and its inductive action upon H is greater. In this manner as the carriers rotate they add to the charges on the field plates. This does not continue indefinitely. The field plate C being much larger than the carrier G has a much greater *capacity*. This property is defined later but for the present we may say (in a figurative sense) that C requires more electricity to fill it up than does G but once that it is filled up no more will flow into it from G. However, induction continues to act and the unappropriated charges on the carriers are now taken off by the combs Q and R as was explained in the description of the frictional machine, and it is this surplus electricity which we draw from the machine. In this machine, as in the frictional machine, it will be noted that two kinds of electricity are involved in the production of the charge, the initial charge being produced by frictional electricity.

50. Holtz's Influence Machine.—In construction this is a much simpler machine than Toepler's. It consists (Fig. 20) of two circular glass plates face to face, one fixed, the other rotating, two field plates and two combs with adjustable discharging knobs.

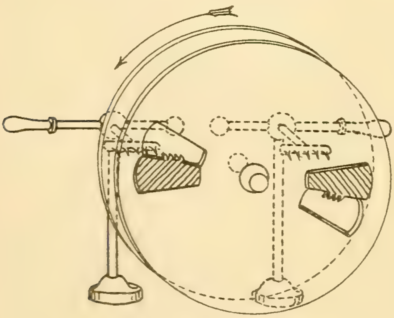


Fig. 20.

At the opposite extremities of a diameter of the fixed plate, window-like openings are cut and on the corresponding side of each of these openings are pasted the paper field plates. Fig. 21 represents an edgewise view of the machine. B is the rotating plate, the direction of its motion being indicated by the arrow. A is the fixed plate with the windows and C and D are the paper field plates. Extending from the field plates over the edge of the openings are either tongue-like strips as shown or else a series of sharp metal points. The operation of the machine in detail is as follows:

The discharging knobs G and H are placed in contact.

The field plate C is given a small initial charge, say positive. This induces a negative charge on F and repels a positive charge to E.

An electric wind escapes from F upon B and, as explained in Par. 43, charges the surface of B negatively.

The positive charge on E induces a negative charge in D. A positive electric wind escapes from E upon B and neutralizes the negative charge brought along the surface from F.

A positive wind escapes from the point of D and charges the inner surface of B positively. As this positive charge approaches C a negative wind escapes from C and neutralizes it. The escape of this negative electricity from C leaves C more highly charged positively and C exerts more induction upon F.

This, as explained above, makes D more highly charged negatively and so on, the "building up" continuing as the plate B rotates.

Finally, when the discharging knobs are separated, a large positive charge is induced in E and a corresponding negative one in its knob G, while a large negative charge is induced in F and a corresponding positive one in H and the attraction between the two in G and H is sufficient to drive sparks across the gap between the knobs, this gap being much shorter than the distance between C and D.

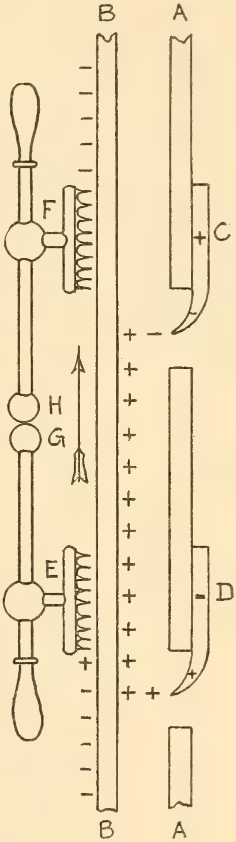


Fig. 21.

Influence machines are sensitive to atmospheric moisture but not to the same extent as the frictional machines, one reason being that the glass plates of the influence machines may be coated with varnish which in a measure prevents the deposition of moisture while in the frictional machines the plates must be kept free.

Those influence machines which employ appropriating brushes are self-exciting, that is, the slight friction of these brushes is enough to start the machine in operation when the plate is re-

volved, but the machines of the Holtz type must be given an initial charge.

51. Electrical Diagrams.—The illustrations (Figs. 19 and 21) in the preceding paragraphs are examples of a class of figures termed *diagrammatic* which are largely used in the study of electricity. In these the main object is to bring out clearly the essential arrangements, connections and principles and to this end, when necessary, details are omitted, the rules of perspective are ignored, proportions are distorted and relative positions changed. Conventional signs are frequently used, a simple character standing for a piece of apparatus like a cell or for a complicated machine like a dynamo. Many examples will be noticed in the following pages.

CHAPTER 7.

LAWS OF ELECTRIC ATTRACTION AND REPULSION.

52. Coulomb's Torsion Balance.—At various points in the preceding pages it has been shown that charges differ from one another in quantity and that the force of electric attraction and of repulsion varies both with the quantity of the charges and with the distance between the charged bodies. In the present chapter

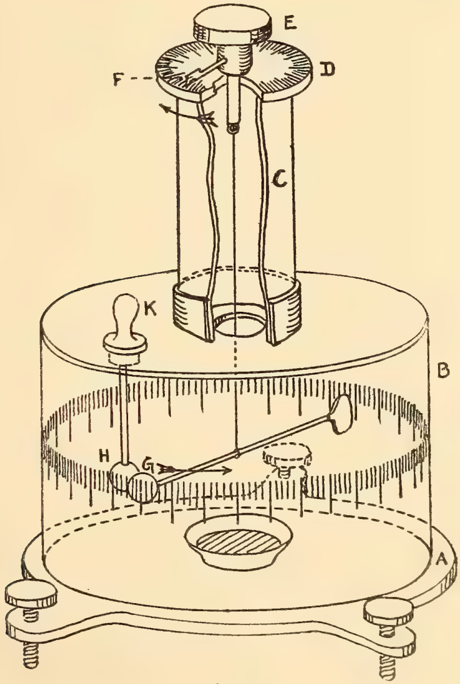


Fig. 22.

we shall see what are the laws governing this attraction and repulsion and also how and by what units charges may be measured.

The first exact experimental determinations of the laws of electrical attraction and repulsion were made by Coulomb with

an instrument called by him the *torsion balance.* This is shown in Fig. 22 and consists of a vertical glass cylinder B graduated in degrees around a belt a little below its middle and covered with a top which is pierced with two circular openings, one in the center and a smaller one near the edge. Around the central opening stands a second and smaller vertical glass cylinder C (represented in the figure as being partly cut away). This smaller cylinder carries on its top a metal cap D graduated around its edge in degrees and pierced in its center with a small hole in which fits a metal spindle which may be turned by means of the milled head E. Projecting from the shoulder below the milled head is the pointer F which travels over the graduated edge of the cap and thus indicates the number of degrees through which the spindle has been turned. Hanging from the spindle is a delicate silver wire to the lower end of which there is attached so as to swing in the plane of the graduations a needle of shellac. At one end of this there is a gilded pith ball G, about four-tenths of an inch in diameter, and at the other end a sufficient counterweight to hold the needle horizontal. In the second opening in the cover of the larger cylinder there fits a handled stopper K from which extends downward a needle of shellac, or of paraffine-coated glass, terminating in a second gilded pith ball H of the exact size of the first. The centers of the two balls lie in the same horizontal plane. Finally, the instrument stands upon a bed plate A furnished with levelling screws by means of which the silver wire can be brought to coincide with the axis of the larger cylinder.

The operation of the instrument is as follows: It is first carefully levelled and then the milled head E is turned until the ball G is just tangent to the ball H. In this position the plane through the suspending silver wire and the center of the ball G passes through the zero of the graduated scale on the larger cylinder. K is now removed, a charge is imparted to H and K is then reinserted. As H touches G the charge on H is distributed between the two balls. Having similar charges H and G repel each other and G (in the case represented in the figure) swings off to the right and as it does so twists the suspending silver wire. Now there is a definite law that when a body such as a wire is twisted by a force, its elastic limit not being exceeded, the resistance offered to the twisting, or the tendency to untwist, increases directly with the angle through which it is twisted and consequently the angle

through which it is twisted is directly proportional to the force exerted. The force which will twist a wire through ten degrees is exactly double that which will twist it through five degrees. As G moves to the right the resistance of the wire to twisting increases and as the distance between G and H increases the repelling force grows weaker until finally a position of equilibrium is reached, G comes to rest, and the angle through which it has turned can be read from the scale on the surface of the cylinder.

53. The Law of Inverse Squares.—By means of the torsion balance Coulomb demonstrated that electric attraction and repulsion followed the law of inverse squares, or that *the force exerted between two charged bodies varies inversely as the square of the distance between these bodies.* Two charged bodies which at a certain distance repel each other with a certain force will repel each other with only one-fourth of this force if the distance be doubled, or one-ninth if it be trebled, etc. His experiment was conducted as follows: The balls H and G (Fig. 22) were charged as explained in the preceding paragraph and let us suppose that the movable ball G was repelled until it swung through an angle of 16 degrees. By turning the milled head E in the direction shown by the arrow an additional twist was put upon the silver wire and the ball G was gradually forced back towards H. When G had thus been twisted back to within 8 degrees of H it was found that the pointer F of the milled head had travelled over 56 degrees of the scale on the cap D. The total angular torsion on the wire was consequently $8+56=64$ degrees. The force exerted in the two cases was, therefore, as 64 is to 16, which is the same as four to one. For small angles the chords bear to each other practically the same ratio as their arcs, hence at sixteen degrees the balls were twice as far apart as at eight degrees, or as the distance between the balls was divided by two the force between them was multiplied by 2×2 and this conforms to the law of inverse squares. These results are confirmed by experiments based upon other methods.

In the foregoing illustration the figures were selected to fit the demonstration but to obtain such accurate results in practice requires very careful manipulation and the observance of many precautions. The most troublesome source of error is loss of a portion of the charge during the progress of the experiment. The shellac needles to which the balls are fastened are non-conductors when free from hygroscopic moisture but a film soon deposits

upon them from the air and leakage of charge results. To remedy this there is placed in the instrument a small saucer containing quicklime or calcium chloride or sulphuric acid which substances have a great affinity for water and thoroughly dry the air inside of the cylinder.

A similar experimental demonstration can be made in the case of the attraction between unlike charges but the manipulation is much more difficult. The two balls must separately be given charges of the opposite kind, they attract each other and a condition of unstable equilibrium exists. Should they touch, their charges are neutralized and the process must be rebegun.

From the foregoing it will be seen that electric attraction and repulsion follow the law of central forces. In order that the law of inverse squares should be strictly true, the charged bodies must be small spheres, so small as to approximate points, and should be at such distance apart that in comparison with this distance their own dimensions are negligible. To other bodies the law does not apply. The force between two charged flat discs near together does not vary with small variations in the distance.

54. Variation of Force with Charges.—*The force exerted between two charged bodies varies as the product of the charges.* Reflection will show the truth of this second law. If two charged bodies repel or attract each other and the charge of either one be doubled or trebled, the repulsion or attraction must likewise be doubled or trebled. If the charge of the second one be now doubled or trebled, the existing force will be doubled or trebled, that is, the original force will be multiplied by four or six or nine. This law may be demonstrated by the torsion balance. It will be remembered that the two balls G and H (Fig. 22) are of exactly the same size, therefore, no matter what charge we start with, as soon as the balls have touched they (in accordance with the principle stated in Par. 45) divide the charge equally and we have two similar and equal charges. We may determine the angular repulsion between these, then withdraw the fixed ball H, touch it to a third and equal ball thereby halving its charge, return H to the cylinder, determine the new angular repulsion and hence the variation in the repulsion with the variation in the charge.

55. Variation of Force with Intervening Medium.—Those non-conducting substances which surround charged bodies and through

which electric effects are transmitted were termed by Faraday "*dielectrics.*" The force of attraction or of repulsion between charged bodies varies with the nature of the dielectric. Thus two small similarly-charged spheres which at a certain distance apart in air repel each other with a force of so many dynes will, if kept at the same distance and immersed in oil, repel each other with a force only one-half as great, or, if separated by an equal thickness of mica will repel each other with a force only one-sixth as great. The force between two charged bodies in air is not varied by compressing or by rarifying the air and for this reason and on account of the absence of any absolute measure we use air as our standard. The ratio of the force exerted between two charged bodies in a certain dielectric to the force exerted between the same bodies with the same charges at the same distance apart in air may be called the *dielectric coefficient of repulsion* and is the coefficient by which the force exerted between two charged bodies in air would be multiplied in order to obtain the force between the same two bodies under the same conditions in the medium to which the coefficient pertained. For oil this would therefore be 1/2, for mica 1/6, etc.

For gases and liquids this coefficient might be determined by the use of Coulomb's balance as explained above but it is obvious that this method could not be applied to solids. However, we shall see later on (Par. 91) how it may be otherwise determined and at the same time it will be shown why it is written in the form of a fraction or as $1/k$.

In problems involving forces exerted between charged bodies in other media than air, the appropriate value of $1/k$ should be used and when in discussions in the following pages this coefficient does not appear it is to be understood that the dielectric is air.

56. Unit Quantity of Electricity.—Representing by f the force of attraction or of repulsion, by q and q' the two charges and by d their distance apart we may combine the three laws discussed above and express them mathematically thus

$$f = \frac{1}{k} \cdot \frac{q \times q'}{d^2}$$

Since, as was explained in Par. 10, electricians have agreed to follow the C. G. S. system of units, f in this expression must be measured in dynes and d in centimeters. In the torsion balance

where the two gilded pith balls were of equal size, q and q' are equal, and since the dielectric is air, $1/k=1$, hence the above expression becomes

$$f=\frac{q^2}{d^2}$$

If we assume f to be one dyne and d to be one centimeter we obtain $q=1$, whence follows at once the definition: *An electrostatic unit of electricity is that quantity which when placed at a centimeter's distance in air from a similar and equal quantity repels it with a force of one dyne.*

The expression "in air" is essential to this definition as is also the term "electrostatic" for, as we shall see later (Par. 228), there is another and different unit of quantity, *the coulomb,* which is based upon current relations. The coulomb is three billion (3×10^9) times as large as the electrostatic unit.

CHAPTER 8.

ELECTRIC FIELD.

57. Electric Field.—We have seen that a charged body attracts non-electrified bodies and others with opposite charge and repels those with similar charge, therefore, in the space around an electrified body all bodies experience a force either of attraction or of repulsion and this space is called the *field* of the charge. As we recede from the charged body the force falls off rapidly and to fix its limits with more definiteness we define *the electric field as that space surrounding a charged body in which the force of attraction or of repulsion is perceptible.* If there be more than one charged body involved each produces a certain effect and they have a resultant field. The medium within the limits of a field is not passive or inert but takes part in the transmission of the electrical effects and is subjected to certain mechanical strains. Between two oppositely charged bodies there is a tension as if they were being pulled together by invisible rubber bands and at the same time a stress at right angles to the bands pushing the bands apart.

58. Intensity of Field.—It may aid the beginner in his conception if he consider a field as analogous to a current of water. In the electric field there is no matter in actual movement but in a sense there is a flow of force and light charged bodies, such as pith balls, if their charges are all of one kind, will be swept along in one direction just as corks are carried by a river. In order that a charged body be acted upon it must be in the field, just as the corks to be carried along must be in the stream. Finally, as we may measure the strength of a stream by the push it exerts upon a board of unit area inserted in it, so we measure the strength or intensity of a field by the push it exerts upon a unit charge placed in it. We therefore define *a unit field as that field which acts with a force of one dyne upon a unit charge placed in it.* It follows from the deduction of Par. 56, that the field produced at a distance d in air from a charge q must be q/d^2. In any other medium than air the field must be q/kd^2. If we say that a field has a strength of

three we mean that it will pull or push such a unit charge with a force of three dynes. If the charge itself be not unity, the force with which it is acted upon is equal to the product of the charge by the strength of the field.

59. Direction of Field.—Suppose that we have a horizontal sheet of glass in whose center there is a charged metal sphere (Fig. 23).

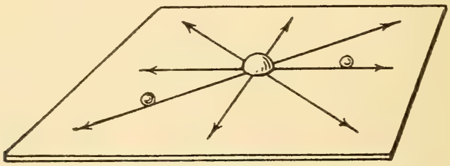


Fig. 23.

If a small pith ball be released upon the glass anywhere near the sphere, it will first be attracted to the sphere, will become charged and will then be shot away in a radial line. The force acts along these lines and they therefore indicate the direction of the field. Since opposite charges would move in opposite directions we, by convention, define *the positive direction of a field as that direction in which a free positive charge would move.*

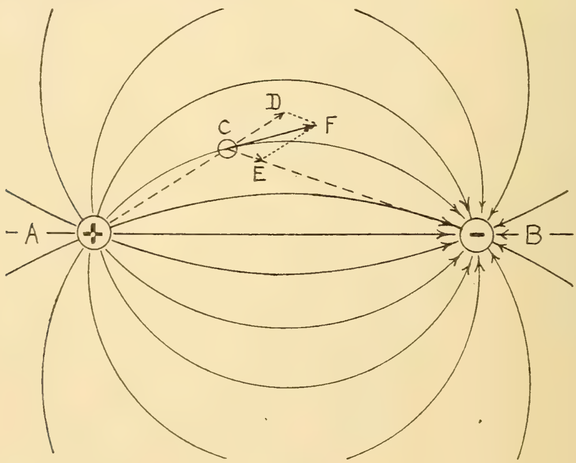


Fig. 24.

If we continue this experiment, substituting for the single sphere two placed some distance apart and charged, one positively, the other negatively (Fig. 24), the pith ball will no longer follow

rectilinear paths but curves emerging from one sphere and entering the other. These curves indicate the direction of the resultant field at the successive points through which they pass. A positively-charged ball at C is repelled by A along the line CD and attracted by B along the line CE. A being the nearer, the force CD is greater than CE and the ball moves along the resultant CF which indicates therefore the direction of the field at the point C. The space about the two spheres may be regarded as permeated with similar lines symmetrically distributed around the line joining the two centers.

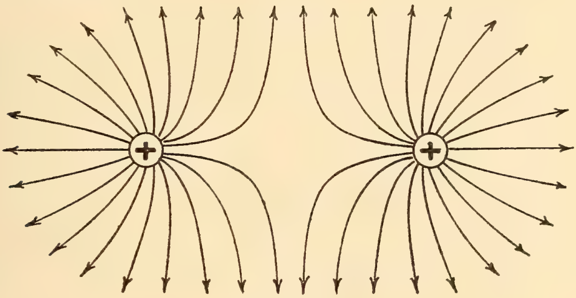


Fig. 25.

Had the two spheres contained like charges, the paths would have been as represented in Fig. 25.

60. Lines of Force.—These lines indicating the direction of the resultant electric force at the points in the field through which they pass are called *lines of force.* They start from a positively-charged surface and terminate upon a negatively-charged surface. They therefore have opposite charges at their two ends and never extend between bodies with like charges. They never penetrate below the surface or pass through a conductor. They are always perpendicular to the terminal surfaces at the points of origin and termination, otherwise there would be a component parallel to the surface and a movement of electricity along this surface would result. They never intersect, for in that case two tangents could be drawn at one point, that is, there could be two resultants at one point which is an absurdity. It follows from the foregoing that every electric field consists of non-conductors and is bounded by conductors.

61. Graphic Representation of Intensity of Field.—In mechanics, in order to treat graphically, to discuss mathematically and

to interpret geometrically problems involving parallel forces distributed over a surface or among the particles of a mass we, by convention, represent the direction and the intensity of the forces by the direction and length respectively of a right line and for the entire system may substitute a single resultant whose length is the sum of the lengths of the separate components and whose point of application is the center of gravity of the surface or of the mass. In the case of electric fields however, the intensity varies from point to point and in general the lines of force are not parallel, therefore, instead of representing this intensity by the length of a resultant, we agree to represent it by the number of lines of force per unit of area, the area being taken perpendicular to the lines. By convention, therefore, *a unit field is that field which contains one line of force per square centimeter of cross-section.*

It is not meant by this convention that in moving about in a unit field the force is experienced at intervals of one centimeter only and that the intervening space is blank, any more than by representing the attraction of gravity upon a body by a single line we imply that there is no gravity in the adjacent space. In a similar manner we might consider beams of light as made up of a number of parallel lines or rays and might agree to measure the intensity of the beam by the number of rays per centimeter of cross-section. Two beams of light passing through circles of the same size may differ in intensity and therefore include a different number of rays, yet on a cross-section of each the illumination is uniformly and continuously distributed, so two fields may differ in intensity yet in each the force exists at every point.

In representing lines of force graphically the positive direction, or direction in which a free positive charge would move, should always be indicated by arrow-heads.

We conclude by saying that *lines of force are those imaginary lines in a field which by their direction indicate the direction of the resultant field and by their number indicate its intensity.*

62. Tubes of Force.—Another convention which avoids this apparent intermittent distribution of the lines of force and which is much used in mathematical discussions of electric fields is that of *tubes of force.* There are supposed to originate from the surface of a positively-charged body certain tubular surfaces various in cross-section and frequently curved but lying side by side like the

cells of a honeycomb and including within themselves all the space about the body. Their walls are parallel to the lines of force of the field and therefore at every cross-section of one of these tubes the same number of lines would be cut. They terminate upon a negatively-charged surface. If the portion of the surface of the charged body included in the base of the tube contains one unit of electricity, the tube is called a *unit tube*. It follows from this conception (and also from Par. 31) that the quantities of electricity upon the terminal surfaces of a tube are equal and opposite and a further consequence is that in the case of two parallel planes near together, one of which is charged, the tubes at a distance from the edges are parallel and the surface density upon the central portions of the two planes equal and of opposite signs. This principle is utilized in the attracted disc electrometer described later (Par. 101).

It is difficult to represent these tubes graphically and we generally do so by drawing a single line supposed to be the axis of the tube, so that after all we resort to lines of force.

63. Lines of Force from Unit Charge.—If in Fig. 26 A represents a unit charge and B a similar and equal charge at a distance of one centimeter from A, B will be repelled with a force of one dyne. A unit field is that field which acts with a force of one dyne upon a unit charge placed in it. A is surrounded by its own field and B is in it, therefore at B there is a unit field. The same is true for every point at a distance of one centimeter from A, that is, the surface of a sphere with A as a center and a radius of one centimeter is a unit field. From Par. 61 there is in a unit field one line of force per square centimeter of cross-section. The surface of this sphere is 4π square centimeters and since each contains one line of force, 4π lines of force radiate from a unit charge.

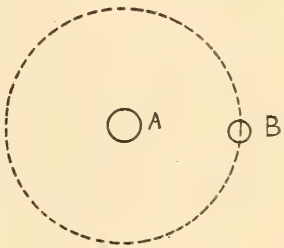


Fig. 26.

64. Gauss' Theorem.—If around one or more charged bodies a closed surface be drawn, the number of lines of force which pierce this surface is equal to 4π times the total charge included inside the surface. This follows at once from the preceding paragraph. Each unit charge has 4π lines of force radiating from it,

therefore from a charge q there would radiate $4\pi q$ lines. This is one way of expressing Gauss' Theorem, a principle of frequent employment in mathematical discussions of electrostatic problems. An example of the application of this theorem is given in the following paragraph.

65. Field about a Uniformly-Charged Sphere.—Let O (Fig. 27) be the center of a uniformly-charged sphere, its surface density being δ, and let P be an external point at a distance D from this center. Through P pass a sphere with O as a center. If the charge on the original sphere be q, then according to Gauss' Theorem $4\pi q$ lines of force pierce the sphere P. The area of the sphere P is $4\pi D^2$ and the distribution of the lines of force is uniform, therefore the number of lines per square centimeter is $4\pi q/4\pi D^2$, or q/D^2. But (Par. 61) this measures the intensity of the field at P or, in other words, measures the force with which a unit charge at P is acted upon, whence we see that *the charge upon a uniformly-charged sphere acts upon external points as if it were concentrated at the center.*

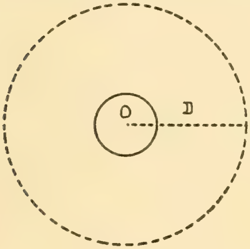


Fig. 27.

If the external point be indefinitely near the surface of the sphere the force exerted will be q/R^2. Substituting for q its value $4\pi R^2\delta$, this becomes $4\pi\delta$, that is, the field very near the surface of a charged sphere is equal to 4π times the surface density of the charge. Coulomb extended this theorem to include charged bodies of any shape.

66. Field near a Uniformly-Charged Plane.—Let AB (Fig. 28) be a uniformly-charged plane, its surface density being δ; to find the force exerted upon a unit positive charge at P at a distance D from the plane. Let PC be the perpendicular from the point to the plane. With C as a center and radii y and $y + dy$ describe a zone. The area of the zone is $2\pi y.dy$. The

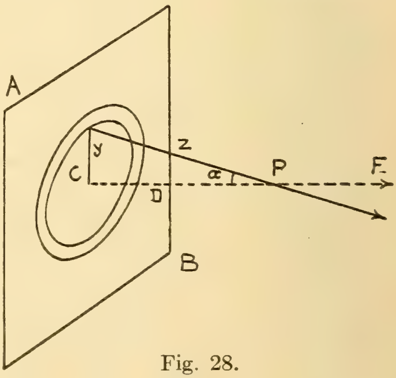


Fig. 28.

charge upon this zone is $2\pi y.dy.\delta$. The force exerted at P by this charge is

$$\frac{2\pi y \,.\, \delta}{z^2} \,.\, dy$$

The normal component in the direction PF is

$$\frac{2\pi \,.\, \delta \,.\, y \,.\, \cos\alpha}{z^2} \,.\, dy$$

The integral of the components from all the zones will give the total force. To prepare the above expression for integration $\cos\alpha$ and z^2 must be expressed in terms of y.

From the figure $z^2 = D^2 + y^2$ and $\cos\alpha = \dfrac{D}{z} = \dfrac{D}{\sqrt{D^2 + y^2}}$

Hence $$df = \frac{2\pi D \,.\, \delta \,.\, y}{\sqrt{(D^2 + y^2)^3}} \,.\, dy$$

And integrating $$f = -\frac{2\pi \,.\, D \,.\, \delta}{\sqrt{D^2 + y^2}} + C$$

Taking this between the limits $y = \infty$ and $y = 0$

$$f = 2\pi\delta \text{ dynes}$$

In this expression D does not appear, so that the force is independent of the position of P with respect to the plane. If the charged plane be not of indefinite extent, the expression is still approximately correct if P be so near the plane that the dimensions of the plane as compared to this distance are very great.

67. Force Exerted upon an Internal Point by a Uniformly-Charged Sphere.—Consider a uniformly-charged insulated sphere remote from other bodies. Let P (Fig. 29) be any point within such a sphere and let AB be any line drawn through this point. Let the tangents at A and B represent the traces of the tangent planes at those points. The line AB makes equal angles with these planes. With P as a vertex describe about PB as an axis a slender cone EPF. Prolong its elements beyond P thus describing a second cone GPH. Suppose the bases of these cones to be charged, the surface density being the same as that of the sphere. They are similar since they

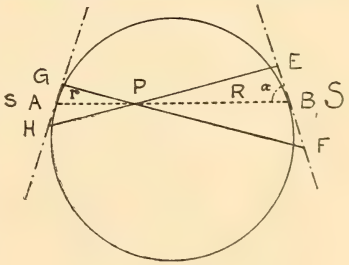


Fig. 29.

have equal solid angles at the vertices and their axes make equal angles α with their bases. Let $PB=R$, $PA=r$, the area of the base $EF=S$, that of $GH=s$, the charge on $EF=Q$, that on $GH=q$. The force exerted by Q upon a unit charge at P is $Q \cdot \sin\alpha/R^2$, that exerted by q is $q \cdot \sin\alpha/r^2$. The charges on the bases of these cones being of the same surface density

$$Q : q = S : s$$

hence the above expressions are proportional to S/R^2 and s/r^2, respectively. The cones being similar

$$S : s = R^2 : r^2$$

whence $S/R^2 = s/r^2$, or the forces exerted upon P are equal and opposite.

As the cones are made smaller their bases approach coincidence with the surface of the sphere. The whole surface of the sphere can be thus divided up by pairs of cones, the effects of the charges upon whose bases exactly neutralize one another, therefore, *the charge upon the surface exerts no force at an internal point.*

This is true whatever the shape of the conductor or surface distribution of the charge but in only a few cases can the conditions be given a sufficiently simple mathematical expression to permit of ready proof.

This fact was shown experimentally by Faraday. He constructed of tin-foil and wire-netting an insulated cubical chamber into which he entered with his most delicate electroscopes. The chamber was then charged so highly that great sparks and brush discharges were escaping from the corners, yet his instruments gave no indications at all.

68. **The Charge Resides on the Surface.**—The proof of the statement in Par. 38 that the charge resides on the surface of an insulated conductor follows from the foregoing. Suppose that we might have an insulated sphere with a charge distributed uniformly throughout its substance. This charge will induce on surrounding objects a charge of the opposite kind. The attraction between these charges will cause the charge in the sphere to move out to the surface. The portions of the charge upon the surface mutually repel each other and thus spread over the entire exterior. No part of the charge could be crowded off into the interior of the sphere for we have just seen that the charge on the surface exerts no force in the interior.

CHAPTER 9.

POTENTIAL.

69. Cause of Movement of Electric Charges.—If a charged conductor be connected to the earth it will be instantly discharged. If two equal insulated spheres containing unequal charges be brought into contact there will be a flow from the greater charge to the lesser until the two charges are equalized and equilibrium established. If the spheres be of unequal size yet contain equal charges a portion of the charge of the smaller sphere will flow to the larger. Finally, if these unequal spheres have charges of the same surface density a portion of the charge of the larger will flow to the smaller. The movement is therefore not entirely determined by difference in the size of the conductors or by inequality either of the charges or of surface density and we naturally ask why does it take place. It is produced by what is designated *a difference of electric potential.*

70. Physical Analogues of Electric Potential.—It has been remarked that one of the reasons why the study of electricity is difficult for the beginner is that although we have a sense of weight, of force, of direction, of velocity, etc., we are devoid of an electric sense and therefore such expressions as intensity of current, quantity of electricity, electric pressure, electric potential, etc., are pure abstractions. To convey a physical conception of these and to aid in our explanations we are compelled to resort to analogies. In explaining electric potential it is frequently compared to temperature and to water level.

Making the first comparison, it is not size or shape of the bodies or quantity of heat contained but difference of temperature which determines whether heat shall pass from one body to another, the flow taking place from the body whose temperature is the higher. Thus a red-hot nail loses heat when dipped into a bucket of hot water, although the water may contain several hundred times more units of heat than the nail; and no matter how they differ in size there is no net transfer of heat between two bodies at the same

temperature. So with electricity, there is always a flow when conductors at different potentials are brought into contact, the flow (of positive electricity) taking place from the conductor of higher potential, and there is no flow if their potentials are the same.

Again, if two vessels containing water (Fig. 30) are connected by a pipe there will be a flow from the vessel in which the water stands at the higher level and this is irrespective of the actual amounts of water in the two. There will be no flow if the level in the two is the same.

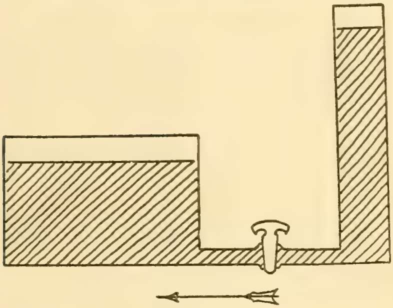


Fig. 30.

These analogies can not be carried too far. For example, it will be noted that a change in temperature is accompanied by a change in volume and often by a change in state, but conductors show no such changes when their charges are varied.

71. Mechanical Potential.—Consider a cord (Fig. 31) attached to a weight W and running over a pulley. By the expenditure of a certain amount of work on the free end of the cord the weight can be raised against the force of gravity through a vertical distance to a new position W'. In this new position it has a certain amount of stored up energy, or ability to do work, or potentiality, for if the free end of the cord be released the weight will drop back to W and in doing so will, if proper mechanical arrangements be made and losses by friction be not considered, do as many foot-pounds of work as were expended originally in raising it to the position W'.

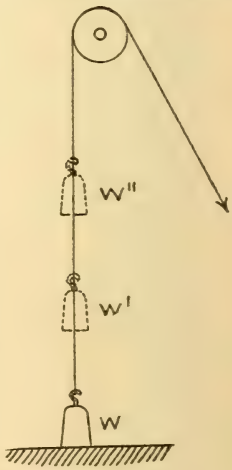


Fig. 31.

To raise it to W'', higher than W', would require a greater expenditure of energy but also its potential energy at W'' would be correspondingly greater than that at W'. We see then that its potential energy, or for brevity its *potential*, varies with its posi-

tion with respect to the surface of the earth (more strictly with respect to the center of gravity of the earth) and at every different level we reach it has a corresponding different potential, always exactly measured by the amount of work expended in moving the weight from the surface of the earth to that level.

Potential as thus explained is not an inherent property of the weight for in its various changes of position the weight in itself does not change. It sometimes becomes desirable to compare the potential which a body has at one point with that which it would have at another point and we therefore speak of the potential of this body *at* the point, or simply of the *potential of the point,* but points in space have no potential and we mean the potential which the body when moved to the point acquires due to the work expended in the movement.

In ordinary mechanical problems the force of gravity at any one spot is considered constant and the potential of a body varies directly with the vertical distance through which it is raised, therefore it suffices to give the height and this height in feet multiplied by the weight of the body in pounds gives the foot-pounds by which the potential of the body is measured. However, should we take this force as following the law of inverse squares, the amount of work done in raising a body one foot from a certain level would differ from the amount done in raising this same body one foot from some other level. The relative potentials of the two points would not in this case be given directly by their heights but by the work expended in raising the same weight to the respective points. Logically therefore we would compare the potentials of points by comparing the work expended in moving a unit weight against the force of gravity and from the surface of the earth to the respective points.

Theoretically, since it is neither raised nor lowered, no work is expended in moving a body about on a level. Every point on such a surface has the same potential and it could therefore be called *an equipotential surface.* A unit difference of potential exists between two levels when a unit of work must be expended in moving a unit weight from one to the other.

72. Electric Potential.—We arrive at a definite conception of electric potential by a course of reasoning parallel to the foregoing. Suppose A (Fig. 32) to be a positively-charged insulated sphere and B a small sphere with a unit positive charge. B will be

repelled by A, the force varying inversely as the square of the distance. At an infinite distance the force would be zero; at a great distance it would be very small but as the distance be-

Fig. 32.

comes small it would increase rapidly. Should we begin with B at an infinite distance and push it up towards A the work done at first would be very, very small but would increase as we approached A and at any point as P the potential would be exactly measured by the work expended in bringing B up to that point. We therefore say that *the electric potential at any point is measured by the amount of work that must be spent in bringing up to that point from an infinite distance a unit of positive electricity.* Since we use the C. G. S. system, electric potential as thus explained is measured in *ergs.*

Had the unit charge upon B been negative, its potential at P would have been negative and measured by the work expended in pushing it back to an infinite distance.

From the above it follows that *the difference of potential between any two points is measured by the work expended in moving a unit of positive electricity from one point to the other.* Hence also, a unit difference of potential exists between two surfaces when it requires the expenditure of one erg to move a unit positive charge from one to the other.

Parallel to the case of mechanical potential, a surface every point of which is at the same potential is an *equipotential surface.* Such a surface is that of any conductor in which no electricity is in movement.

73. Zero Potential.—Electricity not being matter, we recognize it and measure it and its dynamical properties only by its effects. If all bodies about us were at the same potential there could be no movement of electricity among them and hence, with the exception of mutual repulsion, there would be none of the manifestations which we use in measurements. Repulsion of like charges depends solely upon the quantity of the charges, their distance apart and the medium in which they are situated and would be the same no matter how high or how low their common potential, therefore,

there is no means of determining absolute potential but only relative potential, or, as it is usually expressed, "*difference of potential.*" Fortunately, there is no need of knowing the absolute potential, just as in utilizing water power it is not necessary to know the height above the sea but it is essential to know the difference of level. A point at an infinite distance from all charged bodies would be at zero potential but for convenience the potential of the earth is taken as an arbitrary zero. This no more means that the absolute potential of the earth is zero than that taking the melting point of ice as zero implies that a lower temperature does not exist or the taking of the sea level as zero means that we could not go to greater depths.

74. Potential at a Point due to a Charge.—If in Fig. 33 the charge at P be unity, that at A be Q and the distance between

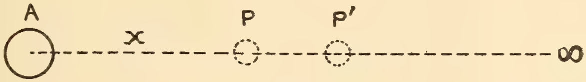


Fig. 33.

A and P be x centimeters, the force at P will be Q/x^2 dynes, the work performed by the unit charge in moving from P to P', a distance dx, will be

$$\frac{Q}{x^2} \cdot dx \text{ ergs}$$

and the total work performed in moving from P to an infinite distance will be

$$\int_{x=x}^{x=\infty} \frac{Q}{x^2} \cdot dx = \frac{Q}{x} \text{ ergs}$$

Hence the work expended in the opposite direction in moving the unit charge from infinity up to P will also be Q/x ergs. But from Par. 72, this measures the potential at the point P. Therefore, the potential at any point due to a charge is equal to the charge divided by the distance between the charge and the point.

An important corollary of the foregoing is that the potential at any point due to more than one charged body is equal to the sum of the potentials at that point due to the bodies taken separately.

75. Expression for Electric Force.—An expression for the electric force acting upon charged bodies may be deduced as follows: Work is equal to force × path, hence

$$\text{force} = \frac{\text{work}}{\text{path}}$$

The work performed in pushing a unit of positive electricity from one point to another is equal to the product of the electric force by the distance between the points. But from what we have seen above (Par. 72) this work measures the difference of potential between the two points, therefore the

$$\text{electric force} = \frac{\text{difference of potential}}{\text{distance between the points}}$$

This is correct only on the assumption that the force has been constant throughout the path but it is the exception when such is the case. However, the nearer we take the two points together the nearer we get to the true value of the force, hence, designating the difference of potential between the two points by V and the distance between them by x we have at the limit

$$\text{electric force} = \frac{dV}{dx}$$

or *the electric force at any point is equal to the rate of change at that point of potential per unit of length.*

76. Electro-Motive Force.—In the example of mechanical potential in Par. 71 above, if the cord be only partly paid out the weight will fall a corresponding distance, the tendency always being for the body to move from a point of high potential to one of lower. In the case of electricity there is a like tendency, and the insulation of a charged body may be regarded as analogous to the cord since it restrains the charge from flowing from the body to another of lower potential. If the charged body be connected to a body of lower potential it is analogous to paying out the cord, and if it be connected through a conductor to the earth the effect is analogous to cutting the cord.

In the illustration of electric potential, if the little sphere pushed up from an infinite distance and containing the unit positive charge be released it will be pushed back, the charge and the sphere both moving. If instead of releasing the sphere, it be connected, say through a conducting wire, with the earth, the charge alone will be pushed back along the wire to a point of zero poten-

tial. In this case no movement of matter is involved but only of the charge. If new charges be supplied to the little sphere as fast as the previous charges flow away, it will be kept at a constant potential and the successive charges following along the wire will constitute a continuous stream. This is what is known as *current electricity* and is discussed later.

Mechanical force is defined as that which moves or tends to move or tends to produce a change of motion in matter. In the case of the movement of electricity however no matter is involved. The first force might therefore be named "matter-motive force," the second in contra-distinction, is named *"electro-motive force,"* and can be defined as that force which moves or tends to move electricity. It is represented in symbols as *E. M. F.*

77. Practical Unit of Electro-Motive Force.—Reverting to our comparison of potential to water level, the flow of water is produced by a force and this force is the pressure due to the "head" or difference of level between the surface of the water and the outlet. So the flow of electricity is produced by the electro-motive force which in turn is caused by the difference of potential between the two ends of the path. The difference of level in the case of water is measured in feet, the corresponding pressure is measured in pounds per square inch, and for any given difference in level the pressure in pounds per square inch may be obtained by multiplying this difference expressed in feet by the factor .434. In the case of water the cause and effect are so closely connected that we often hear such expressions as "a pressure of 30 feet."

The practical unit of electric pressure or of electro-motive force is called the *volt* and will be defined later. Difference in potential expressed in ergs is, for reasons given later, converted into the corresponding electro-motive force in volts by multiplying by 300. Similar to the case of water it has become usual to confound cause and effect and it is customary to speak of a difference of potential of so many volts. Some writers even go to the extent of stating that difference of potential and electro-motive force are two names for one and the same thing. In view of this, insistence upon the distinction becomes academic and of no practical importance and hereafter will not be dwelt upon.

78. Summary.—The gist of the preceding discussion upon potential is that whenever a charge of electricity is produced, it

may be regarded as brought up from infinity or from a point of zero potential and whenever a difference of potential is developed, the charge must either have been pushed against a repulsion or pulled against an attraction. In either case, just like a spiral spring which has been compressed or extended, it has a tendency to fly back and can be retained in its position only by a continuation of the push or pull or by the interposition of an insulator. The more the mechanical or chemical energy expended in bringing up the charge, the greater its potential energy or the greater its tendency to fly back when released. The potential of the charge is measured by the work in ergs spent in bringing up a portion of it equal to one positive unit. The force with which the unit charge when released would be pushed back, or the electro-motive force, is measured in units called *volts* whose number is obtained by multiplying the ergs by 300.

CHAPTER 10.

ELECTROSTATIC CAPACITY.

79. Electrostatic Capacity.—At several points in the preceding pages reference has been made to *electric capacity*. The word "capacity" in its application to electricity is used in a sense quite different from its ordinary acceptance and necessarily so, as the following will show. The capacity of a vessel is the volume which it will contain and is fixed once for all. If by the capacity of a conductor we meant the amount of electricity which could be imparted to it, the term would be indefinite for as conditions vary the same conductor could contain very different amounts. Resorting to analogy, conductors can be compared to vertical

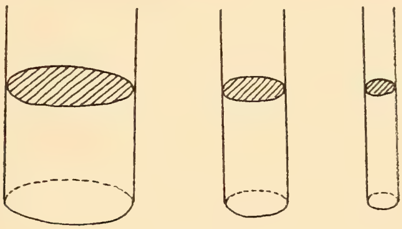


Fig. 34.

cylindrical vessels differing in cross-section and of indeterminate height (Fig. 34). The amount of water which could be placed in any of these vessels would depend upon the cross-section, upon the height to which the inflowing liquid could be raised by the supply pump and also upon the strength of the material, that is, the height to which the cylinder could be filled before the pressure caused the bottom or sides to yield. Hence, keeping the cross-section constant, the capacity might be varied by using a more powerful pump or a stronger material for the vessel. The only basis of comparison in terms of contents would therefore be the amount of water that would cause the pressure per square inch on the bottom to increase a definite amount, or, since this pressure varies directly as the head, the amount of water that would raise the level in the vessel a certain distance, say one foot. If one

gallon raises the level one foot in a certain cylinder and it requires two gallons to do the same in another, the second cylinder may be said to have twice the capacity of the first. The same amount of water will raise the level more quickly in a small cylinder than in a large one.

If various insulated conductors be connected to a charged body of higher potential, electricity will flow from the source into them until a common potential is reached. The small conductors will receive least for less is required to raise their potential to the common level. Upon reaching the common potential the flow will cease but should the potential of the source be increased the flow will again begin and continue until a new common potential is reached. This can be continued, the potential of the conductors steadily rising, but finally the strain on the medium surrounding the conductors becomes so great as to overcome its dielectric strength (see Par. 93), there is a breakdown and a discharge occurs. Hence the total quantity of electricity which can be transferred to a conductor, besides varying with the size of the conductor also depends upon the difference of potential between the conductor and the source of electricity and upon the dielectric strength of the surrounding medium and is therefore indefinite. On the other hand, *the capacity of a conductor is measured by the quantity of electricity which must be imparted to it to raise its potential one unit,* and is perfectly fixed and definite.

If a charge Q imparted to a body raises its potential V units, then a charge Q/V would raise its potential one unit. But, by the preceding definition, this is the measure of the capacity of the body, and representing the capacity by K, we have the relation between these three quantities given by the expression

$$K=\frac{Q}{V}$$

80. Capacity of a Sphere.—The capacity of most bodies must be determined by actual measurement but for a few of simple geometrical form it may be calculated. The capacity of a sphere may be determined as follows. In Par. 74 it was shown that the potential at a point due to a charge Q at a distance x from the point is Q/x. If the charge Q be upon the surface of a sphere it acts as if concentrated at the center of the sphere (Par. 65), and hence the distance between the charge and the point must be

measured from the center of the sphere. Therefore, the potential of a point infinitely near the surface of the sphere (that is, the potential of the sphere itself) is $V = Q/R$. In other words, the potential of a sphere varies directly as the charge and inversely as the radius. In the above expression if $V = 1$, Q must be equal to R, that is, to maintain unit potential as R varies, Q must vary in the same ratio and preserve numerical equality with R. We also see that *the capacity of a sphere varies directly as its radius.* This may be shown directly by substituting in the expression for capacity, $K = Q/V$, the above value $V = Q/R$, whence we obtain

$$K = R$$

or the number of units of electricity required to raise the potential of a sphere by unity is equal to the number of centimeters in the radius of the sphere. A unit charge would therefore raise by unity the potential of a sphere of one centimeter radius and such a sphere is said to have unit capacity.

Certain interesting consequences follow from the foregoing, two of which we shall now notice.

81. Case of Two United Spheres.—If two unequal spheres be placed in contact or be joined by a conductor and a charge be imparted to either they will come to a common potential, or will share the charge in proportion to their capacities, which, from the preceding paragraph, is also in proportion to their radii. Suppose the radius of A (Fig. 35) to be twice that of B, the charge upon A will be twice the charge upon B.

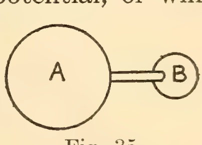


Fig. 35.

The surfaces of these spheres being to each other as the squares of their radii, the surface of A is four times that of B. The surface density of the charge on A is therefore as $2/4$, that of the charge on B is as $1/1$, or the surface density on B, although B has the smaller charge, is twice that on A. If B is very small as compared to A, its surface density will become very great and we have seen (Par. 41) that if the surface density exceeds 20 units per square centimeter a discharge will take place. This is the explanation of the action of points already described (Par. 42).

82. Case of Two Coalescing Spheres.—Suppose two equal charged spheres, A and B (Fig. 36), should coalesce producing a resultant sphere C. If the radius of A be r and that

of C be R, since the volume of a sphere $= \frac{4}{3}\pi r^3$ we have

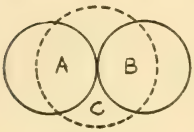


Fig. 36.

$$2 \times \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi R^3$$

Hence $R^3 = 2r^3$ or $R = \sqrt[3]{2} \,.\, r = 1.26\,r$.

Hence, since the capacity of a sphere varies directly with its radius, it will require 1.26 times as large a charge to raise the potential of the sphere C one unit as is required to raise that of A or of B one unit. But by the coalescing of the spheres C receives twice as great a charge as A or B, or .74 times more than necessary to bring it to the same potential, and hence its potential is greater than that of A or B.

It is known that evaporation is accompanied by the production of electricity, the vapor being charged. As the vapor begins to condense, the molecules unite into globules, these microscopic globules into larger ones and these into still larger ones until drops of rain result. By this coalescing the potential is enormously increased until a final point is reached when a disruptive discharge, a flash of lightning, takes place. This is an explanation which has been advanced to account for thunder storms.

83. **Condensers.**—In the discussion of capacity in Pars. 79 and 80 above, the conductors were supposed to be remote from all other bodies. Should the conductor to which the charge is given be near to a second, this last being connected to the earth, a very different state of affairs will result. A charge imparted to the first will repel from the second into the earth a similar and almost equal charge and induce and attract into it an opposite and almost equal charge. In Par. 74 it was stated that the potential at a given point due to more than one charged body is equal to the sum of the potentials at that point due to the bodies taken separately. The potential of the first body is therefore the sum of the potentials due to its own charge and to the induced charge and these being of opposite signs the resultant potential is much less. The potential being less, a greater charge is required to raise the potential of the first body a certain amount than was required when this body was remote from all others, in other words, its capacity is increased. We see then that the capacity of a conductor is increased by the proximity of another which is earth connected, and since a greater charge can now be given to it before a given change of potential is produced, such an arrangement is called a *condenser*. The earliest

form of a condenser was the *Leyden Jar* which we shall now consider.

84. Invention of the Leyden Jar.—The invention of the Leyden jar is in dispute, the merit having been claimed for three or more persons. Priestly, noted as the discoverer of oxygen, has left a contemporaneous account of the event which is in substance as follows: Dr. Muschenbroek of Leyden in experimenting with static electricity was much troubled by the rapidity with which his conductors lost their charge and ascribed this loss to some "effluvium" in the surrounding air. He therefore thought to protect his charged body by surrounding it by a non-conducting vessel which would shield it from the atmosphere. To test this, he poured some water into a glass jar and holding the jar in his left hand he led a charge into the water by a wire attached to the prime conductor of the crude machine he was using. After giving the handle of the machine a few turns he attempted to disengage with his right hand the wire from the prime conductor but as he touched it there was a flash and he was subjected to a strong convulsive shock. In a letter describing this experience he states that he felt himself struck in his arms, shoulders and breast so that he lost his breath and was two days before he recovered from the effects of the blow and the terror. He added that he would not take a second shock for the whole Kingdom of France.

This experiment was quickly repeated by other investigators. It was soon found that no appreciable charge could be given to the jar unless it were held in the hand and that the amount of the charge varied with the amount of the surface touched by the hand. This led to the substitution of a metallic outer covering. It was next discovered that the charge did not increase in proportion to the amount of water in the jar but rather in proportion to the area of the surface wetted and this led to the substitution of a lining of tin-foil. Finally, it was found that, other conditions being the same, the thinner the jar the greater the charge that it could be given.

85. The Leyden Jar.—The usual form consists of a wide-mouthed glass jar (Fig. 37) coated inside and out for about two-thirds of its height with tin-foil. It is closed with a stopper of insulating material through which passes a brass rod terminating above in a knob and below in a small chain which dangles long enough to touch the tin-foil lining.

To charge the jar, the outer coating must be connected to earth either by being placed upon a wire or chain, one end of which is grounded, or by being held in the hand and afforded a path through the body. The knob is then held to the prime conductor of a machine in operation and in a very short while the jar is charged.

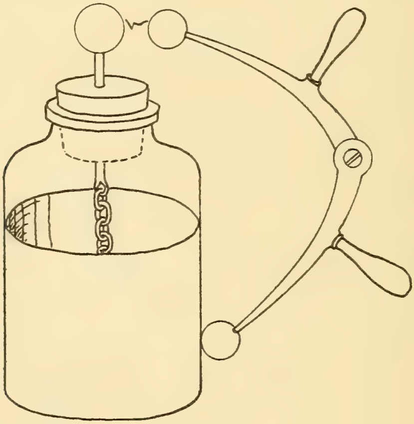


Fig. 37.

The inner lining receives the same kind of charge as is generated by the machine; the outer coating receives a charge of the opposite kind. It can not be charged indefinitely. As we continue to turn the handle of the machine, a point will be reached when the tension between the two opposite charges becomes so great that either the glass of the jar will be pierced or else a discharge will occur by the charge creeping up the surface of the glass to the mouth of the jar and thence down to the outer coating.

A jar once charged will remain so for some time. The inner and outer charges are mutually bound and can not be removed by touching the inner or the outer coatings separately, but if they be touched simultaneously by any body which will afford a path between the two charges, the jar is instantly discharged. Since the effect of the discharge through the body is disagreeable and may be dangerous, use is made of a *discharger*, a knobbed conductor, hinged at the middle like a pair of tongs and furnished with glass handles. It is held by the handles while, as shown in Fig. 37, one knob is touched to the knob of the jar, the other to the outer coating.

86. Explanation of Leyden Jar.—Reflection and experiment will show that the jar form of this apparatus is unimportant and that the essential parts are two sheets of conducting material separated by a thin non-conducting sheet. A window-pane set on edge with a sheet of tin-foil pasted in the center on each side is as efficient as a jar of equal area of glass and foil. Such an arrangement was called by Franklin a "fulminating pane." If more rigid metal sheets be substituted for the tin-foil and if they be mounted upon an insulating support, the glass may be replaced by a thin layer of air and the apparatus is then called an *air condenser*.

The arrangement shown in Fig. 38 enables us to examine the action of a condenser under various conditions. *A* and *B* are

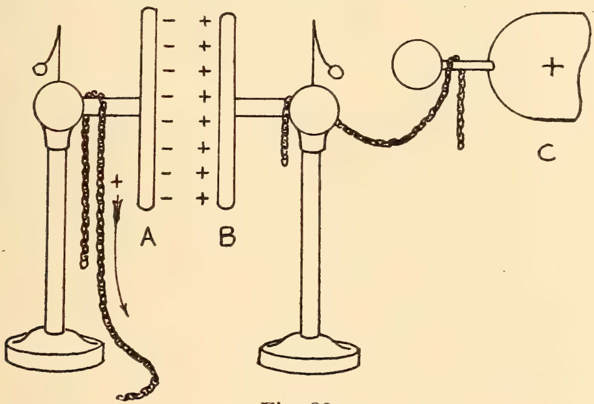


Fig. 38.

vertical metallic plates mounted upon insulating stands by which the distance between them may be varied. *A* is connected by a chain or wire to the earth and *B* is connected to the prime conductor *C* of a machine which we shall suppose is positively charged. At first let *A* be remote from *B*. *C* being at a higher potential than *B*, a charge will flow into *B* and *B* and *C* will reach a common potential. If now *A* be moved up near *B*, the charge on *B* will induce a negative charge on *A* and repel a positive charge into the earth. The potential of *B* is the sum of that due to its own charge and that due to the charge on *A*. This last being negative, the potential of *B* is lowered and more charge will flow into *B* from *C* until *B* and *C* are again brought to a common potential. Each additional quantity that flows into *B* from *C* will induce a corre-

sponding quantity of negative electricity in A, the joint effect of the two being to reduce the potential to which B, if remote from A, would be raised and thus a much greater charge can be given to B than would otherwise have been possible.

If the chains be now disconnected from A and from B and if A and B be drawn apart, the pith balls attached to the supports will be repelled more strongly, as if A and B had received greater charges. This may be explained either by the fact that as the distance between A and B increases, the two charges are not so strongly bound mutually and tend to spread, or that the effect of the negative charge on A upon the potential of B becomes less and the potential of B increases.

If A and B be pushed closer together the pith balls will again drop down. The conclusion is that other things being equal the capacity of a condenser varies inversely as the distance apart of the conducting surfaces.

87. Location of Charge of a Condenser.—In the course of some experiments with a Leyden jar which contained water instead of an inner coating of tin-foil, Franklin, having charged the jar, poured out the water into another vessel and expected

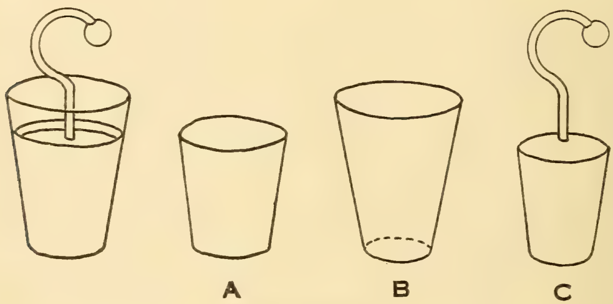


Fig. 39.

thus to obtain the liquid highly charged. His tests however giving no marked results, he thought to repeat the experiment and poured fresh water into the jar when, to his surprise, he found the jar to be almost as highly charged as in the beginning. He concluded that the charge, since it remained behind, could not have been distributed in the liquid and must have been spread over the surface of the glass. To demonstrate this he constructed a jar with movable coatings (Fig. 39). After this jar has been charged, the

inner coating C may be lifted out by inserting a glass rod in the hook and then the glass B may be taken out of the outer coating A. C and A may now be shown to have no appreciable charge either separately or together, but if the jar be reassembled it will give almost as large a spark as it would have given just after charging. The coatings therefore serve merely as paths by which the charge is conducted about over the surface of the glass and the surface of this glass is the seat of the charge. This may also be shown with the condenser represented in Fig. 38 if a sheet of some non-conducting material be inserted between the plates but not if the medium between be air or gas.

We saw in Par. 60 that every electric field consists of non-conductors and is bounded by conductors, and elsewhere (Par. 57) it was stated that the medium within the limits of a field is not passive or inert but takes part in the transmission of the electrical effects and is subjected to certain mechanical strains. Of this there are many proofs. For example, if a beam of polarized light be passed through a piece of glass not under mechanical strain no effect is produced but should the glass be strained, then the beam on emergence will, if allowed to fall upon a white surface, produce certain color effects. Such a beam passed through a piece of glass placed in an electrical field will reveal the presence of strains. Again, if shortly after a Leyden jar has been discharged, the discharger be again applied, an additional spark may be obtained and sometimes even a third. The production of this *residual charge* may be hastened by tapping the jar. This is sometimes explained by saying that a portion of the charge has soaked into the glass but it is perhaps better to say that the material has been strained so near its elastic limit that, like an overloaded spring when the load is removed, it does not return instantly to its primary position. There is no residual charge in an air condenser. Also when a Leyden jar is charged and discharged rapidly a number of times the glass grows warm just as does a spring when rapidly compressed and extended. Finally, the discharge of a jar, while apparently a simple phenomenon, is in reality complex and by the application of instantaneous photography to the image of the spark in a rapidly rotating mirror (Par. 688) it can be shown to be in the nature of an oscillation, sparks of decreasing intensity passing back and forth just as a released spring vibrates with decreasing amplitude back and forth across its neutral position. This, with

the proof that the charge of a Leyden jar lies on the surface of the glass, would seem to justify us in saying that the real seat of the charge is along the bounding surfaces of the non-conductor enclosed within the limits of the field and that the energy of the charge is due to the stresses set up in this medium; the conductor therefore plays a minor part.

88. Capacity of a Spherical Condenser.—The capacity of a condenser is measured by the quantity of electricity that must be imparted to one plate (the other plate being connected to the earth or "grounded" and hence at zero potential) to raise its potential unity. For many condensers the capacity must be measured, for others it may be calculated. For example, let it be required to determine the capacity of a spherical condenser. Let A (Fig. 40) be a metallic sphere surrounded by the metallic sphere B and separated from B by a thickness of air t. If R be the radius of A, that of B is $R' = R + t$. Let B be connected to earth. The potential of B is therefore zero. If a charge Q be imparted to A, a charge $-Q$ will be induced upon the inner surface of B. The potential of A due to its own charge is Q/R (Par. 80); the potential of A due to the charge on B is

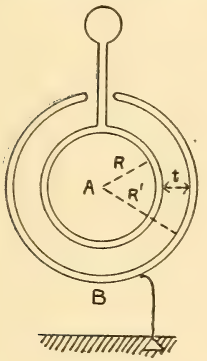


Fig. 40.

$$-\frac{Q}{R+t}$$

The resultant potential of A is (Par. 74)

$$V = \frac{Q}{R} - \frac{Q}{R+t} = \frac{Qt}{R(R+t)} = \frac{Qt}{RR'}$$

And since (Par. 79) $K = Q/V$

$$K = \frac{QRR'}{Qt} = \frac{RR'}{t}$$

or the capacity varies directly as the area of the conducting surfaces and, as was shown in Par. 86, inversely as the thickness of the layer of air separating these surfaces.

A conducting sphere of one centimeter radius has unit capacity, that is, one electrostatic unit raises its potential unity. If it be surrounded by a concentric conducting sphere connected to the

earth and leaving an air space of one millimeter (1/25 of an inch) between the two, its capacity becomes 11, that is, eleven units of electricity must now be imparted to it to raise its potential unity. The appropriateness of the term "condenser" is hence apparent.

89. Capacity of a Plate Condenser.—Let AB (Fig. 41) be a plate of glass of thickness t upon the opposite sides of which are pasted equal circular discs, E and F, of tin-foil, one of which, as F, is connected to the earth. Let the radius of these discs be R. If now a positive charge be imparted to E it will induce and bind an equal opposite charge upon F and repel into the earth an equal positive charge. If the surface density of E be δ, that of F (as shown in Par. 62) will be $-\delta$. A unit positive charge placed between E and F will be repelled from E with a force of $\frac{1}{k}.2\pi\delta$ dynes (Pars. 66 and 55) and attracted to F with an equal force, the total force being $\frac{1}{k}.4\pi\delta$. The work done in moving this unit charge from F to E, a distance t, is $\frac{1}{k}.4\pi\delta t$. According to what was shown in Par. 72, this measures the difference of potential between F and E, hence

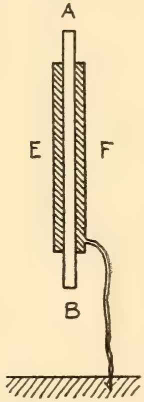


Fig. 41.

$$V' - V'' = \frac{1}{k} . 4\pi\delta t$$

F being connected to earth, its potential V'' is zero, hence the potential of E is

$$V' = \frac{1}{k} . 4\pi\delta t$$

But (Par. 79) the capacity $K = Q/V$, hence

$$K = k . \frac{Q}{4\pi\delta t}$$

The face of the disc E is πR^2, the charge upon it is $\pi R^2\delta$. Substituting this for Q in the above expression we get

$$K = k . \frac{R^2}{4t}$$

that is, the capacity of a condenser is different with different dielectrics and, as has already been shown, varies directly with the area of the conducting surfaces and inversely as their distance apart.

90. Dielectric Capacity.—The fact that the capacity of a condenser varies with the medium between the plates may be shown by a simple experiment. If the air condenser (Fig. 38) be charged to a certain potential and then, without altering the charge or the distance apart of the plates, a slab of paraffine be inserted between them, the potential will immediately drop. If mica be used the drop will be even greater. Since the potential is reduced the condenser will require a greater charge to bring it to its original potential, that is, by substituting for air these other media its capacity is increased.

Since without changing the geometrical arrangement of a condenser but by substituting one dielectric for another we alter its capacity, and since we have seen that the charge resides on this dielectric and not on the conducting plates, we naturally associate the idea of capacity with the dielectric itself and therefore speak of *dielectric capacity.* We use air as the standard of comparison and when we say that the dielectric capacity of mica is six we mean that a condenser in which mica is the dielectric has six times the capacity of one with air as the dielectric but otherwise precisely similar. The dielectric capacity of a substance is therefore measured by the ratio of the capacity of a condenser in which the substance is employed as the dielectric to that of the same condenser in which air has been substituted for the substance. This ratio is represented by k in the last expression in the preceding paragraph. This factor k is sometimes called the *dielectric coefficient* since it is the coefficient by which the capacity of an air condenser must be multiplied to obtain the capacity of the same condenser in which the corresponding dielectric has been substituted for air. Reference to Par. 55 will show that this is the reciprocal of what was there called the "dielectric coefficient of repulsion," whence it follows that in a medium whose dielectric coefficient is k, the force exerted between charged bodies is $\frac{1}{k}$th as much as the force exerted between these bodies in air.

91. Determination of Dielectric Capacity.—In Faraday's determination of dielectric capacity he used spherical condensers similar to the one represented in Fig. 40 but with the opening in the outer sphere closed by an insulating stopper through which the stem of the inner sphere passed. The outer sphere was supplied with a stop cock by which the air between the spheres could be drawn off and liquids or gases introduced, also the outer sphere

could be separated into halves when it became necessary in inserting or removing other materials. Two of these condensers of equal size were taken. In one air was retained as the dielectric; into the other was introduced the substance whose dielectric capacity was to be determined. Suppose the space in the second one to be filled with oil. The air condenser was now charged to a certain potential which was carefully measured by the torsion balance. The outer coatings of the two condensers were next placed in contact, either directly or through a third body, and were thus brought to a common potential. Finally, the inner coatings were brought into contact. The air condenser, being at a higher potential, gave up a portion of its charge to the oil condenser until equality of potential was reached. If the capacities of the two condensers were equal, the charge would be divided equally between them and the resultant potential would be one-half that of the original potential. If the capacity of the oil condenser were greater than that of the air condenser, the oil condenser would take more than half the charge and the resultant potential would be less than half the original potential. If the capacity of the oil condenser were less than that of the air condenser, the resultant potential would be greater than one-half of the original. In either case, the resultant potential having been measured, the dielectric capacity is calculated as follows. Let Q be the original charge of the air condenser, V its original potential, V' the potential of both condensers after division of the charge, K their capacity when used as air condensers and k the dielectric capacity of the oil.

From Par. 79 we have

$$K = \frac{Q}{V}, \text{ whence } Q = V K$$

The charge in the air condenser after contact is

$$Q' = V' K$$

The charge in the oil condenser is

$$Q'' = k\ (V' K)$$

The sum of the separate charges must be equal to the original charge, hence

$$V K = V' K + k\ (V' K)$$

whence

$$k = \frac{V - V'}{V'}$$

92. Dielectric Capacity of Various Substances.—The dielectric capacity of many insulating materials has been measured and some of the accepted determinations are given in the table below. There is wide variation in the results obtained by different investigators and this is due to the fact that the capacity of a condenser is greater if the charge be slowly imparted than if it be suddenly applied and as suddenly withdrawn. In the first case the medium yields and accommodates itself to the stress put upon it. By the so-called, instantaneous method of determining dielectric capacity, the condenser is charged and discharged several hundred times per second and the determinations are less than those obtained by the slow methods. The dielectric capacity of a vacuum is about .94; that of the various gases differs from that of air in the third or fourth decimal place only.

Table of Dielectric Capacities.

Paper	1.5	Mica	4.0 to 8
Beeswax	1.8	Porcelain	4.4
Paraffine	2.0 to 2.3	Glycerine	16.5
Petroleum	2.0 to 2.4	Ethyl Alcohol	22.0
Ebonite	2.0 to 3.2	Methyl Alcohol	32.5
Rubber	2.2 to 2.5	Formic Acid	57.0
Shellac	2.7 to 3.6	Water	80.0
Glass	3.0 to 10.	Hydrocyanic Acid	95.0

93. Dielectric Strength.—The quantity of electricity which must be imparted to a condenser to raise its potential unity depends upon the capacity of the condenser. If the plates of a condenser be connected to two objects between which unit difference of potential is maintained, the condenser will receive the charge which is the measure of its capacity. If the difference of potential between the two objects be doubled, the condenser will receive a charge twice as great and so on. In other words, as has been stated in Par. 79, the quantity of electricity which can be transferred to a condenser depends upon its capacity and also upon the difference of potential maintained between the two plates. By increasing this difference of potential, a greater and greater charge can be given to the condenser but this can not go on indefinitely for as the potential increases, the strain upon the dielectric increases until finally it is pierced by a spark and the condenser is discharged. The resistance which a medium offers to piercing by

the spark is called its *dielectric strength* and is measured by the maximum difference in potential in volts which a given thickness (one centimeter) of the medium will stand before piercing occurs. It is difficult of accurate determination since it is affected by temperature and pressure, by the size and shape of the bodies between which the sparks pass and by the manner in which the electric pressure is applied, that is whether constantly in one direction or alternately in opposite directions.

The dielectric strength of air has been investigated by a number of observers. A minimum difference of potential of 300 volts is required to produce a spark at all, even across a space of less than .01 of an inch. Sparks pass more readily between points than between bodies of other shapes. The strength increases with the density of the air, whether produced by falling temperature or by increasing barometric pressure. If air be under a pressure of 500 pounds per square inch, it can be hardly pierced at all. On the other hand, a vacuum offers an equal resistance. To throw a spark between two points an inch apart requires about 20,000 volts and to produce a 15-inch spark requires 145,000 volts. To pierce one centimeter of paraffine requires 130,000 volts, one centimeter of ebonite about 200,000 and one centimeter of mica about 350,000.

94. Commercial Condensers.—Condensers are used, as will be explained later, in certain electrical measurements, in telegraphy

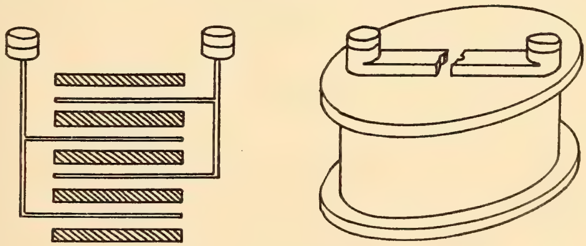


Fig. 42.

and in the production of high potential electricity by means of induction coils. They are usually constructed of alternate layers of tin-foil and mica or of tin-foil and waxed paper pressed tightly together and thus including a large surface in very small bulk. The alternate sheets of foil are connected as shown diagrammatically in Fig. 42 (in which the shaded spaces represent the paper

and the heavy lines the foil) and the whole is contained in a rectangular or cylindrical case provided with the proper terminals. The one represented in the figure is of invariable capacity but by connecting the sheets of foil together in groups attached to separate terminals it is possible to use at will different fractions of the entire condenser.

95. Practical Unit of Capacity.—The practical unit of capacity, *the farad*, is defined as the capacity of that body whose potential is raised one volt by one *coulomb* of electricity. The coulomb will be defined later (Par. 228) but we have already seen (Par. 56) that it is three billion (3×10^9) times as large as the electrostatic unit of quantity. We have also seen (Par. 77) that the electrostatic unit of potential is equal to 300 volts. Since one electrostatic unit of quantity raises the potential of a sphere of one centimeter radius 300 volts, one coulomb would raise the potential of such a sphere to $3 \times 10^9 \times 300$, or nine hundred billion (9×10^{11}) volts, and a sphere of 9×10^{11} centimeters radius would be raised one volt by one coulomb and would therefore have a capacity of one farad. The radius of such a sphere is about 5,600,000 miles or about 1,400 times as large as that of the earth. A farad is therefore so great that in practice one-millionth of a farad (or a *micro-farad*) is used. An isolated sphere of 9×10^5 centimeters radius (about 5.6 miles) would have a capacity of one micro-farad. A mica-tin-foil condenser containing about 25 square feet of tin-foil, has also a capacity of about one micro-farad.

Since a sphere of 9×10^5 centimeters radius has a capacity of one micro-farad, a sphere of one centimeter radius (or a sphere of unit electrostatic capacity) has a capacity of

$$\frac{1}{9 \times 10^5} = \frac{1}{900,000} \text{ micro-farad}$$

96. Work Expended in Charging a Condenser.—In Par. 72 it was shown that the potential at a point was measured by the work done in bringing up to that point from an infinite distance, or from a point of zero potential, a unit charge. If the potential be V, we mean that the work done in bringing up the unit charge is V ergs. The work done in bringing up a charge Q would therefore be QV ergs, although the potential of the point would still remain V, that is, the assumption is that the charge brought up does not increase the potential of the point. The potential in this case is analogous

to the head of a body of water which body is of such extent that its level is not appreciably altered by the pumping up of additional quantities. However, the case is different if the charge is to be brought up to a body of limited capacity. Suppose we have a sphere of unit capacity and at zero potential. At first sight it might seem that to transfer to this sphere from zero potential a certain charge would not require the expenditure of any energy. But suppose the charge to be brought up by successive portions. The first portion could be brought up without the expenditure of energy but would raise the potential of the sphere and would repel the second portion as the latter approached. These two portions would repel the third still more strongly and so on, the work required to bring up the successive portions increasing in regular progression. The potential in this second case is analogous to the head of water in a narrow vessel, each portion that is added raising the level and thus increasing the work which must be expended to bring up the succeeding portion.

To determine the amount of work in bringing up in this manner by n successive portions a charge Q. The first portion Q/n would raise the potential of the unit sphere to Q/n. To bring a unit charge to a point of such potential would, from the definition of potential, require an expenditure of Q/n ergs, therefore to bring up a charge of Q/n will require Q/n times as much or Q^2/n^2 ergs. The second portion would therefore require an expenditure of Q^2/n^2 ergs and the potential would become $2Q/n$. Similarly, the third portion would require $2Q^2/n^2$ ergs and the potential would become $3Q/n$ and so on. To bring up n portions would require a total expenditure of

$$\left(\frac{Q}{n}\right)^2 + 2\left(\frac{Q}{n}\right)^2 + 3\left(\frac{Q}{n}\right)^2 + \quad . \quad . \quad . \quad + (n-1)\left(\frac{Q}{n}\right)^2 \text{ergs}$$

$$= \{1 + 2 + 3 + \quad . \quad . \quad . \quad + (n-1)\}\left(\frac{Q}{n}\right)^2 \text{ergs}$$

The sum of this series obtained by applying the formula

$$\Sigma = \frac{n}{2}(a + l)$$

in which a is the first term, l the last, and n the number of terms (in this case $= n-1$),

is
$$\frac{n-1}{n} \cdot \frac{Q^2}{2} \text{ergs} = \left(1 - \frac{1}{n}\right)\frac{Q^2}{2} \text{ ergs}$$

which when n increases indefinitely becomes

$$\frac{Q^2}{2} \text{ ergs}$$

and the corresponding potential

$$V = \frac{nQ}{n} = Q$$

which last also follows directly from the fact that the sphere is of unit capacity.

Since $Q = V$, the above expression for the work may be written

$$\tfrac{1}{2} QV \text{ ergs}$$

that is, the work done in bringing up from zero potential to a body of limited capacity, likewise at zero potential, a charge Q by which the potential of the body is raised to V is just one-half the work done in bringing up the same charge from a point of zero potential to a point whose potential is V.

97. Energy of a Condenser.—If the body to which the charge is brought is of capacity K instead of unity, the expression $\frac{1}{2} QV$, since $Q = VK$, may be put in the form $\frac{1}{2} Q^2/K$, that is, if a charge Q be given to a condenser of capacity K, the work spent is proportional to the square of the charge and inversely proportional to the capacity of the condenser.

If the condenser be discharged it will give out as much energy as was expended in charging it and therefore the expression $\frac{1}{2} Q^2/K$ also represents the energy of discharge or the energy of the condenser.

If for Q we substitute its value VK, the expression becomes

$$\tfrac{1}{2} V^2 K$$

that is, the energy of a condenser varies as the square of its potential and as its capacity. This principle is utilized in the quadrant electrometer to be described later (Par. 103).

CHAPTER 11.

ELECTROSTATIC MEASUREMENTS.

98. Electrostatic Measurements.—The electrostatic quantities which we most frequently desire to measure are quantity of charge and difference of potential. Of these two, the latter is the more important but if we may measure either one we may determine the other indirectly. Thus, if an unknown charge raises the potential of a certain conductor by a given amount, we have only to find out how much its potential is raised by a unit charge and can then determine at once the quantity of the unknown charge, or similarly, can determine the potential to which a known charge will raise a given conductor.

99. Unit Jars.—At first, attempts were made to measure charges directly by means of what were called "*unit jars.*" These were small Leyden jars, their outer coatings connected with a knob which could be made to approach or recede from the knob communicating with the inner lining. By adjusting the air gap between these knobs a greater or a lesser charge could be given to the jar before a discharge took place. They were used to measure the charge imparted by a machine to a large Leyden jar or to a battery of these jars. One was inserted between the machine and the knob of the large jar. Obviously no charge could pass to the large jar until the unit jar filled up and discharged and the amount was determined by counting the number of sparks.

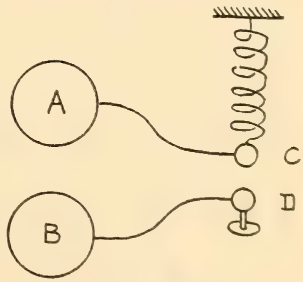


Fig. 43.

100. Principle of Electrometers.—Instruments for measuring differences of electrostatic potential are called electrometers. The principle upon which they operate will be understood from the following. Suppose A and B (Fig. 43) to be two bodies between which there exists a difference of electrostatic potential which we desire to measure. For one reason

or another it is generally impracticable to measure the difference of potential between the bodies themselves and we therefore have to transfer the potentials to the parts of our instrument. Let *C* and *D* be two small spheres, *D* fixed and *C* attached to a spring which can be extended or compressed and which has a scale from which the force producing the extension or the compression can be read. If *A* and *C* be connected by a wire they will at once attain the same potential and the charge imparted to *C* will vary directly with the potential of *A*. Likewise if *B* be connected with *D*, *D* will attain the potential of *B* and acquire a charge proportional to this potential. *C* and *D* will now attract or repel each other with a force which can be read from the scale and which is proportional to the product of the charges which in turn are proportional to the potentials. But *C* and *D* are of the same potentials as *A* and *B*, respectively, and therefore this force is proportional to some function of the difference of potential between *A* and *B*. If *A* or *B* be very small, they would part with a considerable portion of their charge when connected to *C* and *D* and the resultant potential would be less than the original potential, but usually *A* and *B* are so large that the small loss of potential can be neglected. For example, *B* is frequently the earth, in which case *D* is of zero potential.

Coulomb's torsion balance, already described, may be used as an electrometer, the removable ball being touched to the body whose potential is required and thus obtaining a charge proportional to that potential, but the usual form of electrometers use plates or flat moving parts instead of the spheres described above.

101. The Attracted Disc Electrometer.—The attracted disc electrometer was invented by Snow Harris but perfected by Lord Kelvin. Its essential parts are shown diagrammatically in Fig. 44. *AB* is a lever pivoted upon a tightly stretched horizontal wire *CD*. At one end is a counterpoise *B*, at the other end a fork *A* which embraces an upright *E* and across which there is stretched a fine hair. From the fork there is suspended so as to hang horizontally a circular disc *G* which moves with a minimum clearance inside of a fixed ring *R*. A portion of this ring is represented in the diagram as cut away. Below the disc and ring is a circular plate *P* insulated by being mounted upon a glass stem which in turn is attached to a brass support. The plate *P* can be raised or lowered by turning the micrometer screw *H*, which is so arranged that the plate

is always kept strictly parallel to the disc *G* and which permits the distance through which *P* has been moved to be read with great accuracy. Upon the upright *E* there are two black dots and when the lower surface of *G* is exactly in the plane of the lower surface

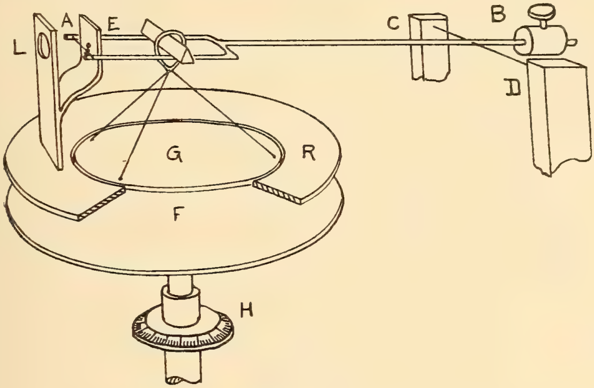


Fig. 44.

of *R* the hair at *A* is just between these dots. There is a lens *L* by which the position of the hair is observed and it is said that an error of as little as 1/50,000 of an inch can be detected and corrected. *G* and *R* are connected electrically by means of a wire from *R* to *D*. By moving the counterpoise *B* or by twisting the wire *CD*, the disc *G* is given an initial position slightly above the ring *R*. Small weights are then placed upon *G* until the lower surfaces of *G* and *R* lie in a common plane. From the weights used the force in dynes to effect this is determined. The weights are then removed. If now a charge be given to *G* it will induce an opposite charge in *P*, *G* and *P* will attract each other and *G* will be drawn downward. By varying the position of *P* the downward pull on *G* can be so adjusted that the plane of the lower surface of *G* coincides with that of the lower surface of *R*. At this point, the force of attraction equals the force in dynes as determined by the weights.

102. Theory of Attracted Disc Electrometer.—In Par. 40 we saw that a charge imparted to a flat disc was uniformly distributed over the central portion but much denser around the edges. When *G* and *R* are in one plane they practically constitute one surface. The surface density over the movable disc *G* is therefore quite uniform and the excessive density is confined to the fixed ring *R* which on this account was called by Lord Kelvin the "guard ring."

To measure the difference in potential between two bodies, R (and hence G) is connected to one and P to the other. Let V' be the potential of P and V'' that of G. The surface density of G is δ and that of the induced charge upon P is $-\delta$. The difference of potential, $V'-V''$, is measured (Par. 72) by the amount of work done in moving a unit positive charge from P to G, a distance D. The force exerted upon a unit charge placed between P and G is (Par. 66) an attraction of $2\pi\delta$ by one and a repulsion of $2\pi\delta$ by the other, or a total force of $4\pi\delta$. The work therefore is

$$V' - V'' = 4\pi\delta D$$

Again, every unit charge upon G is attracted by P with a force of $2\pi\delta$ dynes. If S be the area of G, the charge upon G is $S\delta$ and the total force of attraction is

$$F = 2\pi\delta^2 S$$

Whence

$$\delta = \sqrt{\frac{F}{2\pi S}}$$

Substituting in the expression for $V'-V''$, we have

$$V' - V'' = D\sqrt{\frac{8\pi F}{S}}$$

F is determined in dynes from the weights as described above, D is in centimeters, S is in square centimeters and $V'-V''$, the difference in potential, is in absolute electrostatic units. As explained in Par. 77, if this be multiplied by 300 it is converted into volts.

Since S is constant and F may be kept constant, the expression $\sqrt{\frac{8\pi F}{S}}$ is a constant and can be determined once for all. The difference of potential between G and P is therefore directly proportional to the distance between the plates when the instrument is balanced.

The actual distance between the plates is difficult of measurement. If P be connected to some other charged body whose potential is V''' and the apparatus be balanced we have

$$V''' - V'' = D'\sqrt{\frac{8\pi F}{S}}$$

Subtracting this from the expression above we have

$$V' - V''' = (D-D')\sqrt{\frac{8\pi F}{S}}$$

that is, the difference of potential between two charged bodies, each being compared to a third, is proportional to the difference in the distance between the plates in the two observations and this difference in distance is easily and accurately determined from the micrometer screw.

By using the earth as the third body, that is, by connecting G to the earth, V'' in the above becomes zero.

There are many refinements used in connection with this instrument but it is not necessary to describe them here.

103. The Quadrant Electrometer.—The quadrant electrometer of Lord Kelvin is a more sensitive instrument than the

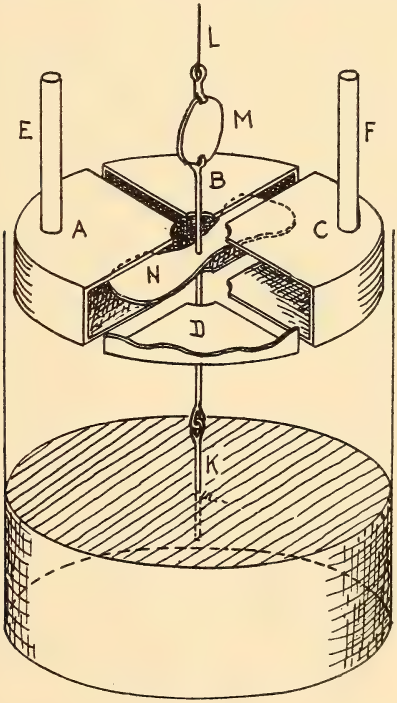


Fig. 45.

foregoing. It is shown diagrammatically in Fig. 45. A flat cylindrical brass box is cut into quadrants A, B, C and D (this last is represented as cut away to show the interior) which are fastened to the top of the apparatus (not shown) by the glass pillars E, F, etc. The diametrically opposite quadrants $A-C$ and

$B-D$ are connected by wires (Fig. 46). Within the box is a flat needle N of light aluminum plate which is fastened rigidly to an aluminum wire extending above and below. The needle is suspended by one or by two fibres of silk or by a single fibre of quartz L attached to the upper end of this wire. To the lower end of the wire there is fastened a platinum wire which dips into some sulphuric acid in a glass jar. This jar, which also serves as a case for the lower part of the instrument, has an outer coating of tin-foil and with the sulphuric acid within is thus a Leyden jar. The acid also keeps the air in the jar dry and prevents loss of charge by moisture. The needle swings midway between the top and bottom of the box and symmetrically over the separation between the quadrants. Upon the wire above the needle there is fastened a small circular concave mirror M. The angle through which the needle turns is determined either by the reflection of a beam of light from this mirror upon a scale or by observing in the mirror by means of a telescope the reflection of a printed scale fastened just above the telescope. These methods of determining the angle of deflection are described in detail later on; the former in connection with the mirror galvanometer (Par. 377), the latter in connection with the suspended coil galvanometer (Par. 378). There project from the glass case terminals, called "electrodes," one of which connects with each pair of quadrants and one with the acid of the jar.

To use the instrument, one pair of quadrants is connected to one body, the other pair to the second body between which the difference of potential is to be measured. The quadrants thus acquire the potentials of the respective bodies. The Leyden jar is then charged until the needle has a high potential V_3 which by certain arrangements, not necessary to describe here, is kept constant during the measurement. If the charges are of the same kind, mutual repulsion exists between the charge on the needle and those on the adjacent quadrants and the needle moves toward the quadrant of lesser charge, that is, of lower potential. The deflection of the needle is read from the mirror and the difference of potential is proportional to this deflection.

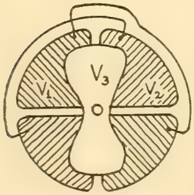


Fig. 46.

This instrument is sufficiently delicate to measure differences of potential almost as small as .01 of a volt.

104. Theory of the Quadrant Electrometer.—Figure 47 represents a cross-section of the needle and two adjacent quadrants, the potentials being as marked and V_3 being much greater than either of the others. V_1 V_3 constitute a condenser, V_2 V_3 another. The energy of a condenser (Par. 97) is $\frac{1}{2}V^2K$ in which V is the difference in potential between the two plates and K is its capacity. The energy of V_1 V_3 is therefore

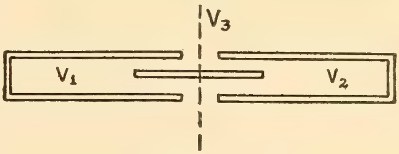


Fig. 47.

$$\tfrac{1}{2}K(V_3 - V_1)^2$$

and that of V_2 V_3 is

$$\tfrac{1}{2}K(V_3 - V_2)^2$$

V_3 being symmetrically suspended with respect to V_1 and V_2 as it swings to the right or left it increases the surface embraced by one by exactly the same amount as it decreases the surface embraced by the other and as its edges still remain well inside of V_1 and V_2 it increases the capacity of one condenser and decreases by an equal amount that of the other. Let this increment of capacity for a unit angular motion of V_3 be denoted by k; the decrement will be $-k$. The change in the energy of V_1 V_3 for an angular movement θ will therefore be $\frac{1}{2}k\theta(V_3 - V_1)^2$ and that of V_2 V_3 will be $-\frac{1}{2}k\theta(V_3 - V_2)^2$. The total change in the energy of the system will be

$$\tfrac{1}{2}k\theta(V_3 - V_1)^2 - \tfrac{1}{2}k\theta(V_3 - V_2)^2$$

The force moving the needle $= \dfrac{\text{energy}}{\text{path}} = \dfrac{\text{energy}}{\theta}$ is therefore

$$F = \tfrac{1}{2}k(V_3 - V_1)^2 - \tfrac{1}{2}k(V_3 - V_2)^2$$

or the force between the needle and each quadrant is proportional to the square of the difference of potential between the needle and the respective quadrant.

Simplifying the foregoing expression we have

$$F = k(V_2 - V_1)\left(V_3 - \frac{V_1 + V_2}{2}\right)$$

or the force tending to turn the needle is proportional to the difference of potential between the quadrants and also to the difference between the potential of the needle and the average of the potentials of the two quadrants.

Since V_3 is kept constant and is very large as compared to V_1 and V_2, $\left(V_3 - \frac{V_1 + V_2}{2}\right)$ may be taken as a constant and the expression for the force becomes

$$F = a(V_1 - V_2)$$

The force being counterbalanced by the torsion of the suspending fibre, the difference of potential, $V_1 - V_2$, between the two bodies being examined is proportional to the deflection as indicated by the mirror. By using a known difference of potential the constant a may be determined once for all.

PART II.

MAGNETISM.

CHAPTER 12.

MAGNETS.

105. Natural Magnets.—Of the four important ores of iron the richest is that one whose chemical formula is Fe_3O_4. This when pure is a heavy black mineral, often coarsely crystalline but also frequently massive. It occurs in beds in many widely scattered localities and from it a large part of the iron and steel of commerce is made. Some specimens of this ore possess the remarkable property of attracting and picking up small particles of iron and steel. If such a specimen be dipped or rolled in iron filings, the filings will adhere to it like a mossy growth. This property has been known for nearly 3,000 years and because the best specimens came from the vicinity of the town of Magnesia in Lydia they were called by the Greeks *magnetis lithos* (Magnesian or Magnetian stone), whence are derived our name *magnet* and the mineralogical term *magnetite* or magnetic iron ore. To distinguish these magnets from those prepared artificially they are usually called *native* or *natural magnets.*

106. Lodestones.—About 800 years ago an additional property of magnets, equally as remarkable as the first, became known to European nations. If an oblong or elongated magnet be arranged so that it is free to rotate in a horizontal plane (as for example by suspending it by a thread or by placing it upon a floating cork or by balancing it upon a pivot) it will take up a north and south position, the same end always returning to the north, no matter what may be its primary position. This property was quickly utilized in navigation and since these magnets thus led the mariner about over the seas, they were called *lodestones* (leading stones).

107. Fables of the Ancients.—In contemplating the mystical power of attraction of magnets, the ancients gave free rein to their imagination and gravely recorded and copied from each other's writings the most wonderful statements about magnets. They were by some reputed to be endowed with life and to possess a soul. A magnet was supposed to protect from witchcraft. If held in the hand it cured cramps. The power of a weakening magnet could be restored by soaking it in the blood of a buck while if it were rubbed with garlic it lost its power. It also lost its power when in the presence of a diamond. If pickled in salt with a certain fish, the *remora* or sucking fish, it acquired the property of attracting gold and silver and could thus be used to fish up treasure from the deepest wells. At various points in the Eastern Seas were islands of lodestone so powerful that they pulled the nails from the sides of vessels and thus caused their loss. In those parts ships had to be built with wooden pegs. In India there were side by side two mountains, one of lodestone so powerful that if a person with iron nails in his shoes stepped upon it he could not raise his feet to take a second step, the other of a substance which repelled iron so strongly that such a person found it impossible to place his foot upon the surface. We can not now understand the state of mind of these writers, for very simple experiments would have readily shown the absurdity of their statements.

108. Doctor Gilbert.—Such for near 2,000 years remained the state of knowledge until, as has already been stated (Par. 12), a certain Doctor Gilbert in the reign of Queen Elizabeth undertook a series of investigations of the properties of the lodestone. In 1600 he published his work, *De Magnete Magneticisque Corporibus* (On the Magnet and Magnetic Bodies), in which he described his experiments, wonderful for their simplicity and in some directions well nigh exhaustive. Anticipating the Baconian system, he accepted no statements about magnets until he had confirmed these statements by his own experiments and he was thus able not only to sweep aside the mythological rubbish which until then passed current but also to bring forward many facts, hitherto unknown. In short, by his researches he brought to light the majority of the truths and principles upon which our present knowledge of magnetism is based.

109. Artificial Magnets.—If a bar of iron or of steel be rubbed or stroked in a certain manner (see Par. 162) by a lodestone, the bar acquires magnetic properties. Steel is found to be more retentive of magnetism than iron and is accordingly used. The bar thus magnetized may in turn be used to produce magnetism in others. There is another and better method, in which an electric current is used to produce magnets, but an explanation of this method must be deferred until later (Par. 164). These artificial magnets, on account of their strength, of the ease with which they may be prepared and of the readiness with which they may be given any desired shape, have quite displaced lodestones and are the ones referred to in the following pages. The commonest forms are bars and the so-called needles. These last are usually thin, elongated, losenge-shaped magnets with a socket at the center by which they may be pivoted upon a sharp point (Fig. 48). In the best needles the socket is jewelled.

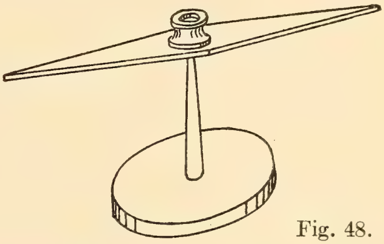


Fig. 48.

110. Magnetic Poles.—In pursuing a certain line of investigation, Gilbert caused to be cut from a lodestone a regular sphere to which he applied the name *terrella* (little globe). When this terrella was rolled in iron filings they adhered to it in tufts, not however uniformly over its surface but upon two restricted areas at the opposite ends of a diameter. These regions he designated as the *poles* of the terrella.

If a bar magnet or a magnetic needle be dipped in filings, they will adhere only to the regions at the ends, and these regions are likewise called poles.

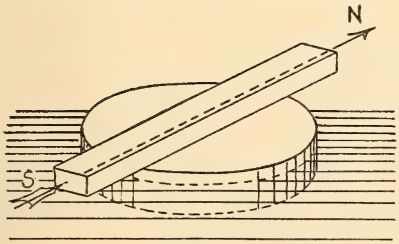


Fig. 49.

If such a magnet be balanced upon a cork which in turn floats in a vessel of water (Fig. 49) it will oscillate in a horizontal plane and finally come to rest in a north and south position. The same end of the magnet always points north and is therefore called the north pole, the other end being called the south pole. The fact that this one end always points north shows

that it must differ from the other, but so far as the attraction of iron filings and the lifting of iron weights is concerned, the two ends are of identical properties. The north and the south poles are frequently designated positive and negative, respectively.

111. The Poles Inseparable.—Should a slender bar magnet (Fig. 50) be broken in half, it will be found that each half is a complete magnet and has a north and a south pole nearly as

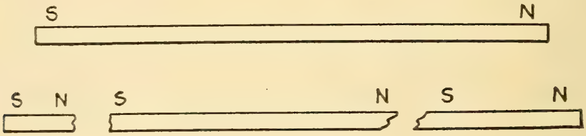


Fig. 50.

strong as those of the original magnet. If one of these halves be again broken, the fragments will each have a north and a south pole and so on. In other words, it is impossible to get a separate north or south pole unaccompanied by an equal pole of the opposite kind. Explanation of this fact will be given later (Par. 152).

112. Magnetic Attraction.—If a bar magnet be dipped into iron or steel filings and then be lifted, the filings will be found to

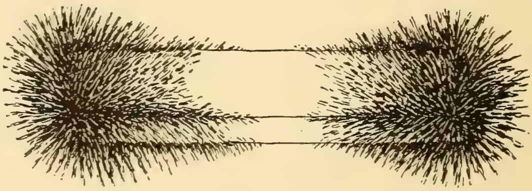


Fig. 51.

cling to it like a thick mossy growth (Fig. 51). Upon examination the following peculiarities will be noted.

1st. As already stated, the filings do not adhere all over but mainly in the region of the poles and none at all in the central portion of the magnet.

2nd. The individual filings cling to the magnet by their ends rather than by their sides and at each pole radiate from an internal focus near the end of the magnet.

3rd. The filings cluster more thickly along the edges and corners of the magnet than along the flat surfaces.

4th. Where the filings are thickest, it will be found that those which cling to the magnet may have others clinging to them in turn, and these may have still others, forming, as it were, chains.

113. The Attraction Takes Place Through Intervening Bodies.—The magnetic attraction takes place at a distance and through space, although it falls off rapidly as the distance increases. Fine filings will leap up to reach a strong magnet held above them. Furthermore, the attraction is propagated through intervening objects. A bar magnet inserted in a glass tube will attract filings through the glass. If filings be sprinkled upon a thin board or a slate or a sheet of glass or of brass, a magnet moved about beneath will drag after it the filings on top. There is but one screen for the magnetic force and that, as will be explained later (Par. 143), is a comparatively thick plate of iron or steel.

114. The Attraction Mutual.—If a small iron bar be floated upon a cork in a basin of water, the bar and cork will move about in pursuit of a magnet held near. If the bar and the magnet be made to change places, the magnet will follow about after the iron bar.

115. Action of Magnets upon Each Other.—The mutual action of magnets is most easily studied by means of a magnetic needle. If when the needle has come to rest, its north end be approached

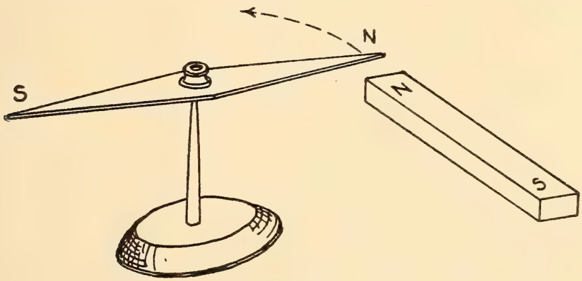


Fig. 52.

by the north end of a bar magnet (Fig. 52), it will be repelled and move off. On the other hand, its south end will be attracted by the north end of the magnet. If the bar magnet be turned end for end and its south end be held to the north end of the needle, the latter will be attracted, and if it be held to the south end, this end will be repelled. We see then that magnetic poles follow a law

similar to that given for positive and negative charges of electricity (Par. 24), that is, *like poles repel and unlike poles attract each other.*

If two bars of similar shape and size attract each other we would know that one of them was a magnet but without other test could not tell which. If they repelled each other we would know that they were both magnets.

116. Why a Magnetic Needle Points North and South.—Suppose that upon a bar magnet resting on a horizontal surface there be placed, as shown in Fig. 53, a magnetic needle. The north pole of the magnet will repel the north pole of the needle but will attract its south pole; the south pole of the magnet will repel the south pole of the needle and attract its north pole. The needle will in consequence take up a position parallel to the axis of the bar magnet but with its poles in reverse direction. Similar experiments led Gilbert to the discovery that *the earth itself is an immense magnet,* its poles being in the neighborhood of, but not coinciding exactly with, the geographical poles. A magnetic needle will therefore take up a position approximately in the plane of the earth's magnetic axis for the same reason that the needle in the above experiment poised parallel to the axis of the bar magnet.

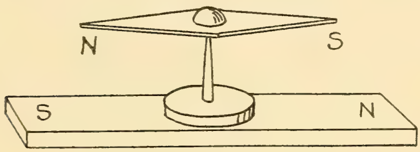


Fig. 53.

117. The Poles Misnamed.—Gilbert called attention to a fact following directly from his discovery, that is, that the pole of the needle which is attracted by the earth's north magnetic pole (and which we have called its *north* pole) should strictly be called its *south* pole. Subsequent writers in view of this have sought to avoid confusion by using the terms "north-seeking pole" and "south-seeking pole," but it is thought that the shorter expressions are sufficiently sanctioned by custom and that no ambiguity will arise if in the following pages the pole of the needle which points north be designated its north pole, the other, its south pole.

118. Magnetization by Induction.—A soft iron nail touched to a bar magnet will cling to it. If a second nail be now touched,

not to the magnet but to the first nail (Fig. 54), it will cling to this nail and even a third nail may be attached to the second and so on. If while thus dangling the several nails be tested, each will be found to possess polarity, the upper ends being of opposite polarity to that end of the magnet to which they are clinging, the lower ends being of the same polarity. If the magnet be removed the chain of nails will fall apart. The magnet therefore influences the nails so that for the time being they themselves are magnets. This is the explanation of the chains of filings referred to in Par. 112. The phenomenon is called magnetization by induction.

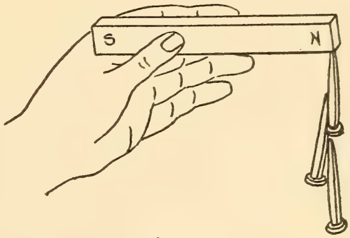


Fig. 54.

119. Induction Takes Place Through Space.—Actual contact is not necessary for induction. A piece of iron or steel placed anywhere in the vicinity of a magnet becomes temporarily a magnet. This fact is clearly shown by the following experiment. A soft iron bar *AB* (Fig. 55) free from magnetism is arranged

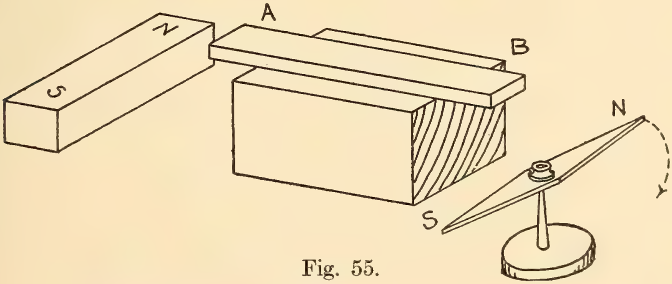


Fig. 55.

upon a convenient support. Near the end *B* but not so near as to be attracted into contact is placed a needle. If the north pole *N* of a bar magnet be approached to the end *A* of the iron bar, but not actually touching the same, the north pole of the needle will be repelled from *B*. The bar *AB* becomes a magnet by induction, the end *B* becoming the north pole and repelling the north pole of the needle. To show that the repulsion of the needle is not due to the direct action of the bar magnet, if *AB* be removed the effect of the bar magnet upon the needle is almost negligible.

120. Magnetic Attraction Explained.—The foregoing enables us to explain magnetic attraction. A piece of iron or steel near a magnet becomes a magnet by induction. The near end of the piece is of opposite polarity and hence attracted; the farther end is repelled but the attraction is stronger than the repulsion (magnetic attraction and repulsion will shortly be shown to follow the law of inverse squares) and the piece, if free to do so, will move bodily up to the magnet. As in the case of electric charges, induction precedes attraction.

121. Other Magnetic Substances.—We have heretofore mentioned only iron, steel and the lodestone as being magnetic substances. Two other metals, nickel and cobalt, are noticeably magnetic, though much less so than the above mentioned, and many substances are feebly so, so feebly however that the property can be detected only by the most delicate apparatus and for practical purposes may be neglected. Among these substances are some of the salts of iron, and oxygen, especially when liquefied.

Gilbert carefully distinguished between magnets and magnetic substances. A magnet exerts its attraction at certain portions only, has polarity and its poles will repel similar poles of a second magnet. A magnetic substance, such as soft iron, has no polarity, attracts either pole of a magnet at any portion of its surface and does not attract other magnetic substances.

122. Diamagnetism.—It has long been known that some substances, notably bismuth and antimony, are repelled from the poles of a magnet, the bodies placing themselves so that their longer axis is at right angles to the magnet. Explanation of this phenomenon can not be given until the subject of electro-magnetics is reached (Par. 402). The repulsion is very feeble and delicate instruments are required to detect it. Faraday investigated the magnetic properties of many bodies and those which are attracted he called *paramagnetics;* those which are repelled, *diamagnetics.* The majority of liquids, except those containing in solution the salts of iron, are feebly diamagnetic. The subject is of theoretical interest only.

CHAPTER 13.

MEASUREMENT OF MAGNETIC FORCES.

123. Coulomb's First Law.—The first law of magnetic force has already been given (Par. 115) and is that like poles repel and unlike poles attract one another. The second law deals with the variation of this force of attraction or repulsion. Before developing it, we must get some preliminary notion of what is meant by the strength of magnets.

124. Lifting Power of Magnets.—At first sight it might seem that a simple way to determine and compare the strength of magnets would be to ascertain the weight which they could support. Various pieces of apparatus have been devised for this purpose. For example (Fig. 56) the magnet is held vertically in a frame and supports by its attraction an iron piece or armature *A*. Attached to this armature is a hook from which hangs a receptacle into which fine sand is slowly poured. When the accumulated weight reaches a certain point the armature is torn away and with the receptacle drops to a table placed just beneath to receive it. The total weight supported by the magnet is then determined by weighing.

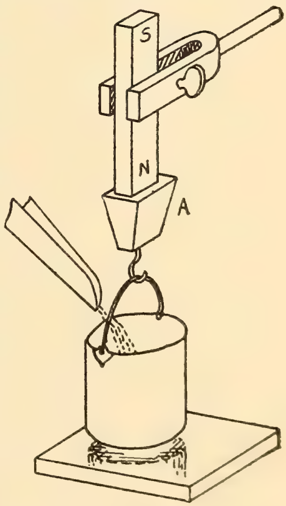


Fig. 56.

When, however, the conditions of this experiment are varied, it will be seen that these results are of but little value. The following will make this clear.

(a) If one end of a bar magnet be rounded and the other be squared, the rounded end will lift a greater weight than the squared end, and this although it can be shown that the two ends are of equal magnetism. The weight lifted therefore varies with the shape of the pole.

(b) If the magnet be bent into a horseshoe shape so that both poles concur in the lifting, instead of the weight being just twice

what it was before for a single pole, it may be three or even four times greater. The weight lifted therefore varies with the shape of the magnet.

(c) If the weight be applied very gradually the magnet will support more than it would if it were applied suddenly. If a magnet be loaded to nearly the maximum point and the load be left in position for a day, the weight may then be gradually increased until it considerably exceeds the original maximum. Once, however, that the armature is torn away, the lifting power of the magnet drops back to what it was formerly.

(d) Within certain limits, the larger the armature the greater the weight lifted.

(e) The weight lifted varies with the character of the iron or steel of which the armature is composed.

(f) Retaining the same weight and composit on, a greater weight will be lifted if the armature be of a compact shape, such as a cube, than if it be a flat disc. The weight lifted therefore varies with the size and shape of the armature.

In the last four cases above we see that although the magnet itself does not vary, the weight lifted fluctuates through a wide range. We can not say which of these weights should be taken to measure the strength of the magnet nor is it practicable to give mathematical expression to the heterogeneous conditions enumerated and deduce formulæ from which this strength might be calculated. What we have done therefore is not to measure the *strength* of the magnet but its *lifting power* under certain given conditions.

A small bar magnet should lift from 15 to 25 times its own weight. In the Paris Exhibition of 1882 there was shown a magnet which supported 76 times its own weight. Thompson states that the lifting power of a good steel magnet may amount to 40 pounds per square inch of pole surface. Electro-magnets, to be described later, are much more powerful, the lifting power reaching 200 pounds per square inch.

125. Strength of Magnets.—If we examine the force with which one magnet attracts or repels another, the two being at some distance apart, we find that it is not affected by shape of poles or length of exposure to each other's influence, etc., but is to a great extent independent of the varying conditions mentioned in the preceding paragraph. The force with which magnetic poles

interact is therefore selected as the measure of their strength. We are thus naturally led to enquire what is precisely a magnetic pole and how is the force between two poles measured.

126. Magnetic Pole Defined.—In the preceding pages we have used the word *pole* to designate rather vaguely the terminal portions of a magnet, the regions in which the magnetic force is most marked. In mathematical discussions it is desirable to treat a pole as if it were a focus or the point of application of the resultant of the magnetic forces at that particular end of the magnet. This point may be approximately located as follows. In a bar magnet the magnetic forces are symmetrically distributed around its axis and the pole must consequently lie upon this axis. In Fig. 57 let MN represent one-half of the bar magnet which is supported

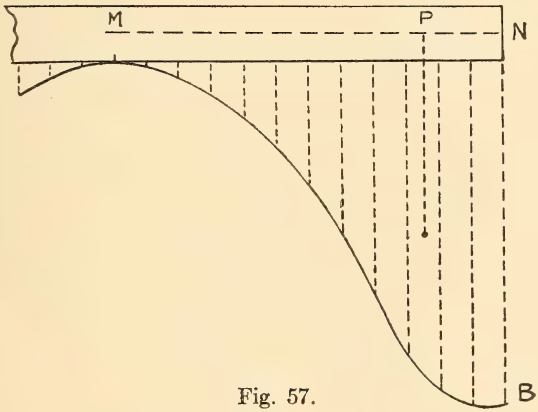


Fig. 57.

horizontally. With a pencil mark off this half in equal divisions. Cut a small soft iron wire into a number of short pieces of equal length (and hence of equal weight). Apply the end of one of these pieces to one of the divisions of the bar and then other pieces to the first piece until the accumulated cluster drops off of its own weight. Note the particular division and the corresponding number of pieces. Repeat this for each of the divisions, then construct a curve MBN in which the divisions are the abscissæ and the ordinates are laid off to a scale to represent the number of pieces of wire supported. The pole is on the axis of the magnet and approximately opposite the center of gravity of the triangular figure MBN.

In short thick magnets the poles are distributed over a considerable area but for long slender bars they approach the ends and approximate the hypothetical point or focus. According to Fleming, the poles of a bar magnet are about one-twelfth of its length from the ends. For shorter and thicker bars this distance may amount to one-sixth or even one-fifth.

It will be shown later (Par. 146) that for certain magnetic measurements the exact location of the pole is immaterial.

Although it is impossible to get a magnetic pole unaccompanied by an equal and opposite pole, yet in a long slender magnet the poles are so far apart that in many experiments the effect of the more distant one may be neglected and the results are as if we were dealing with a single or "free" pole.

127. Measurement of Magnetic Forces.—The measurement of magnetic forces is not entirely a simple matter. Two magnets, *A* and *B*, exposed so each other's influence are each acted upon by four forces. The north pole of *A* is repelled by the north pole of *B* and attracted by its south pole; the south pole of *A* is repelled by the south pole of *B* and attracted by its north pole. In addition, each magnet is acted upon by the earth's magnetic poles so that each is subject to eight forces.

In most cases the forces are comparatively feeble. They must therefore be measured by comparing them with, or by balancing them against, other forces, likewise feeble, whose variation follows some readily determined law. The forces used for comparison are—

(a) A known magnetic force, usually that of the earth. There are two methods of comparison, both of which will shortly be described (Pars. 129, 146).

(b) The torsion of a suspending thread, as in Coulomb's torsion balance, already described (Par. 52). The law in this case is that the force varies directly as the angle through which the thread is twisted.

(c) The force of gravity applied through a *bifilar suspension.* A magnet is suspended in a horizontal position by two parallel threads. If it be deflected the threads must be twisted from a vertical to an oblique position and the magnet must therefore be raised. The law in this case is that the force varies directly as the sine of the angle through which the magnet is twisted.

128. Coulomb's Second Law.—The second law of magnetic force comprises two statements. First, the force exerted between two magnetic poles varies directly with the product of their strengths, and second, this force varies inversely as the square of the distance separating them. The distance between the poles is supposed to be so great that they may be regarded as points.

The first of these statements hardly requires proof since it follows at once from the fact that the action between poles is mutual and that if we double or treble the strength of either one we double or treble the force exerted between them. Its truth may easily be shown experimentally. The second statement is proved experimentally by one of the methods now to be described.

129. Method by Oscillations.—From mechanics, the time of oscillation of a simple pendulum, its angular displacement being small, is given by the equation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

in which l is the length of the pendulum and g is the acceleration due to gravity. The force acting upon the pendulum is mg, m being its mass. The above expression may be written

$$T = 2\pi\sqrt{\frac{ml}{mg}} = 2\pi\sqrt{\frac{ml}{\text{force}}}$$

whence

$$\text{Force} = \frac{4\pi^2 ml}{T^2} = \text{constant} \times \frac{1}{T^2}$$

But $\frac{1}{T}$ is the number of oscillations n in a unit of time, hence

$$\text{Force} = \text{constant} \times n^2,$$

or the force producing pendular vibrations is proportional to the square of the number of vibrations executed in a unit of time. Any convenient interval of time may be taken as the unit.

If a magnetic needle AB (Fig. 58), whose position of rest is along the magnetic meridian NS, be pushed aside through an angle δ and then released, it will be acted upon by forces tending

to return it to its primary position but in swinging back it will acquire a momentum which will carry it very nearly an equal angular distance beyond *NS* and will then swing in the opposite direction and so on, that is, the needle will act as a double pendulum in a horizontal plane and if δ be small will execute oscillations whose period is practically constant. The forces which tend to restore the needle to its position in the meridian are due to the interaction of the poles of the magnet and those of the earth. The earth being a sphere and its poles being located beneath its surface, their action lines are oblique to the plane of oscillation of the needle and only the horizontal component of the forces along these lines affects the oscillations. This horizontal component of the earth's magnetism is usually designated by the letter H. Within the limits of space covered by the average experiment H is constant. If the strength of the poles of the needle be represented by m, the force acting upon each pole will be mH, represented in Fig. 58 by AD and BC and, from what we have seen above, this force is proportional to the square of the number of oscillations executed by the needle in a given time. How this principle may be utilized in measurements will be shown in Par. 131.

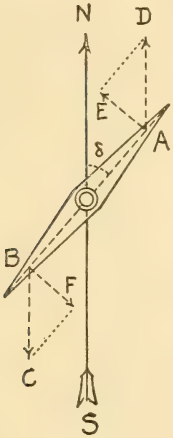


Fig. 58.

130. Magnetic Moment.—The active components of the forces AD and BC (Fig. 58) are AE and BF, each of which is equal to $m.H.\sin\delta$. These constitute a couple whose moment is $m.H.\sin\delta.l$, l being the distance between the two poles. The product ml is called the *magnetic moment* of the needle and in formulæ is represented by M. Although the exact position of the poles, and consequently the distance l, is most often unknown, M itself may be determined by experiment and is used in certain magnetic measurements to be described later (Pars. 148, 149, 150).

131. Experimental Proof of Law of Inverse Squares.—In Fig. 59, A is a very small magnet, less than half an inch in length, suspended in a paper stirrup by a single fibre of unspun silk and at rest in the magnetic meridian. The resistance of the silk fibre being very slight, if the magnet be started in oscillation it will continue so for from five to ten minutes. It is given a slight im-

pulse and the number of oscillations executed in a given interval, say one minute, is counted. Suppose this number to be 10. A slender bar magnet B is now placed in the same meridian, its axis in the prolongation of the axis of A. (This experiment may also be performed with the magnet B in a vertical position, its pole being in the meridian and horizontal plane of A.) B must be placed at such distance from A that for small angular deviations of A the action lines of B are sensibly parallel. This is also one of the reasons for keeping A very small, the other being that if A be small its poles are more nearly the same distance from the pole of B. A is again set in motion and if the poles of the bar magnet

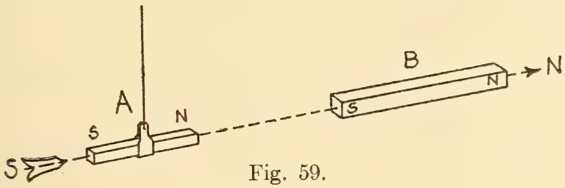


Fig. 59.

coincide in direction with those of A, the oscillations will be more rapid. Suppose that now 12 are executed in one minute. The force due to the horizontal component of the earth's magnetism is to the combined force of this component and that of the pole of B as 100 is to 144. Let B now be pushed up towards A until the distance between its pole and that of A has been halved. A set in motion will now be found to execute about 16.5 oscillations per minute. The total force upon A in the first place is to that in the second as $(12)^2$ is to $(16.5)^2$ or as 144 is to 272. The force due to B alone is as (144–100) is to (272–100) or as 44 is to 172, which is very nearly as 1 is to 4. In other words, as the distance is halved the force is quadrupled, which is in accordance with the law of inverse squares.

In the above proof the distance between the poles of the two magnets must be known and as the exact position of the poles themselves is not precise, their distance apart is apparently uncertain; however, this distance may be assumed as nearly as possible and one or two trial experiments thereafter will show what the correct distance should be.

132. Proof of Law of Inverse Squares by Coulomb's Balance. —Coulomb also proved this law by means of the balance which bears his name. A description of the actual experiment would be

somewhat long and it will suffice to say that in the apparatus as represented in Fig. 22, a slender bar magnet took the place of the shellac needle G and a second one took that of KH. The instrument was set up so that with no torsion on the suspending fibre, the horizontal needle and the opening K in the glass cover lay in the same magnetic meridian. The experiment was then conducted as explained in Par. 52, due allowance being made for the effect of the earth's magnetism.

133. Unit Magnetic Pole.—Magnetic poles differ in strength. We may consider that there is more magnetism concentrated at the stronger pole or may assume that there are magnetic poles of unit strength and that a greater number of these are gathered at the stronger pole. A definite conception of a unit pole may be obtained from the following. Coulomb's second law may be expressed thus,

$$f = \frac{m \times m'}{d^2}$$

in which, since we are using the C. G. S. system, f is the force in dynes between the poles, m and m' the strength of the respective poles and d their distance apart in centimeters. If the poles be of equal strength this becomes

$$f = \frac{m^2}{d^2}$$

Finally, if f becomes one dyne and d one centimeter, we have $m = 1$, or *a unit magnetic pole is that pole which when placed at a distance of one centimeter from a similar and equal pole repels it with a force of one dyne.*

CHAPTER 14.

THE MAGNETIC FIELD.

134. Magnetic Field.—In the space around a magnet all poles experience forces of attraction and of repulsion and this space is called the field of the magnet. As we recede from the magnet these forces diminish in accordance with the law of inverse squares and, to fix its limits more definitely, we define a magnetic field as that space surrounding a magnet in which magnetic force due to this magnet is perceptible.

135. Direction of Magnetic Field.—As an aid to the conception of a magnetic field we may resort to the same analogy as in the case of the electric field (Par. 58) and compare it to a current of water. In a magnetic field there is no matter in actual movement but there is in a certain sense a flow of force and magnetic poles, placed in the field, are swept along just as light objects are carried by a stream. Since free north poles would be carried along in one direction and free south poles in the opposite direction we by convention define *the positive direction of a magnetic field as that direction in which a free north pole would move.*

136. Intensity of Magnetic Field.—Just as we might measure the strength of a current by the force with which it pushes a board of unit area placed in it, so we agree to measure the intensity of a magnetic field by the force with which it acts upon a unit pole placed in it and we define *a unit magnetic field as that field which acts with a force of one dyne upon a unit pole placed in it.* If we say that a magnetic field has a strength of three, we mean that it will act with a force of three dynes upon a unit pole placed in it. If the strength of the field be H and that of the pole be m, the force with which the field acts upon the pole is Hm dynes. From the foregoing and from Par. 128 it follows that the field at a distance d from a pole of strength m is m/d^2.

137. Magnetic Lines of Force.—In Fig. 60 let P be a point in the field of the bar magnet NS and for simplicity of construction suppose that at this point the distance SP is twice the distance

NP. Suppose a free north pole to be placed at the point *P*. It will be repelled from *N* along *NP* and attracted towards *S* along *PS*. In the case assumed the distance *NP* being only one-half of *PS*, by the law of inverse squares the repulsion along *NP* is four

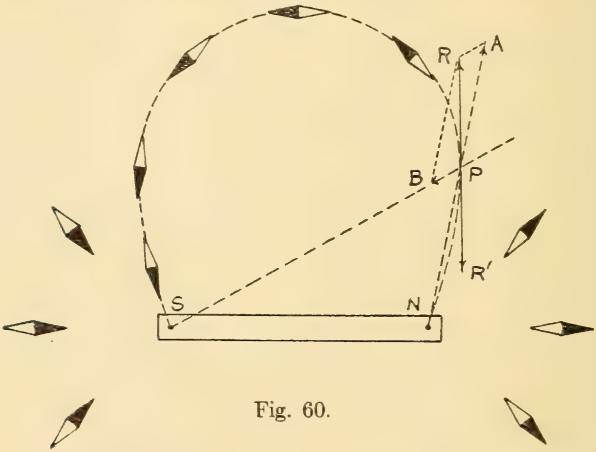


Fig. 60.

times as great as the attraction along *PS*. Lay off *PB* any convenient distance and *PA* four times as great and complete the parallelogram. *PR* is the resultant at the point *P* of the magnetic forces of the two poles *N* and *S* or, in other words, the free north pole at *P* will be urged along the resultant *PR* with a force proportional to *PR*.

Suppose this free north pole to move along *PR* a very small distance. In doing so it will move away from *N* more rapidly than it does from *S*. This will cause the repulsion from *N* to grow weaker and the attraction to *S* to grow relatively stronger and the path of the pole will bend around towards *S*. In its successive positions therefore, the pole will follow a curve which at every point indicates by its direction the direction of the resultant of the magnetic forces at that point. This curve is called a *magnetic line of force.*

If instead of a free north pole a free south pole had been placed at *P*, it would have been urged with an exactly equal force in an exactly opposite direction *PR′*, and in its path would have traced out the same curved line but in a reverse direction. By convention (Par. 135) we define the positive direction of a magnetic line of force as that direction in which a free north pole would move. In

our diagrams the positive direction of these lines is always indicated by an arrowhead placed upon the lines.

138. Mapping Lines of Force.—If at any point P (Fig. 60) there be placed a very small magnetic needle, its north pole would be urged in the direction PR, its south pole in the direction PR', and the needle will take up a position approximately tangent to the line of force at the point P. If a sufficient number of these little needles be placed one after the other, as shown in Fig. 60,

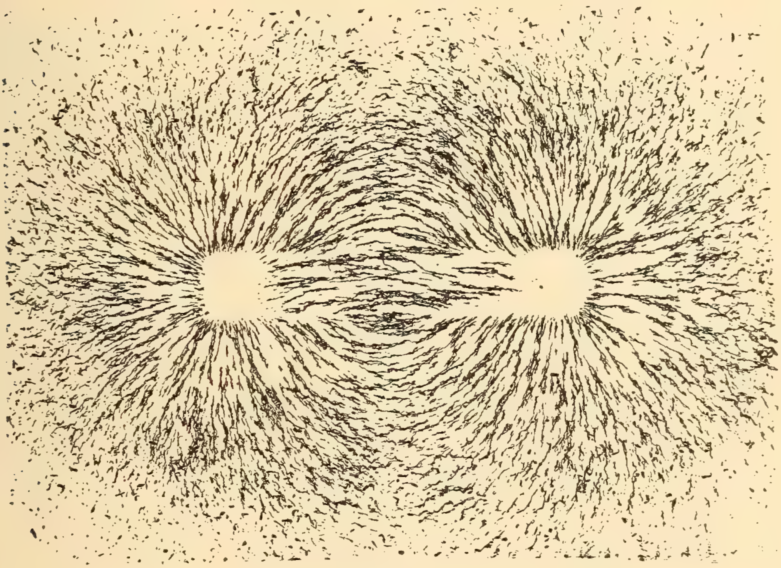


Fig. 61.

the successive tangents which they indicate will serve as an envelope and will mark out the line of force, approximating more and more closely to it as their length is decreased and number increased. Finally, if the entire space about the magnet were strewn closely with the little needles a number of lines of force would be traced.

This condition may be realized practically as follows. A sheet of glass, of stiff paper or of any non-magnetic body is placed upon a magnet and is then sprinkled with fine iron filings. From what we have already seen (Par. 120) each individual filing becomes for the time being a magnet, but these little magnets are not free to

move since their weight holds them with friction against the surface over which they are sprinkled. If the sheet be given a gentle tap the filings are jarred and for a minute interval of time are bounced up into the air. Being now freed from the friction which held them in place, they move under the influence of the magnetic forces and after a few repetitions of the jarring they gather along well marked lines as shown in Fig. 61.

139. Permanent Record of Magnetic Figures.—Several ways have been described by which these magnetic figures, or curves traced by the filings, may be recorded permanently. The following is simple and convenient. Upon a soft pine board about a foot square the magnet is placed and its outline is traced with a pencil. With a chisel this outline is then hollowed out until when in position the upper surface of the magnet is on a level with that of the board. The board is then taken into a subdued light and there is pinned upon it, prepared surface up, a sheet of blue-print paper about 8×10 inches. Iron filings are then sprinkled over this paper and the board is tapped on the under side until the magnetic figures come out as desired. Better results are obtained if before using the filings they are passed through two sieves, one to separate the dust-like particles and the other those of too large size. The board with the filings in position is then exposed in a strong sunlight for from three to five minutes, the rays falling as nearly perpendicular to the paper as possible. It is then carried back to the subdued light, the filings poured off and the paper thoroughly washed in clear water. The resulting blue-print is then dried.

140. Use of Magnetic Figures.—These magnetic figures are of assistance in the study of magnetic fields and often enable us to grasp at a glance conditions which might otherwise require considerable mathematical analysis to develop. For example, they show in a striking manner how the field between two mutually attracting poles differs from that between two that mutually repel. Fig. 62 represents the field between two dissimilar poles. In this the lines of force are seen to pass from one to the other as if pulling them together. At the same time these lines are bowed out revealing the existence of the crosswise pressure causing them to separate. Fig. 63 shows the field between two similar poles and it does not require a great stretch of the imagination to conceive

of the lines of force as hands placed palm against palm and pushing each other back. Further examples of the use of these figures will be met in subsequent pages.

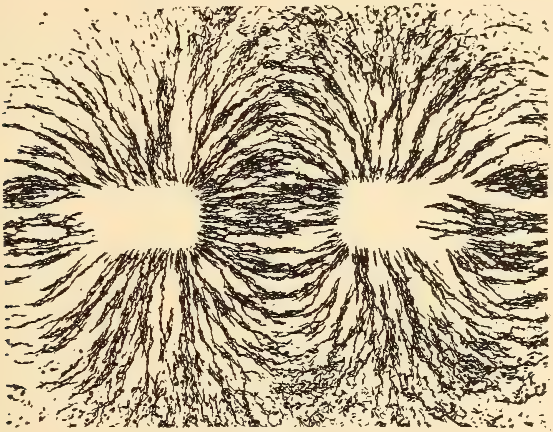


Fig. 62.

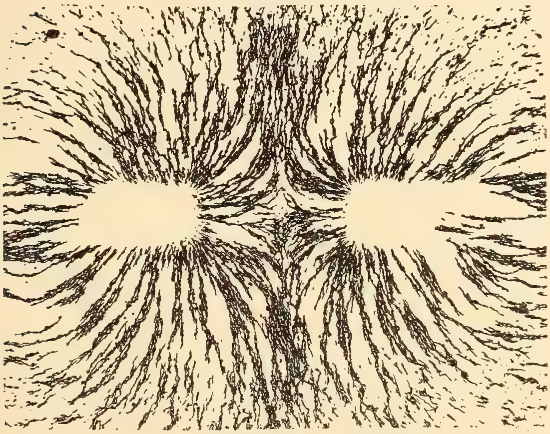


Fig. 63.

141. Compounding Magnetic Fields.—The magnetic fields hitherto considered are those surrounding a single pole or pair of poles and are symmetrical with respect to the single pole or to the line joining the two poles. Should these fields be intersected by another, the resultant field would be obtained by compounding

the two and would in general be unsymmetrical. The earth's field most often produces distortion in others but its strength being comparatively feeble, the distortion is not revealed in the magnetic figures produced with filings. If, however, the field be mapped as follows the effect of the earth's field becomes evident. Place a bar magnet in the center of a sheet of paper and then in contact with the magnet place one of the little compass needles one centimeter in length and mounted in a glass-covered brass case. With a pencil make a dot at the far end of the needle, then shift the compass until the near end of the needle is over this dot and

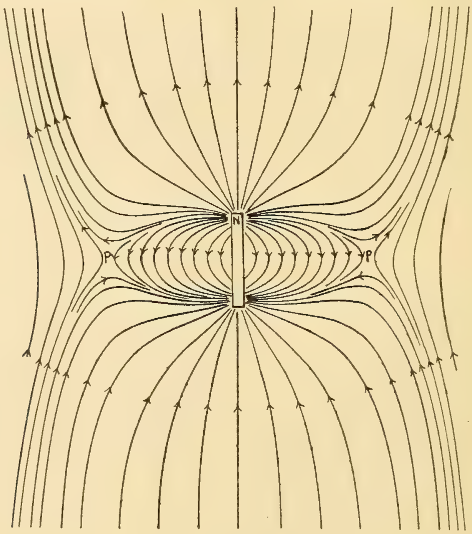


Fig. 64.

again make a dot at the new position of the far end of the needle and so on to the limits of the paper. Connect these dots by a continuous curve. Start with the compass at some other point along the magnet and make a second chain of dots and so on until the whole space about the magnet has been marked off. Figs. 64 and 65 represent fields traced in this way, Fig. 64 with the north pole of the bar magnet pointing north, Fig. 65 with the north pole pointing south. In each case immediately around the magnet the strength of its field overpowers that of the earth but the strength of the magnet's field falls off rapidly as the distance from it increases while the earth's field is constant over a considerable

area and at a distance from the magnet the earth's field has the ascendancy. At the spots marked P these two forces neutralize each other and the needle will vacillate and come to rest in any position. These two figures, though different, are both symmetrical since the bar magnet was designedly placed in a north and south position. Should it be placed in any oblique position the symmetry will be destroyed.

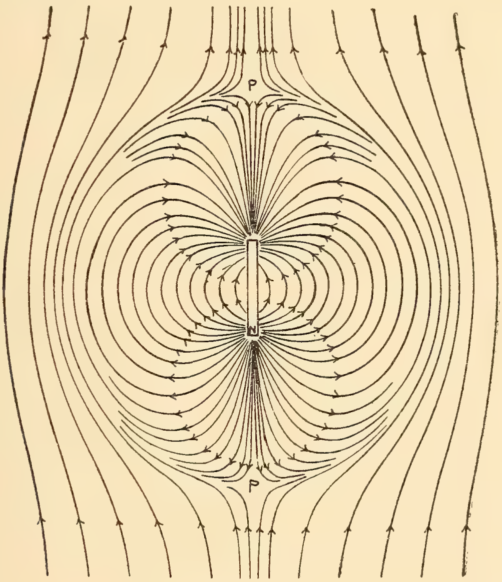


Fig. 65.

142. Properties of Magnetic Lines of Force.—In some of their properties magnetic lines of force are similar to electric lines of force but in others they differ widely. They agree with electric lines of force in having a tension along their length, or a tendency to shorten, and also a pressure at right angles. They also never intersect. They differ from electric lines of force in that they are closed curves, that they penetrate all substances whether conductors or not and that they do not necessarily, or even generally, leave or enter a surface at right angles. Being a closed curve, a complete magnetic line of force lies partly in the magnet and partly in the surrounding medium. While the majority of these lines emerge near the poles, many, as shown in Figs. 61 and 66, emerge along the sides of the magnet. The lines within the magnet

are designated collectively as the *magnetic flux* ana this flux is evidently a maximum at the mid-section of the magnet. This is sometimes otherwise expressed by saying that the *intrinsic magnetism* is a maximum at this mid-section. The intrinsic magnetism is of no effect on outside bodies. Magnetic effects of attraction and repulsion are produced only by those lines of force which emerge from the magnet. This is called the *free magnetism* and is greatest in the neighborhood of the poles.

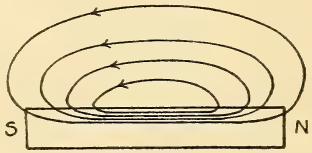


Fig. 66.

It must be noted here that that portion of these lines which lies within the magnet does not conform to the definition of a line of force as given in Par. 137, for which reason these internal lines have been variously designated as *lines of magnetization, lines of magnetic induction,* etc. An internal line of magnetization is, however, always the continuation of an external line of force and the above distinction, although academically correct, is without practical importance. This fact will be brought out more clearly in the subject of electro-magnetics.

That portion of a magnetic body from which lines of force emerge is always a north pole and that portion of such body into which they enter is always a south pole. To this rule there is but one exception. The lines of force of the earth enter at the north magnetic pole and come out at the south magnetic pole. The reason for this exception has already been given (Par. 117).

143. Magnetic Lines Pass Preferably Through Magnetic Substances.—If in the space between the two dissimilar poles in Fig. 62 there be inserted a soft iron block and a magnetic figure be then taken, the lines of force which in Fig. 62 curved out widely will now be seen to have drawn in, as shown in Fig. 67, and pass through the iron block instead of through the air. A simple explanation is that the iron block has become a magnet by induction and the lines of force converge to the nearest poles, but it is sometimes conveniently explained by the statement that magnetic

lines of force travel by preference through magnetic bodies and will avail themselves of such a path whenever the opportunity offers. This principle affords an explanation of certain phenomena and is of considerable practical importance.

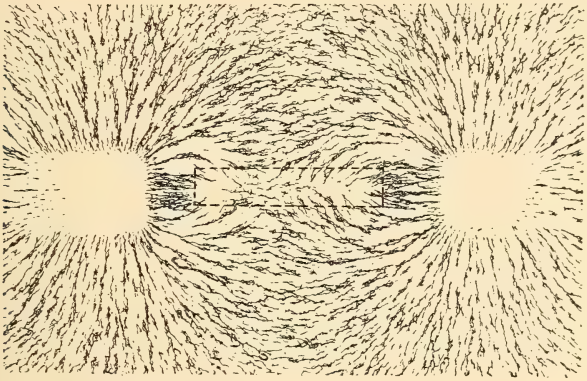


Fig. 67.

It has already been noted (Par. 112) that filings cling to the edges of a magnet rather than to the flat surfaces. This fact is also clearly shown in Figs. 61, 62 and 63. In Fig. 68, *a* and *b* represent end views of a bar magnet. If the lines of force radiated equally from the internal pole they would emerge as shown in *a* and there would be more filings just on top of the magnet than elsewhere since this point is nearest to the pole and consequently

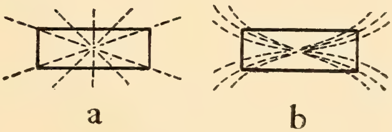


Fig. 68.

at this point the attraction would be strongest. But since the lines prefer to travel as far as possible through the steel, their actual path is as represented in *b* and the filings are thickest where the lines of force are thickest, that is, along the edges.

An essential part of dynamos and motors consists in its simplest form of two powerful magnetic poles embracing between them a cylindrical opening. It is highly important that there should be a

uniform field along the faces of these magnets but owing to the principle above, the lines of force, as shown in Fig. 69 *a*, crowd across at the top and bottom and leave the central portion of the opening thin. If, however, there be inserted in this space a soft

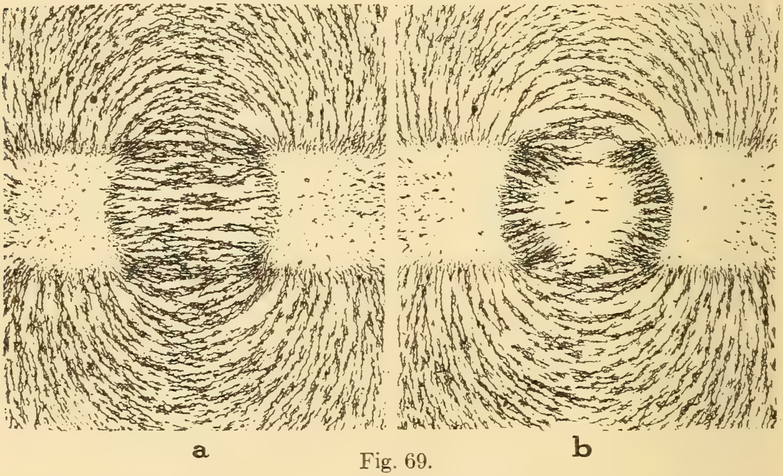


Fig. 69.

iron cylinder, the lines will pass through this cylinder and, as shown in *b*, will produce along the pole faces a dense and uniform field.

If a magnet *NS* (Fig. 70) produces a deflection in a needle *A*, the needle can be screened from this effect by interposing between

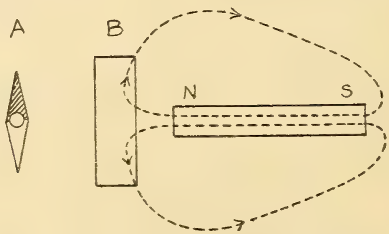


Fig. 70.

it and the magnet a comparatively thick iron plate *B*. The lines of force from *N*, which formerly reached *A*, now travel through *B*, as shown in the figure, and thence back to *S*. If *A* be placed inside of an iron cylinder it may be entirely screened from outside magnetic influences.

If a pivoted iron bar AB (Fig. 71) be placed diagonally across a magnetic field NS it will swing so as to place itself parallel to the field. We may explain this motion as follows. The lines of force of the field, from what we have just seen, turn to one side and, as shown in the figure, run lengthwise through the bar. The tension along these lines produces a couple on AB which pulls it around to parallelism with NS. The law under which this movement takes place is that a magnetic body placed in a magnetic field tends to move so that its longest axis coincides in direction with the lines of force of the field.

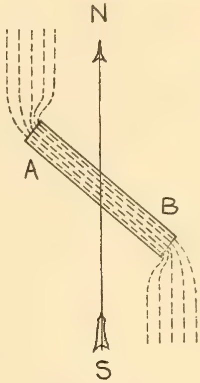


Fig. 71.

144. Law of Maximum Flux.—In Fig. 72, A is a piece of soft iron in a weak field to one side of the strong field B. The lines of force of the strong field move out, as shown in the figure, so as to pass through A and if A be free to move it will be drawn over into the denser field at B. If A be a magnet placed obliquely to B it will be both drawn over into B and turned so that its own lines of force will be parallel to and of the same direction as those of B. It will therefore embrace in lengthwise direction both its own lines of force and those of the field B, or, in general, *a magnetic body placed in a magnetic field tends to move so as to embrace in one direction the maximum number of lines of force.* This is but a particular case of Maxwell's Law, a principle of great importance which will be discussed later (Par. 371).

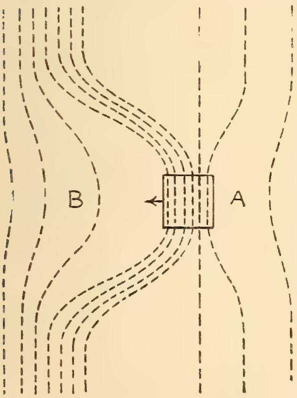


Fig. 72.

145. Graphic Representation of Intensity of Magnetic Field.—For the same reasons as given in Par. 61, it has been agreed to represent graphically the intensity of a magnetic field by the number of lines of force per square centimeter taken perpendicular to these lines. From this standpoint, a unit magnetic field is defined as that field which contains one line of force per square

centimeter of cross-section. A similar course of reasoning to that given in Par. 63 will lead to the conclusion that there radiate from a unit pole 4π lines of force.

146. Comparison of Magnetic Fields. Tangent Law.—There are a number of ways in which magnetic fields may be compared by means of the deflection which they produce in a magnetic needle. If a needle which is poised in the meridian be exposed to such a field at right angles to the meridian, the needle will be deflected through a certain angle. The field draws it to one side with a decreasing moment; the horizontal component of the earth's magnetism pulls it back to the meridian with an increasing moment. A position of equilibrium is finally reached and the angle of deflection may be read from a scale placed beneath the needle.

In general, in instruments which operate thus by a needle moving over a scale, the force which pulls the needle away from its zero position is called the *deflecting force;* the force which tends to restore it to the zero position is called the *controlling force.*

In the following method it is assumed that the action lines of the magnetic force upon the poles of the needle are parallel and that within the limits of the space over which the needle swings the force is constant. In order to realize this condition as nearly as possible the needle is made very small. If the scale of degrees varied with the size of the needle it would become too small to be read with much accuracy, therefore an auxiliary pointer of aluminum or of some other light substance is fastened to the needle and a larger scale may then be used.

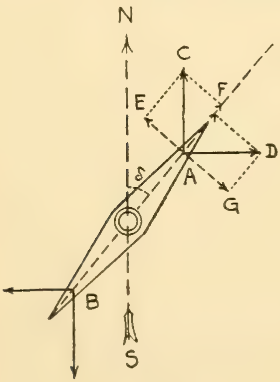


Fig. 73.

In Fig. 73 let AB be a magnetic needle of pole strength m, deflected from the meridian NS through an angle δ by a magnetic field H' acting at right angles to the meridian. At the pole A the controlling force is mH, represented by AC. The deflecting force is mH', represented by AD. An exactly similar set of forces act upon the pole B but to consider them would simply be to repeat what we shall prove for the set at A. The controlling force may be divided into two components, one, AF, in the

direction of the axis of the needle and of no effect so far as rotation is concerned; the other, AE, perpendicular to the needle and active in restoring it to the meridian. From Fig. 73,

$$AE = AC.\sin\delta = mH.\sin\delta$$

Similarly, the deflecting force may be divided into two components, one in the direction AF, the other AG, which $= AD.\cos\delta = mH'.\cos\delta$ and which is active in deflecting the needle from the meridian. When the needle comes to rest these two active components, AE and AG, are equal, hence

$$mH'.\cos\delta = mH.\sin\delta$$

whence

$$H' = H\cdot\frac{\sin\delta}{\cos\delta} = H.\tan\delta$$

or, the magnetic field which acting at right angles to the meridian produces in a magnetic needle a deflection δ, is equal to the horizontal component of the earth's magnetism at that point multiplied by the tangent of the angle of deflection.

It follows direct from the foregoing that different magnetic fields acting at *right angles with the meridian* will deflect a needle through angles whose tangents are proportional to the respective fields. This is known as the *Tangent Law* and is an important principle in certain electrical measuring instruments to be described later (Par. 373).

It will be noted that the deflection produced is independent of the strength and of the length of the needle, or rather of the distance between the poles. Hence, as was stated in Par. 126, the exact location of the poles is immaterial. If, however, the controlling force be non-magnetic (as in Nobili's astatic galvanometer) these factors are of importance.

147. The Sine Law.—Should the deflecting field make a constant angle with the needle instead of with the meridian, a different state of affairs would result. Whatever the constant angle may be, we can always divide the force into two components, one of which is perpendicular to the needle and is the effective one in producing deflection. We may therefore in Fig. 73 consider AG as representing the deflecting force mH'. When equilibrium is reached $AG = AE$, or

$$mH' = mH.\sin\delta,\ \text{or}\ \ H' = H.\sin\delta$$

whence, magnetic fields acting at a *constant angle with the needle*

are to each other as the sines of the respective angles of deflection. This is known as the *Sine Law* and is the principle of one class of galvanometers (Par. 376).

148. Determination of the Strength of a Magnetic Field.—In Pars. 129, 146 and 147, principles have been given which enable us to compare magnetic fields among themselves, that is, to determine how many times stronger or weaker one field is than another, but these do not enable us to make any absolute measurement. The following method of determination of the absolute strength of a magnetic field is due to Gauss.

In Par. 129 we saw that the time of oscillation of a simple pendulum is given by the expression

$$T = 2\pi\sqrt{\frac{m.l}{\text{force}}}$$

Multiplying the expression under the radical sign by l above and below, it becomes

$$\frac{m.l^2}{l \times \text{force}}$$

In mechanics, the sum of the product of the mass of each particle of a rotating body into the square of the distance of the particle from the axis of rotation is called the *moment of inertia* of the body. In the above expression m is the concentrated mass of the pendulum and l is its distance from the point of suspension, therefore, $m.l^2$ is the moment of inertia of the pendulum. Representing this by K, the above becomes

$$\frac{K}{l \times \text{force}}$$

In the case of a needle in a magnetic field, the force is $m.H$, m now representing the strength of the pole of the needle, and the above may be written

$$\frac{K}{l.m.H}$$

But (Par. 130) $l.m$ is the magnetic moment of the needle, hence the expression becomes

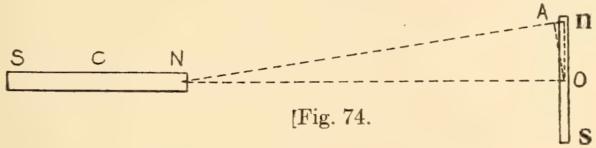
$$\frac{K}{M.H}$$

The expression for the time of oscillation may therefore be written

$$T = 2\pi\sqrt{\frac{K}{M.H}}$$

In this, T can be determined by observation and K by calculation or by experiment.* We therefore have an equation involving two unknown quantities M and H, one of which, H, we wish to determine. If we can obtain another expression involving these same two quantities, we may by combination determine H. The obtaining of this second expression is explained in the two following paragraphs.

149. Turning Moment of One Magnet upon Another.—Let ns (Fig. 74) be a small magnetic needle, its center lying on the prolongation of the axis of the magnet NS and its length perpendicular to this prolongation. Let m' be the strength of the poles



[Fig. 74.

of ns and let m be that of the poles of NS. Let the distance between the poles of ns be $2l$ and that between the poles of NS be $2L$. Let $OC = D$ and $Nn = d$. From the figure $d = \sqrt{l^2 + (D-L)^2}$. The repulsion between N and n is $m.m'/d^2$ and the moment of this force upon ns is

$$\frac{m.m'}{d^2} \times OA$$

The triangles NOn and OnA are similar since they are both right angled and have a common angle NnO, hence

$$OA : On = NO : Nn$$

Hence
$$OA = \frac{On \times NO}{Nn}$$
$$= \frac{l(D-L)}{d}$$
$$= \frac{l\,(D-L)}{\sqrt{l^2 + (D-L)^2}}$$

* The moment of inertia of a bar magnet is

$$K = \left(\frac{(\text{length})^2 + (\text{breadth})^2}{12}\right) \times \text{mass}$$

that of a cylindrical magnet is

$$K = \left(\frac{(\text{length})^2}{12} + \frac{(\text{radius})^2}{4}\right) \times \text{mass}$$

Hence the above moment is

$$\frac{m \, . \, m' . \, l \, (D - L)}{[l^2 + (D - L)^2]^{\frac{3}{2}}}$$

The moment of N on s is the same and the total moment due to N is

$$\frac{2 \, m \, . \, m' . \, l \, (D - L)}{[l^2 + (D - L)^2]^{\frac{3}{2}}}$$

The moment due to S is found in the same manner and may be obtained direct from the preceding expression by substituting $D+L$ for $D-L$. Since it acts in the opposite direction to that due to N, the resultant component is

$$\frac{2 . m . m' . l \, (D - L)}{[l^2 + (D - L)^2]^{\frac{3}{2}}} - \frac{2 . m . m' . l \, (D + L)}{[l^2 + (D + L)^2]^{\frac{3}{2}}}$$

If ns be very small, l^2 can be neglected in comparison to $(D-L)^2$, and consequently also in comparison to $(D+L)^2$, and the above expression can be written

$$2 . m . m' . l \left\{ \frac{l}{(D - L)^2} - \frac{l}{(D + L)^2} \right\}$$

or

$$8 . m . m' . l \left\{ \frac{DL}{(D^2 - L^2)^2} \right\}$$

Finally, if the distance between the two magnets be so great that we may neglect L^2 as compared to D^2, the foregoing becomes

$$\frac{8 \, m . m' . l . L}{D^3}$$

But $2mL$ is the magnetic moment of NS and $2m'l$ is that of ns, hence the turning moment reduces to

$$\frac{2 \, M . \, M'}{D^3}$$

If, in accordance with what we have assumed above, ns be very small in comparison to D, the field about ns is uniform, and if ns be deflected through an angle δ, the turning moment becomes

$$\frac{2 \, M . M'}{D^3} . \cos \delta$$

150. Measurement of Strength of Magnetic Field.—From Par. 130 we have seen that if a needle of magnetic moment M' be placed in a field of strength H and be deflected through an angle δ, the moment which tends to restore it to the meridian is

$$M' . H . \sin \delta$$

and from the preceding paragraph we have seen that when such a needle, *ns*, and a magnet, *NS*, are placed in the relative positions

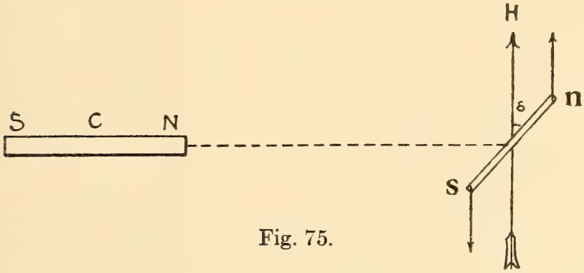


Fig. 75.

as shown in Fig. 75, the turning moment due to the magnet is

$$\frac{2\,M\,.\,M'}{D^3}\,.\,\cos\delta$$

When equilibrium is reached these two moments are equal,

hence $$\frac{2\,M\,.\,M'}{D^3}\,.\,\cos\delta = M'\,.\,H\,.\sin\delta$$

hence $$\frac{M}{H} = \frac{D^3}{2}\,.\,\tan\delta$$

D and δ may be measured directly and we thus obtain a second expression involving M and H, which, when combined with the expression deduced in Par. 148, enables us to determine H.

The needle *ns* being very small, a graduated circle over which its ends might travel could not be read with much accuracy, therefore, the angle δ is usually determined by observing the movement over a scale of a beam of light reflected from a tiny mirror attached to the needle. This method of reading the deflection of a needle will be more fully explained in the description of the mirror galvanometer (Par. 377).

CHAPTER 15.

THEORY OF MAGNETISM.

151. Magnetism.—As in the case of electricity, we must at the outset admit that we do not know what magnetism is. It is not matter yet in its manifestations it must always be associated with matter. Gilbert called attention to the fact that with one magnet we could make hundreds of others and yet the strength and weight of the original magnet would be unaltered. We can conceive of no form of matter which could thus be dipped out or drawn from indefinitely and yet the original source of supply be undiminished. It is not electricity. A charged body placed in a magnetic field is not on account of its charge acted upon in any different way, nor is a magnetized body placed in an electric field attracted or repelled in any different manner on account of its magnetism. An electric current does however produce certain magnetic effects, and mechanical energy expended in moving or varying magnetic fields may be transmuted into electric energy. We may say then that with electric currents we can produce magnetism and from magnetism we can produce electric currents.

Magnetic forces pass with equal ease through the hardest substances, the thinnest gases and a vacuum. The medium concerned in this propagation is therefore considered to be the ether.

152. Molecular Magnetism.—We have already seen (Par. 111) that if a bar magnet be broken across, one surface of this fracture will be of north polarity, the other of south, and this no matter at what point the bar be broken nor how the line of fracture runs across. If these portions be again broken, the resulting fragments will still possess polarity and even if the final result be dust the ultimate particles will still be little magnets. The inevitable conclusion is that the individual molecules are themselves magnets and that they are arranged with their like poles all pointing in one direction. This affords a satisfactory explanation of the fact that the free magnetism resides mainly at the poles, for at any intermediate cross-section the magnetism on one side of the section is

exactly balanced and neutralized by that on the other, consequently, the end layers are the only ones free to cause external effect.

153. Ewing's Theory.—Two hypotheses may be advanced to account for the arrangement of the molecular magnets. First, before a steel bar is magnetized its molecules are unmagnetized and the act of magnetizing imparts to them their magnetism and arrangement. This throws but little light on the matter. Second, the molecules possess magnetism as an inherent property and are always magnetized but are indiscriminately arranged or rather are arranged in little groups satisfying each other's polarity and thus neutralizing each other's magnetic effects and producing little or no external magnetism. The act of magnetizing simply turns these molecules until their like poles point in one direction. The maximum effect would be produced when all of the molecules had been turned and the magnet is then said to be *saturated.*

This theory was first advanced by Weber and later elaborated by Ewing, whose name it bears. The latter showed that it satisfactorily accounts for the known facts of magnetization, especially, as will be seen later (Par. 395), for the varying rate of change of magnetism accompanying a constant rate of change of the magnetizing force. Certain corroborating phenomena are described in the following paragraphs.

154. Magnetization is Accompanied by Molecular Movement. (a) If a small glass tube be filled with steel filings and then subjected to magnetization, the filings will be seen to arrange themselves end to end and thereafter the tube will act as a magnet. This is thought to be analogous to what takes place among the molecules of a magnetic body during magnetization. If the filings be shaken up and disarranged the magnetism disappears.

(b) When an iron bar is suddenly magnetized by an electric current a metallic clink is heard. This could be produced only by a vibration among the molecules of the bar.

(c) When an iron bar is rapidly magnetized and demagnetized it grows hot. This heat could be produced only by internal movement among the molecules.

(d) Magnetization is accompanied by a change in the dimensions of the magnetic substance. An iron bar strongly magnetized increases 1/720,000 of its length but if still more strongly magnetized it contracts again. A bar of cobalt at first diminishes and

then increases. Nickel diminishes from the first. Later investigations show that iron and steel contract along the lines of magnetization and expand across these lines. The change in dimensions must result from movement among the molecules.

155. Freedom of Molecular Movement Facilitates Magnetization.—When a magnetic body is placed in a magnetic field, we may consider that a force is tugging at each of the little molecular magnets endeavoring to turn them so that their like poles will point in one direction. This turning is impeded by the crowding of the molecules or by what may be designated molecular friction. If this crowding or friction be relieved in any way, as by vibration, by heating or by liquefaction, magnetization is rendered much easier.

It has long been known that hard steel is very much more difficult to magnetize than soft iron but that once magnetized it retains its magnetism much better, or, as this last is usually expressed, its *retentivity* is much greater than that of iron. We may consider that the molecules of the rigid steel offer more resistance to turning than do those of the soft iron, and on account of this same rigidity they remain more persistently in the position into which they have been turned. A piece of pure iron loses its magnetism as soon as the magnetizing force is discontinued. The iron generally used in electrical machinery is not absolutely pure and some traces of magnetism persist after the cessation of the magnetizing force. This *residual magnetism* plays an important part in certain electric machines.

156. Magnetization Facilitated by Vibration.—Vibration, however produced, is favorable to loosening up the molecules of a body. Gilbert discovered that if an iron bar held in the magnetic meridian be struck with a hammer it becomes a magnet, but no such effect is produced if the bar be held crosswise.

The steel columns employed so largely in modern buildings are all planted in the meridian and in accordance with the principle stated in Par. 143 are penetrated lengthwise by the lines of force of the earth's field (see Fig. 76). They are subjected to continual vibration and therefore become in course of time highly magnetized. The lower ends of all such columns in the northern hemisphere, being the ends from which the lines of force emerge, are north poles (Par. 142). This also explains the fact which

several centuries ago caused great wonderment, that is, that iron crosses on church steeples and the iron rods of weather vanes are often found to have acquired magnetic properties.

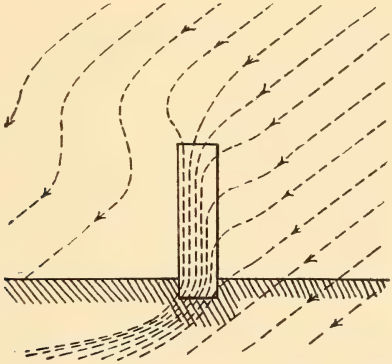


Fig. 76.

157. Loss of Magnetization Facilitated by Vibration.—Reflection will show that the foregoing principle works both ways, that is, if an iron bar be placed in the meridian and jarred it acquires magnetism; on the other hand, if a magnet not in the meridian be jarred it loses its magnetism. Great care must then be observed in handling magnets not to jar them by striking or by dropping or otherwise, as under such conditions they deteriorate rapidly. Even if a magnet be in the meridian when jarred, it loses strength for the earth's field, being much weaker than the magnetic field originally used in making the magnet, can not hold in position all of the molecules when they begin to vibrate.

158. Effect of Heat.—It is known that when a body is heated its molecules are put into more or less violent vibration. When a magnetic body is heated to a red heat the vibrations reach such a pitch that they are no longer controlled by magnetic force, that is, its molecules are dancing about so that the magnetic force can no longer pull them into line, therefore, at a red heat magnets lose their magnetism and magnetic bodies are no longer attracted and can no longer be magnetized. If, however, such heated bodies be allowed to cool in a magnetic field, the molecules as they quiet down take positions in accordance with the magnetic force and the result is that the bodies acquire magnetism. Gilbert found that bars of iron or steel heated to redness and allowed to cool in the

meridian became magnets. If molten cast-iron be run into a mould and cools and solidifies in a strong magnetic field it acquires magnetism.

159. Magnetization Facilitated by Solution.—When a body is in solution it is separated into its individual molecules and these have great freedom of movement. Deposition from solution must take place molecule by molecule. Should a magnetic substance in solution be deposited while in a magnetic field, the molecules should have no trouble in arranging themselves and the resulting body, if our hypothesis be correct, should exhibit marked magnetic properties. This has been confirmed experimentally by depositing iron electrolytically (as electroplating is done) in a strong field.

We are taught by geology that beds of iron ore are accumulated through chemical processes by deposition from solution. Since this deposition takes place in the earth's magnetic field, this affords a reasonable explanation of the occurrence of the lodestone. Gilbert, although he wrote long before the atomic theory had been advanced, evidently had this thought in mind and in Chapter II, Book III of his work gives the following significant experiment. "We once had chiselled and dug out of its vein a lodestone twenty pounds in weight, having first noted and marked its extremities; then after it had been taken out of the earth we placed it on a float in water so it could freely turn about; straightway, that extremity of it which in the mine looked north turned to the north in water and after a while there abode."

CHAPTER 16.

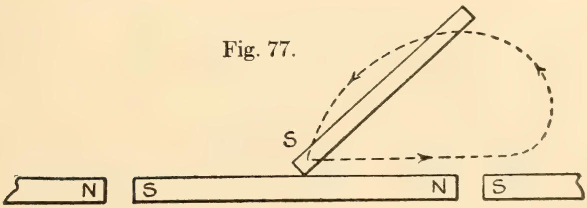
MANUFACTURE OF MAGNETS.

160. Most Suitable Metal for Making Magnets.—We have stated that soft iron is far more easily magnetized than steel but on the other hand its retentivity, or power of retaining imparted magnetism, is very slight. The best permanent magnets are made from glass-hard steel, that is, steel which has been heated to a bright red and then plunged into cold water. Certain metals alloyed with steel improve its magnetic properties and others injure or destroy them. An alloy of tungsten produces magnets of great retentivity, while an alloy of manganese can hardly be magnetized at all and has been proposed for structural work in electrical laboratories.

161. Principle of Manufacture of Magnets.—We have seen that in theory a bar of steel becomes a magnet when its molecules have been turned so that their poles lie in one direction. The manufacture of magnets is based upon this theory, and just as we get the individual hairs of a piece of fur to lie in one direction by combing or by brushing or by blowing upon the fur, so we, in a sense, comb or brush or blow the molecules of the bar of which we wish to make a magnet. The principle of these processes is the same; they differ merely in details of execution.

162. Magnetization by Single Touch.—In this method the bar to be magnetized is placed horizontal and preferably with its ends

Fig. 77.



resting upon or just in front of opposite poles of two magnets as shown in Fig. 77. It may then be stroked from end to end with a magnet, using the pole of the same kind as that near which the

last touched end of the bar is resting. A better way is to begin at the middle of the bar and stroke it, the stroking magnet following the path shown by the dotted line in the figure, then reverse ends of the stroking magnet and stroke the other half of the bar in the opposite direction. Finally, turn the bar over and repeat the strokings upon the other side. The point last touched is of opposite polarity to the pole with which it is stroked.

163. Magnetization by Divided Touch.—This method is in principle precisely the same as the foregoing and differs only in that two magnets are used in the strokings. The opposite poles of the two magnets are placed at the center of the bar (Fig. 78) and

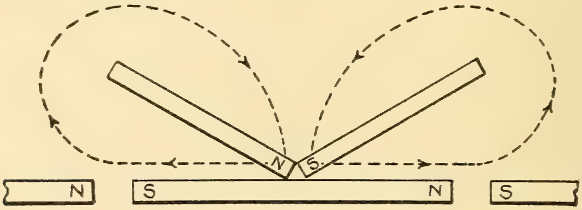


Fig. 78.

are then drawn apart, the magnets following the paths shown by the dotted lines in the figure. After eight or ten strokes, the bar is turned over and the other side is stroked. Sometimes a block of wood is placed between the poles of the stroking magnets and they are held against this and slid back and forth along the bar, being finally removed at the center of the bar. This last is called the method by *double touch.*

164. Magnetization by an Electric Current.—The best method of magnetization is by means of an electric current. An insulated wire is coiled around the bar to be magnetized and a current is

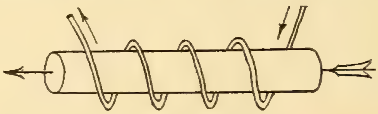


Fig. 79.

sent through the wire. As result of this treatment the bar becomes a magnet. A full explanation of this can not be given without anticipating certain principles which have not yet been developed and it must therefore be deferred until later, but it can be stated that if a current flows through the coiled wire as shown diagrammatically in Fig. 79 and in the direction of the small

arrowheads, there will be produced inside of the hollow of the coil a magnetic field whose lines of force run as shown by the large arrow, that is, inside of this hollow space around which the coil is wrapped lines of force will run like a draught runs up a chimney and an iron or steel bar placed in this field will have its molecules all swept or "blown" into a common direction, and in the case of steel a great part of them retain this new position. They really turn in accordance with the principle given in the latter part of Par. 144.

165. Consequent Poles.—If a magnet be touched at some point between its poles by a second magnet, the molecules around that point may be disarranged to the extent of producing poles intermediate to the original ones. Such intermediate poles are called *consequent poles*. They are usually the result of some accident or error in the process of magnetization. Their presence may be detected by exploring the field about the magnet by means of a small compass needle or by use of the magnetic figures. They may be intentionally produced by stroking the bar in a different manner from that prescribed or, in the electric method, by wrapping a portion of the coil in opposite direction to the rest. If, for example, a steel knitting needle be stroked with the north pole of a magnet, the strokes beginning at each end and terminating at the center, the needle will be found to have a north pole at each end and a south pole at the center.

166. Magnetization Largely Confined to Outer Layers of Magnet.—The process of magnetization effects the outer layers of the steel bar more than it does the interior portions. This may be shown in several ways. If a magnet be placed in acid and its outer layer be dissolved off, its magnetism will be found to decrease at a more rapid rate than its mass. Again, if a number of thin flat pieces of steel, as for example blades of table knives, be bound up in a bundle and magnetized, it will be found when the bundle is taken apart that those on the outside of the bundle are much more strongly magnetized than those on the interior. There are three reasons for this. First, when the magnetism is imparted by stroking, the outer layers act as a magnetic screen for the inner layers (Par. 143), that is, the teeth of our magnetic comb do not reach down into the deeper layers of molecules. Second, when the electric method is used, the field is stronger close to the wire than

it is in the center of the coil so the outer portions of the bar become more highly magnetized. Third, the outer layers act by induction upon the inner layers and tend to produce in them an opposite polarity, thereby weakening them. This last may be shown as follows. If three similar steel bars be magnetized equally and then tied together and used as a magnet, when they are again taken apart the inner one will be found to be weaker than the outer ones.

Since thin ribbon-like bars can be more thoroughly magnetized than thicker ones, the most powerful magnets are made of a

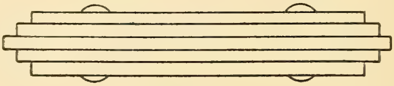


Fig. 80.

number of these separately magnetized and then bound together. Such *laminated magnets* are powerful but, for reasons just explained, their power does not increase in direct proportion to the number of laminæ or strips. It is found best to have the interior layers project, as shown in Fig. 80, slightly beyond the outer layers. Sometimes the ends of such magnets are inserted into soft iron pole pieces.

167. Aging of Magnets.—Even if they are handled carefully and not jarred, magnets grow weaker with time, most probably on account of the inductive effect mentioned in the preceding paragraph, and may take several years to attain a constant state. In certain electrical measuring instruments in which magnets are used, it is of the utmost importance that these retain a constant strength. Such magnets can be put through an artificial process of aging by which the constant state can be reached quickly. This treatment consists in exposing the newly made magnet to a current of steam for some 20 hours, then remagnetizing it and exposing it again to steam for ten hours.

CHAPTER 17.

TERRESTRIAL MAGNETISM.

168. Location of Earth's Magnetic Poles.—We have seen that Gilbert made the discovery that the earth itself is a magnet. Starting from this point we are naturally led to enquire where are its poles, what is the direction and intensity of its field at different localities and, finally, why is it a magnet.

In experimenting with his spherical lodestones or *terrellas* Gilbert located their poles as follows. He laid a short piece of iron wire upon the surface of the terrella near its equatorial region. The wire became a magnet by induction and turning on the polished surface of the sphere, as if on a pivot, pointed toward the pole. The direction in which the wire pointed was marked with chalk and the wire was then shifted to some other position and its direction again marked. These chalk lines prolonged intersected at the poles. Were the earth a homogeneous sphere its magnetic poles could probably be located similarly, that is, the direction in which a magnetic needle pointed at various localities could be determined and these direction lines prolonged would intersect at the poles, but the earth is far from being such a sphere and a series of needles distributed around a parallel of latitude would indicate directions not even approximately converging.

If we should start at any point with a magnetic needle and move it continually in the direction of its length, just as is done in the method described in Par. 141, we would not trace the arc of a great circle but a curved line which if prolonged in both directions would eventually pass through the magnetic poles. Two such lines would by their intersections locate the poles.

Figure 81 represents a portion of the northern hemisphere with a series of such curves which begin at points along the equator ten degrees apart. It will be noted that the north magnetic pole does not coincide with the geographical pole and is in fact nearly twenty degrees, or some twelve hundred miles, south of the latter. It was discovered by Sir J. C. Ross during the arctic expedition of 1829–33 and is located on the Island of Boothia Felix, north of

Hudson Bay, in latitude 70° 5′ north and longitude 96° 43′ west. The south magnetic pole has not been reached. It is located in the antarctic regions in approximately latitude 73° 30′ south and longitude 147° 30′ east, whence it is seen that the two magnetic poles are not at the extremities of a diameter of the earth.

It follows from the foregoing that what we have designated in the preceding pages as the *magnetic meridian*, or the vertical plane through the axis of the poised needle, does not in general pass through the magnetic poles and furthermore changes its direction from point to point.

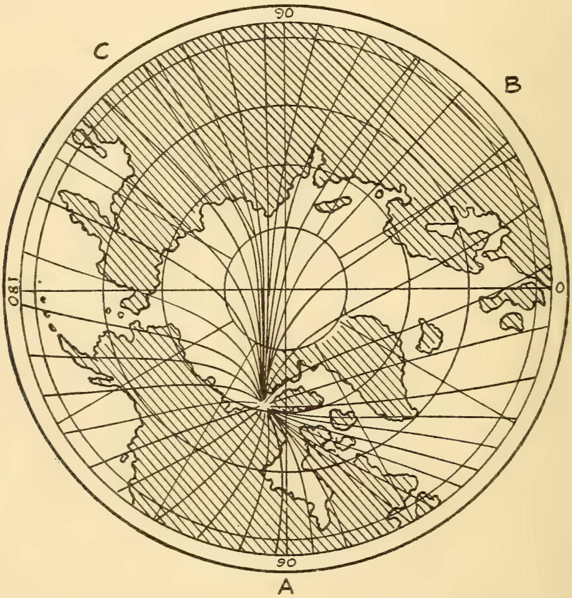


Fig. 81.

169. Magnetic Declination.—A study of Fig. 81 will show that within its limits there are three and only three regions in which the needle points to the geographic north pole. These regions, marked *A*, *B* and *C* on the map, are the western side of Hudson Bay, the vicinity of St. Petersburg in Russia and the eastern portion of Siberia. At other points the needle points either to the east or to the west of the true meridian. Thus, along a parallel from St. Petersburg to Hudson Bay the needle points to the west of the meridian, while continuing from Hudson Bay to Siberia it

points to the east. This deviation of the needle from the true meridian is called the *magnetic declination*. We shall see later that the declination at any locality is slowly changing. In 1905 along a line through Charleston, S. C., Cincinnati, Ohio, Lansing, Michigan, and thence across Lake Superior the needle pointed true north, while in the northeast corner of Maine the declination was 21° west and in the extreme northwest of the State of Washington it was 24° east. The magnetic declination is sometimes called the *magnetic variation*, but there are several kinds of magnetic variation and the term declination is to be preferred.

170. Isogonic Chart.—It is of the utmost importance that navigators should know the magnetic declination at whatever point their vessel may be. For example, if a vessel be off the mouth of the Columbia River and its captain wishes to sail due north, he must steer by compass 22° to the west of north. A knowledge of the declination is also required by surveyors. Information of this kind is often given graphically in so-called *magnetic maps*. One of these, for the year 1905, is shown in Fig. 82 and is prepared by joining by a continuous line all those points at which in that year the declination was the same. The resulting curves are called *isogonic lines* (lines of equal declination) and the map is called an *isogonic chart*. The heavy lines are the *agonic lines*, or lines of no declination; the lighter lines are those of westerly declination; the dotted lines are those of easterly declination. It will be noted that there is one agonic line completely encircling the earth (shown as two in the Mercator's projection used in the chart) and a second one embracing an elliptical area in eastern Asia. This last is called the *Siberian oval*.

Figure 83 is the isogonic chart for the United States for the year 1905 taken from the report of the Superintendent of the Coast Survey for 1906.

171. Magnetic Dip.—In manufacturing needles for compasses and surveying instruments, they are shaped and finished off while the metal is soft, after which they are tempered glass hard and then magnetized. They are carefully balanced before being tempered for afterwards they are too hard to file and grinding would injure their magnetization. Robert Norman, an instrument maker of London, noticed in 1576 that no matter how carefully he balanced his needles they were thrown out of balance after

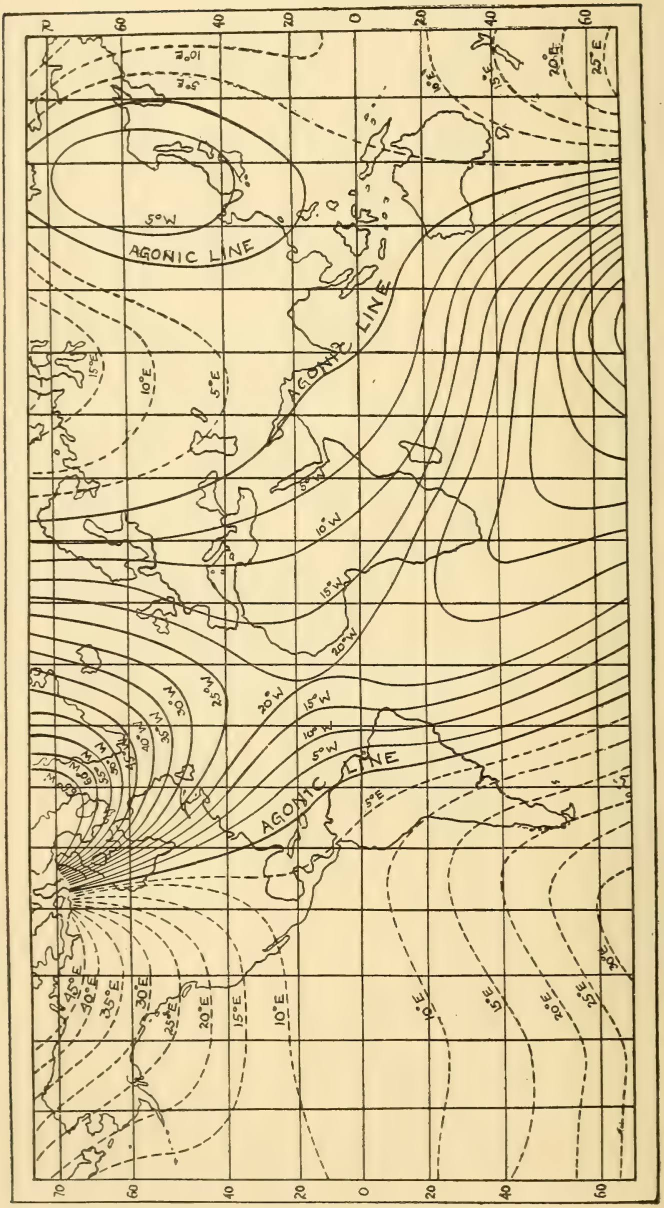


Fig. 82 Isogonic Chart for 1905

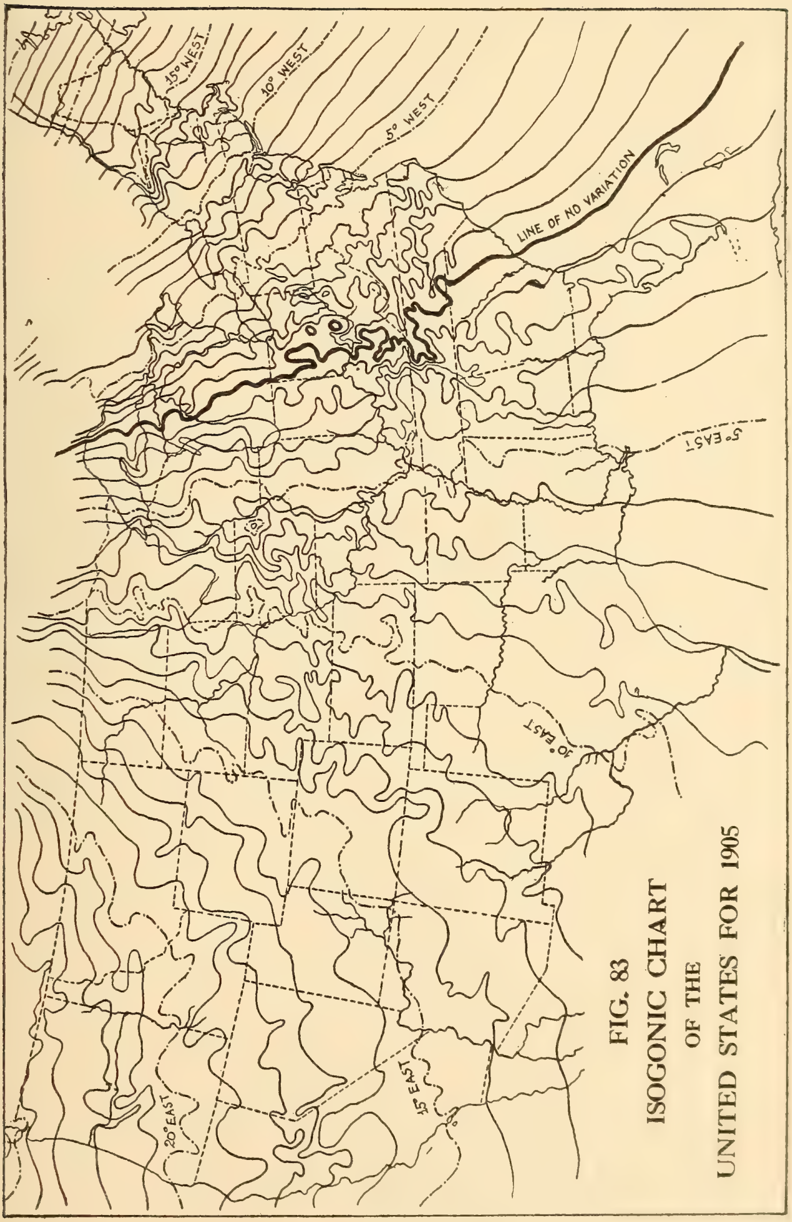


FIG. 83
ISOGONIC CHART
OF THE
UNITED STATES FOR 1905

being magnetized and invariably the north end appeared to be the heavier so that he was compelled to restore the balance by sticking a small piece of wax under the south end. Being angered one day, or as he expressed it "being stroken into some choler," by ruining a needle upon which he had expended a good deal of labor and whose balance he endeavored to restore by cutting off a small piece from the north end, he began to reflect upon the matter and finally made a needle which, before being magnetized, balanced on horizontal trunnions. After magnetization, the north end dipped down until the needle stood at an angle of 72° with the horizontal plane. The angle which the axis of such a needle makes with the horizontal plane is called the *magnetic dip* or *magnetic inclination.* The explanation of the magnetic dip is as follows: The lines of force of the earth's magnetic field not being circles and its poles being at some unknown depth, these lines of force are not parallel to the surface but penetrate it, in other words, they are inclined to the plane of the horizon. A magnetic needle free to move in a vertical as well as in a horizontal plane will place itself tangent to the lines of force and the angle which these lines make with the horizontal plane is the magnetic dip. As in the case of the declination, the dip is slowly changing.

The lack of balance in the needles of engineering instruments is frequently corrected by wrapping a fine silver wire about the south end of the needle.

172. Dipping Needle.—A needle arranged to measure the angle of dip is called a *dipping needle.* One of these is shown in Fig. 84. The needle is ten or twelve inches long and is mounted upon a steel knife-blade axis resting upon polished agate bearings. For still more delicate observations an instrument is used in which the needle is suspended at the center of a complete graduated circle which may be rotated about a vertical axis and which, like a surveyor's transit, is furnished with a slow motion screw by which it may be accurately placed in the meridian. The angles are read by verniers and microscopes and observations are multiplied so as to eliminate instrumental errors. For example, to correct for the error due to the line joining the 90° marks at the top and bottom of the graduated circle not being vertical, the angle marked by the needle is read, the circle is then rotated 180° around its vertical axis, the angle is again read and the mean of these observations is taken. To correct for the error due to the

axis of suspension of the needle not corresponding with the center of the circle, both ends of the needle are read and the mean of these readings is taken. To correct for error due to the magnetic axis of the needle not corresponding with its geometric axis, the above observations are repeated with the needle reversed from back to front and these readings are combined with the former ones. To correct for error due to lack of mechanical balance in the needle, observations are made, the needle is then demagnetized and remagnetized in the opposite direction, placed in position, a second set of observations taken and the means of the two sets combined. There are observed other refinements not necessary to mention here.

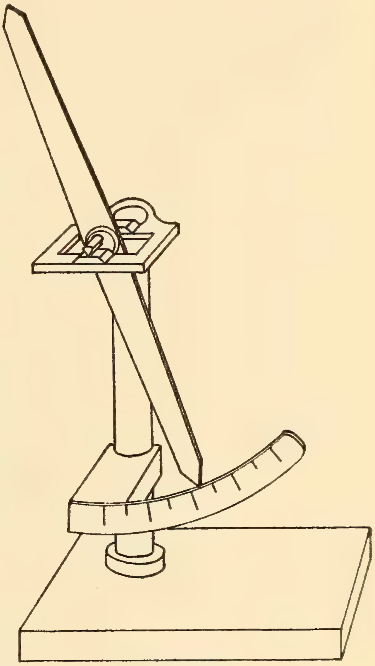


Fig. 84.

173. Isoclinic Chart.—Figure 85 represents a section of the earth by a plane passing through its axis and the north magnetic pole. The arrows represent the position of the dipping needle at the corresponding points. At the magnetic poles the dip is 90° or the

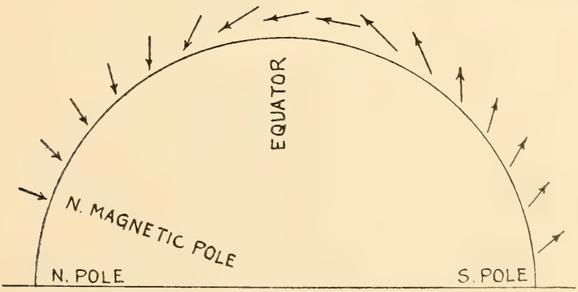


Fig. 85.

needle stands vertical and this was one of the observations by means of which the north magnetic pole was located. Along

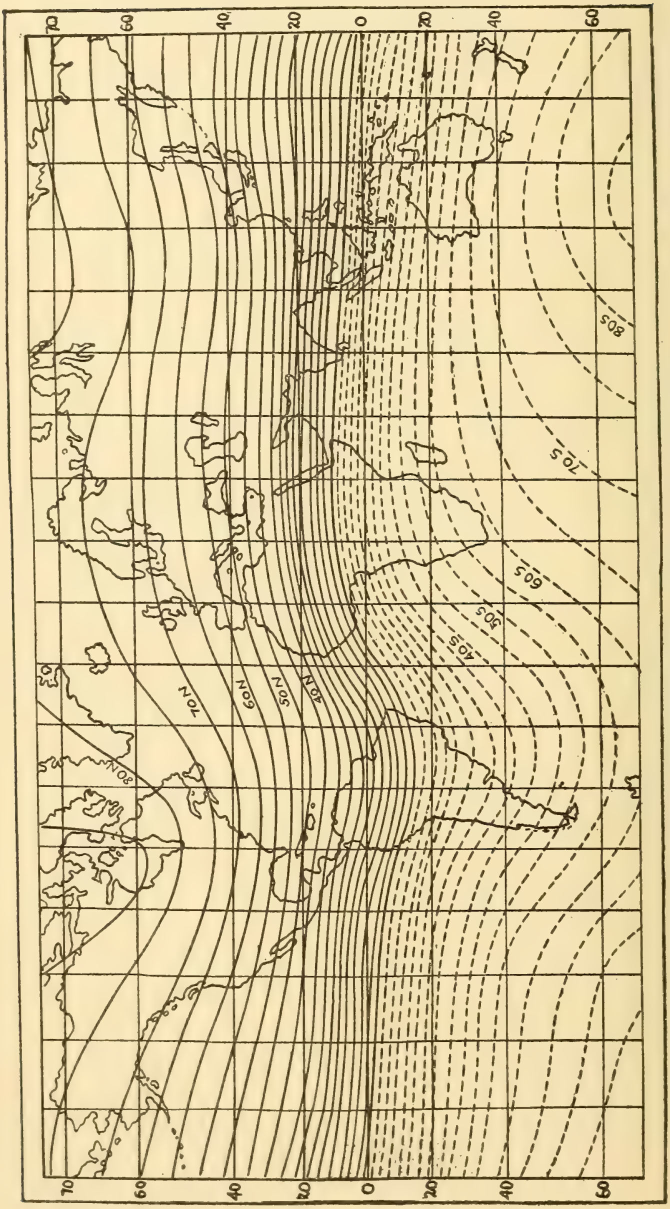


Fig. 86 Isoclinic Chart for 1905

the magnetic equator, which in the western hemisphere lies south of the geographical equator, the dip is zero or the needle lies horizontal. Lines connecting those points on the earth's surface where the dip is the same are called *isoclinic lines*. An examination of the *isoclinic chart*, Fig. 86, will show that these lines run generally east and west but curve irregularly and are not parallel.

174. Magnetic Intensity.—The strength of the earth's field, or the *magnetic intensity*, can not easily be measured directly but by the method outlined in Pars. 148, 149 and 150 we may determine its horizontal component, whence, since the total intensity is equal to this horizontal component divided by the cosine of the angle of dip, the total intensity is readily calculated.

Having determined the horizontal component at one point, it may easily be determined at any other by applying the method by oscillations as described in Par. 129. The same magnetic needle is oscillated for the same period of time at the two places and the number of oscillations counted; the horizontal components at the two places are to each other as the square of the number of oscillations executed in equal intervals of time.

The horizontal component is greatest along the magnetic equator but varies at different points along this line. It is a maximum over a region embracing a part of India, the Malay Peninsula and the Islands of Borneo and New Guinea, its strength being .38, that is, a unit pole placed in the earth's field in this region would be urged in a horizontal direction with a force of .38 dynes. At the magnetic poles the horizontal component is zero and near these points the total intensity is determined from the vertical component instead of from the horizontal. The total intensity increases from the equator towards the magnetic poles. It is however not a maximum at these poles but in each hemisphere at two points or magnetic foci. In the northern hemisphere one of these points is just south of Hudson Bay, the other is in north central Siberia. In the southern hemisphere both points are to the south of Australia. The maximum value in the northern hemisphere is about .65 and in the southern about .70. Just as with the declination and the dip, the total intensity is found to be slowly changing.

Lines connecting points of equal horizontal intensity or of equal total intensity are called *isodynamic lines*, and *isodynamic charts*

are prepared in a similar manner to the isogonic and isoclinic charts.

175. Magnetic Elements.—The declination, the dip and the magnetic intensity at any given point are termed the magnetic elements of that point. As observations are multiplied, our knowledge of these elements and of the laws of their variation correspondingly increases. In the report of the Superintendent of the Coast and Geodetic Survey for 1906, data is presented from accurate observations at 3500 stations, or from every 30 miles square of the U. S. territory. The following table, extracted from this report, gives the declination, dip and horizontal intensity at various localities as determined by observations made in the year ending June 30, 1906.

TABLE OF MAGNETIC ELEMENTS.

1905–1906.

Locality	Declination	Dip	Hor. Intensity
Albany, N. Y	11° 08′ W	73° 50′	.16939
Ann Arbor, Mich	2° 01′ W	72° 51′	.18248
Baltimore, Md	5° 55′ W	70° 42′	.19560
Bangor, Me	17° 28′ W	74° 50′	.15715
Columbia, S. C	0° 0′	65° 35′	.23791
Fargo, N. D	11° 30′ E	75° 35′	.15731
Galveston, Tex	7° 28′ E	58° 37′	.28404
Green River, Utah	15° 40′ E	66° 08′	.23476
Helena, Mont	19° 49′ E	72° 08′	.18548
Honolulu	10° 35′ E	39° 20′	.29566
Joliet, Ill	2° 52′ E	72° 13′	.18857
Key West, Fla	2° 31′ E	55° 03′	.29404
Los Angeles, Cal	15° 14′ E	59° 31′	.26902
Memphis, Tenn	5° 30′ E	65° 47′	.24080
Montreal, Can	14° 40′ W	75° 38′	.15122
New York, N. Y	9° 08′ W	72° 02′	.18690
Philadelphia, Pa	7° 45′ W	71° 04′	.19361
Portland, Ore	22° 44′ E	68° 39′	.21754
San Francisco, Cal	17° 00′ E	62° 43′	.24898
Silver City, N. M	12° 46′ E	59° 51′	.27301
Sitka, Alaska	30° 01′ E	74° 42′	.15494
Washington, D. C	4° 34′ W	70° 28′	.20022

176. Variation of the Magnetic Elements.—The magnetic elements at any locality are far from being constant. They pass through cycles of variation with periods of years, through others with the seasons, still others in each twenty-four hours and finally others at irregular intervals. These variations may therefore be classed as periodic and irregular, the first class embracing the secular, the annual and the diurnal variations. Although all of the elements vary, it is only to the variation in declination, and furthermore only to the secular variation of this element, that any practical importance attaches.

177. Secular Change in Declination and Dip.—For over 300 years it has been noted that the declination and dip were slowly changing. In 1580 at London the declination was 11° 17′ east and was decreasing so that in 1657 the needle pointed true north. The

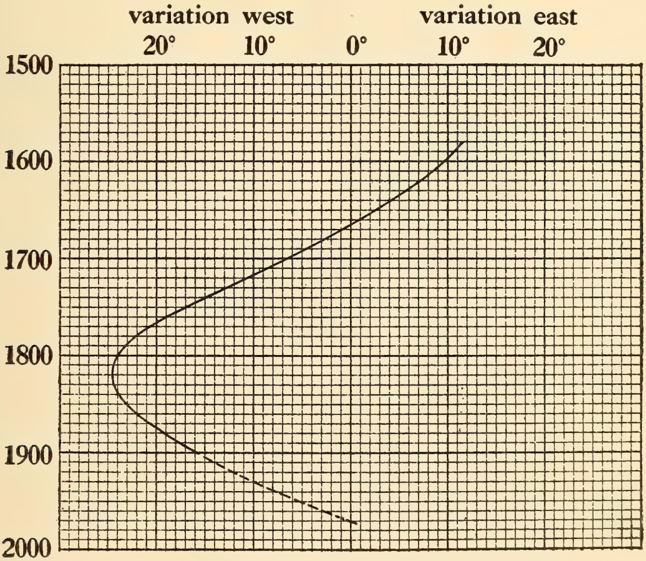


Fig. 87.

movement continued in the same direction until in 1816 a maximum westerly declination of 24° 30′ was reached and retrogression began. In 1900 the declination was 16° 16′ west. This movement is shown graphically by the curve in Fig. 87. In about the year 1976, or some 320 years since the needle last pointed true north, it should again point north, but since the curve shows that the

westerly variation is greater than the easterly, the period of a complete cycle will not be known until the needle moving westward again points to 11° 17′ as in 1580.

The change in declination is accompanied by a change in dip, although the angular range of the dip is much less than that of the declination. At London the total variation in dip has been 7° 33′ while that in declination has been 35° 47′; however, the range in dip is still increasing which is not true of the range in declination. To the eye of the observer placed at the pivot of the needle in London, the north pole of the needle would appear to have traced in a clockwise direction since 1580 about two-thirds of a more or less irregular and flattened oval. This fact, taken alone, would seem to indicate that the north magnetic pole viewed from some point outside of the earth is slowly rotating in a counter-clockwise direction around some undetermined point in the northern regions.

As we travel around a parallel of latitude we find, as has already been shown, that the declination differs at different points and is changing. We also find that both the direction of change and the rate of change vary from place to place. Thus (Fig. 83) across the northeast of Maine in 1905 the declination was 20° west and was increasing 4′ per year; along the agonic line through South Carolina the declination was varying westerly 2′ per year; along a line through Alabama, Illinois and Wisconsin the declination was stationary; along the Mississippi Valley it was increasing easterly 1′ per year; along the crest of the Rocky Mountains it was increasing easterly 3′ per year and on the coast of Oregon, where the declination was 20° east, it was increasing easterly 4′ per year. These changes indicated that the isogonic lines were slowly crowding in upon the agonic line and that the north magnetic pole was moving southward. As observations increase we may be able in time to speak with more certainty of these movements.

178. Diurnal Change in Declination.—The magnetic elements are subject to slight daily changes and these changes are more satisfactorily studied by means of self recording instruments. For instance, a needle suspended by a delicate silk fibre carries a small concave mirror upon which falls a beam of light from an electric bulb. This mirror reflects a brilliant spot upon a roll of photographic paper which is unwound at a known rate by clockwork. As long as the needle is motionless, the trace of the spot

upon the sensitized paper is a straight line but any movement of the needle produces a curve. Fig. 88 represents such a record made near London in 1900. At 8 A. M. the declination was least but increased steadily until about 2 P. M. when it reached a maximum. It then decreased until 8 P. M. when it was nearly stationary for about an hour and then began to decrease again and continued until 8 A. M. A similar record would be made at all points, no matter whether the local declination be east or west, but the direction of movement in the southern hemisphere is the reverse of that in the northern. Along the equator the daily

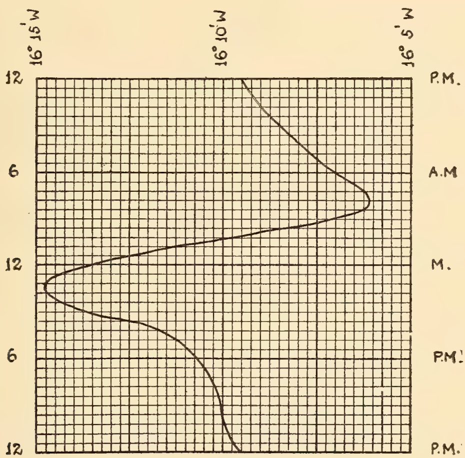


Fig. 88.

range of the needle does not exceed 4′ while in northern Europe it reaches 15′.

The dip, registered in a similar manner, is found to be about 5′ greater in the morning than in the afternoon.

179. Annual Change in Declination. — If the average declination for each month be obtained from the self-registering instruments and these monthly averages be compared among themselves, it will be seen that in the northern hemisphere the needle moves to the west from May to September and to the east from September to May. In the southern hemisphere these movements are reversed and in either case they are but slight.

180. Magnetic Storms.—It has long been known that in addition to the periodic variations described in the preceding para-

graphs, magnetic needles are not infrequently subject to other variations occurring at irregular intervals. If a needle be observed at such a time it will be seen to waver or tremble and to fluctuate through an angle varying from a few minutes to one degree and in extreme cases even to two to three degrees. The variation is only momentary but may be often repeated. Such disturbances are called *magnetic storms.* They occur simultaneously at the most distant points and involve all the magnetic elements. Their effects are best studied by means of the curves traced as described in Par. 178. The record instead of being the sinuous curve as in Fig. 88 is jagged and irregular. These storms occur more frequently at night than during the day and are also more frequent in summer than in winter. They are especially marked during auroral displays and it was for a time thought that the two phenomena were related as effect and cause, but it is now held that they have a common cause.

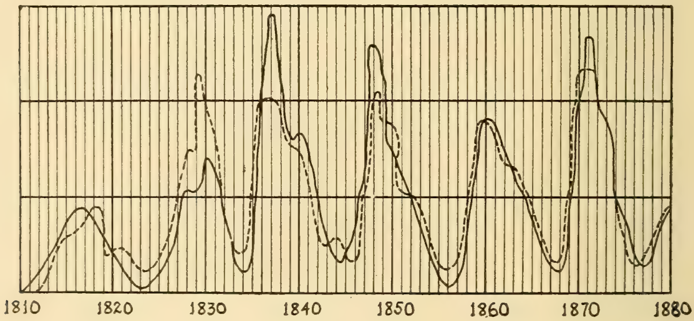


Fig. 89.

In 1852 it was observed that the periods of maximum frequency of magnetic storms coincides with the maximum occurrence of sun spots, both taking place every eleventh year. This coincidence is shown graphically in Fig. 89 in which the full line shows the relative number of sun spots for each year and the broken line the number of magnetic storms. The agreement is too close to be accidental.

181. Theories of the Earth's Magnetism.—There is no accepted theory of the earth's magnetism but since, as we have seen above, its manifestations are periodic in character, these periods corresponding to the diurnal and annual time periods of the earth and

to the eleven year period of the sun spots, the indications are that its source is the sun. The significance of the declination period has not yet been grasped, in fact, as was pointed out (Par. 177), we can not be sure for a number of years to come what is the exact length of this period. Could it be shown that electric currents flowed around the globe from east to west, this, as will be seen in electro-magnetics, would account for the magnetic phenomena and this explanation was advanced and elaborated by Ampere. So-called *earth currents* are known to exist but their direction is along the meridian instead of across it. It is known that electricity is produced both by heat and by evaporation, also that the magnetic properties of bodies are effected by heat, and it is conceivable that the sun as in its apparent motion it sweeps along overhead at the equator at the rate of 1000 miles per hour may produce successive masses of charged vapor which might have an effect similar to a current, and also that the warming of successive portions of the earth's crust may alter its magnetic properties sufficiently to account for the diurnal and seasonal variations. An additional fact which points to this hypothesis is that the isothermal lines, or lines of equal average temperature of the earth's surface, correspond closely in direction with the isoclinal lines.

Faraday, in investigating paramagnetic and diamagnetic bodies, discovered that oxygen is magnetic and that its magnetism increases as it grows colder. He therefore suggested that the oxygen of the atmosphere is naturally magnetic and that the variations produced in its magnetism by the daily and seasonal variations in temperature would afford a satisfactory explanation of the periodic variations of the needle.

Other theories have been advanced but they can not be regarded as much more than speculations.

182. The Mariner's Compass.—In the surveyor's compass, a long, slender needle is pivoted free to rotate within a horizontal circle which is so graduated that the north end of the needle points to the angle which the line of sight of the telescope makes with the magnetic meridian. Since the graduated circle and the telescope rotate together about the vertical axis of the instrument, the needle remaining motionless, the west half of the circle must be marked east and the east half must be marked west.

The mariner's compass (Fig. 90) is differently arranged, the graduated scale being fastened to the needle and rotating with it and hence the interchange of east and west not being necessary. The pointer which indicates the direction in which the vessel is

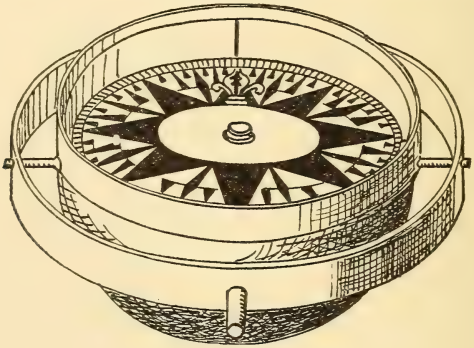


Fig. 90.

sailing is a vertical mark on the inside of the box in which the compass turns. In the compass perfected by Lord Kelvin there turns upon an iridium needle point a central jewelled cup to which is attached by tightly drawn silk threads a thin aluminum ring, six or eight inches in diameter, the whole resembling a wheel of which the cup is the hub and the threads the spokes. Upon the rim is fastened the paper scale divided into the customary 32 "points of the compass," and also with an outer graduation in degrees. The needle proper consists of eight separate needles, slender bars about three inches long, arranged like the rungs of a ladder and fastened to the under side of the silk spokes, being symmetrically placed with respect to the jewelled cup. The compass is contained in a glass-covered, cylindrical copper box, weighted at the bottom and supported on gimbals. To the box itself there are attached two trunnions which rest upon a copper ring concentric with the box. This ring in turn carries two trunnions which are in the same horizontal plane as the first pair and at right angles to them, and these in their turn rest upon a second and outer concentric ring. By this arrangement the compass is kept horizontal no matter how much the ship may roll. In order to slow down the oscillations of the needle, the box is often filled with some thick non-freezing liquid, such as glycerine, and by making a portion of the rim of the compass card hollow, the liquid

will buoy up the card and relieve the pivot of a portion of the weight upon it.

The compass and its box are placed upon a pedestal, called the binnacle, which carries the necessary lamps for reading the compass at night and also supports the magnets and masses of soft iron used in making correction for local disturbances of the needle.

183. Adjustment of Mariner's Compass.—In the construction of vessels the use of iron and steel has largely displaced wood. During the building of a vessel it rests for a relatively long period of time at a constant angle with the earth's field and the continual hammering and vibration to which it is subjected converts it as a whole into a magnet. In addition to this, such vertical columns of steel as the cut water and the stern post become, as explained in Par. 156, magnets whose south poles, for vessels in the northern hemisphere, are at the upper ends and therefore about on a level with the deck upon which the compass stands. When such a vessel is launched the magnetism of the hull may entirely vitiate the indications of the compass. However, it has been found that by means of permanent magnets and of masses of iron, properly placed, compensation may be made for these disturbing influences. For example, reflection will show that a magnetic cut water and stern post produce no variation in the compass when the vessel is sailing in the magnetic meridian, either north or south, but if it be sailing in any other direction in the semicircles to the east or west, the compass will be affected. Since the error produced is in one semicircle always to the east and in the other always to the west, the disturbance is called the *semicircular variation.* It may be corrected by a vertical rod of iron or a sphere of soft iron placed on the opposite side of the binnacle from the vertical magnetic body whose influence is the stronger. Similarly, the magnetism of the hull may be divided into two components, one lengthwise of the ship, the other crosswise, and these can be separately counterbalanced by compensating magnets placed usually in the pedestal of the binnacle. In making these adjustments, the newly launched vessel is anchored in some known position with reference to the magnetic meridian and the needle is brought to its correct reading. The vessel is then swung through an angle of 90° and adjustments again made, and so on around the circle, the process being called *swinging ship.* Magnetic masses in the cargo may cause disturbances of the needle and the magnetism of the hull grows less with

age and varies with the latitude, the vertical component becoming entirely reversed when the magnetic equator is crossed, therefore the navigator checks the indications of his needle by frequent astronomical observations and makes the necessary adjustments when the error becomes excessive.

184. Magnetism to be Reverted to Later.—The subject of magnetism is usually treated more extensively than in the preceding chapters. Thus, a theory of magnetic potential may be developed similarly to that of electric potential. It is thought, however, that enough of the principles have been given to enable the student to follow without difficulty the explanations in the following sections. Moreover, in view of the fact that electromagnets, or magnets produced temporarily by means of the electric current, are for most purposes far more suitable and more largely used than permanent magnets, and that the phenomena and properties of the magnetic circuit are most markedly exhibited and can be most clearly explained by reference to these electromagnets, it is logical that we should first take up the subject of electric currents. Further consideration of magnetism is therefore postponed for the present. (See Chapters 31 and 32.)

PART III.

VOLTAIC ELECTRICITY.

CHAPTER 18.

DISCOVERIES OF GALVANI AND VOLTA.

185. Galvani's Discovery.—The discovery of current electricity, or rather of methods of producing it by chemical means, is ascribed to two Italians, Galvani and Volta, the former Professor of Anatomy at Bologna, the latter Professor of Natural Philosophy at Pavia.

Tradition has it that about 1786 the wife of Galvani being indisposed, her physician prescribed for her a broth of frogs' legs. Some had been procured and skinned preparatory to cooking and lay upon a table near an electrical machine. Galvani's assistant happening to draw a spark from the machine, Madame Galvani noticed that at the same instant the severed legs twitched convulsively and that this was repeated with every spark. She called the attention of her husband to this phenomenon which he immediately proceeded to investigate. We now know that these twitchings were produced by the escape of the charge induced in the legs, which charge was released whenever the machine sparked, but Galvani, who was an anatomist and not an electrician, thought that he was on the verge of discovering the vital principle and continued his researches with this idea in mind. Having one day prepared several pairs of legs for experiment and wishing to place them to one side until they were needed, he hooked a copper wire through the remnant of the back bone and hung the legs to the iron railing of the balcony in front of his window. A light wind was blowing and to his astonishment he saw that whenever the dangling legs came in contact with the railing they were thrown into convulsive movement. Further experiment showed him that in order to produce these movements it was necessary to have two

dissimilar metals in contact and that the greatest effect was produced when the free end of one touched a nerve at the same time that the free end of the other touched a muscle. He attributed these effects to a so-called "animal electricity" whose seat lay at the junction of the nerve and muscle, where, by some unknown vital principle, the nerve became charged positively and the muscle negatively, and, like the Leyden jar, were discharged when connected by the metals. He did not explain why two metals were required.

186. Volta's Investigations.—Volta was not long in hearing of these experiments and, favored by his greater familiarity with what was then known of electricity, pursued a line of investigation which soon satisfied him that the true seat of development of the electricity was not at the junction of the muscle and the nerve but at the point of contact of the two metals. He found that if two dissimilar metals are brought together, one becomes positively charged, the other negatively, that is, they become of different potentials. This electrification by contact may be shown as follows. In Fig. 91, *A* is a light flat needle suspended symmetrically above the gap between the semicircular plates of zinc and copper and free to turn about the vertical axis *X*. If a positive charge be given to *A* and if then the copper and zinc plates be brought into contact at *B*, either by touching them together directly or by

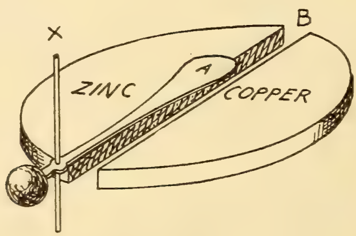


Fig. 91.

laying a piece of wire across the gap, the needle will swing away from the zinc and place itself above the copper, thus apparently showing the zinc to be positively charged or at a higher potential than the copper. Had the needle been charged negatively, it would have swung away from the copper and placed itself above the zinc.

187. Volta's Contact Series.—Further investigation by Volta showed that for a given pair of metals at a constant temperature, this contact difference of potential is constant and is independent of the size of the pieces, of the amount of surface in contact and of the length of time that they remain in contact. For different pairs of metals, however, it varies with the particular ones used, and he was able to draw up a list of these, similar to the list of substances given in Par. 23, so arranged that any one becomes positively electrified when touched to those below it in the series but negatively electrified when touched to those above it. Volta's list comprised seven of the commoner metals. Such a list now would be headed by the alkaline metals, unknown in Volta's time, and would be ended by the non-metal carbon. His observations were merely qualitative but subsequent observers have accurately measured these differences of potential. In the following list the numbers between the names indicate the difference in potential in volts set up between the corresponding pairs of metals when placed in contact:

Zinc
.210 volt
Lead
.069 volt
Tin
.313 volt
Iron
.146 volt
Copper
.238 volt
Platinum
.113 volt
Carbon

The difference of potential between any two metals in the series is the sum of the intervening numbers. Thus, with a zinc and copper pair, the difference would be .738 volts and between zinc and carbon it is 1.089 volts.

Regarding as negative the difference of potential between any pair taken in reverse order from that given in the above list, it follows that the difference in potential between the first and last metals of any number in series depends only upon these two and

is independent of the intervening metals or of the order in which they are arranged. Also, no matter how the intervening metals may be arranged, there is no difference of potential between the ends of a series beginning and ending with the same metal.

The foregoing list might be extended to include other substances than the metals. For example (and this fact is extremely important), a difference of potential is produced between a metal and a liquid when brought into contact and if the liquid attacks the metal chemically, an electro-motive force will act from the metal towards the liquid. Finally, a difference of potential is produced when two liquids come into contact and even between solutions of the same substance when these solutions are of different degrees of concentration.

188. Volta's Contact Theory.—While there is no uncertainty as to the facts as set forth above, there has been much controversy as to the interpretation to be put upon them. According to Volta, when two dissimilar metals are brought together, the surface of contact becomes a seat of electro-motive force which drives positive electricity in one direction from the junction and negative electricity in the opposite, and this separation continues until the force of attraction between the dissimilar charges balances the

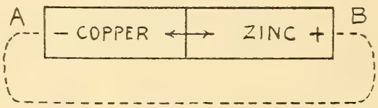


Fig. 92.

force which drives them apart. Thus in the compound bar of copper and zinc, Fig. 92, the zinc end becomes positively charged, the copper end negatively, or, the zinc end is at a higher potential than the copper.

In general, when bodies at different potentials are connected by a conductor, there is a flow of electricity from the one of higher potential to the one of lower, and, unless constantly re-established, the difference of potential disappears. It would therefore seem that in this case if B be connected to A by a wire, a flow of electricity would take place from B to A, but it can be shown that where the difference of potential is produced by contact as above and the metals are at the same temperature, it is not possible to get such a flow. If, for example, the connecting wire be of copper or

of zinc, the effect is the same as if the bar in Fig. 92 had been bent around into a circle until the ends A and B touched, and when these ends touch, a contact electro-motive force is set up equal but opposite to the one already existing and hence just counter-balancing it. If the wire be of some third metal, it follows from Par. 187 that to whichever end of the bar it be connected, the electrical effect is to convert the bar into a compound one consisting of the metal of the remaining end and of that of the wire, and, as shown above, no current would be produced upon completing the circuit.

Independent theoretical considerations lead to the same conclusion, for if a current flowed through the wire joining B and A in Fig. 92, by suitable arrangements, as we shall see later, this current could be made to do mechanical, chemical or thermal work and it is not possible that the mere touching of two metals should be a source of such energy.

In conclusion we may say that even considering the method described in Par. 186, no convincing experimental proof of Volta's theory has yet been devised.

189. Later Theory.—Examination of the series as given in Par. 187 reveals the fact that the metals as therein arranged are in very nearly the order of their chemical affinity for oxygen as determined by the heat produced by the combination of equivalent weights of these metals with that element. The difference of potential between pairs of metals therefore measures the difference of their affinities for oxygen, and its development may be explained as follows. Consider a piece of zinc in air. The molecules of oxygen about it are known to be composed each of two atoms, and, as we shall see later, there is reason to believe that these atoms carry equal and opposite charges of electricity and are held together in the molecules by the mutual attraction of these elementary charges. Under the influence of atmospheric moisture (Par. 281) the zinc slowly tarnishes or oxidizes. The oxygen, in order to combine with the zinc, must first separate into atoms and it is the negatively-charged atoms that enter into the combination, each giving up to the zinc its charge as it does so. The zinc, therefore, becomes negatively charged and is surrounded by a layer of positively-charged oxygen atoms. A piece of copper would behave similarly but having a less affinity for oxygen it would acquire a smaller negative charge and the oxygen about it

would be less highly charged positively. This state of affairs is represented graphically in Fig. 93. No indication of these charges could be detected by an electrometer, for the charges upon the pieces of metal and in the surrounding air being equal and opposite produce no external effect. If, however, the two metals be touched together, they, being conductors, come at once to a common potential, but the air being a non-conductor, that about the zinc

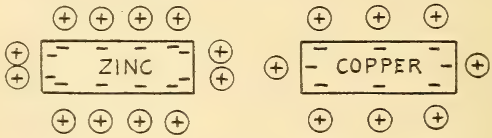


Fig. 93.

is left at a higher potential than that about the copper. We therefore have good reason to believe that the difference of potential between pairs of metals as measured by electrometers is really the difference of potential between the layers of air surrounding the metals and not that between the metals themselves. This view is corroborated by the observed changes in the difference of potential when pairs of metals are surrounded by other gases than air.

190. The Voltaic Pile.—By means of his condensing electroscope Volta demonstrated, as he thought, the difference of potential produced at the ends of a zinc-copper bar but was unable to detect any current in the wires by which he joined the ends of the bar. In Par. 188 above, it has been shown that there is no such current, but Volta, thinking that there was one but so feeble as to elude his instruments, sought some way of multiplying its effect and endeavored to combine the supposed currents from a number of zinc-copper pairs. He began by arranging in a pile a series of discs, alternately copper and zinc, but at once encountered a difficulty. According to his theory, from the junction of the bottom copper disc with the zinc disc above it a positive current ascended, a negative current descended, but when the second copper disc was reached this was reversed, a positive current descended and a negative current ascended, and so on. In other words, the upward currents were alternately positive and negative and alternated in this respect with the downward currents. With an even number of discs the net result was no greater than with two; with an odd number the net result was zero. Since these currents were sup-

posed to originate at the surface of contact of the two metals, if the copper plates were separated from the zinc plate immediately below them, the contrary currents would be eliminated. He therefore inserted between these plates a disc of cloth, Fig. 94, but since cloth is a non-conductor he moistened it with water. Water is a poor conductor (Par. 276) but its ability to conduct is greatly improved by dissolving in it a small amount of salt or of acid. His invention therefore took the final form, as shown in Fig. 94, of a pile of pairs of zinc and copper discs separated by layers of cloth or of blotting paper which had been soaked in brine or in dilute acid. The results far exceeded his expectations. The difference of potential between the top and bottom of the pile varied directly with the number of pairs of discs used. If the top and bottom discs were touched simultaneously, there was experienced a shock, milder than that of the Leyden jar but continuous. By means of wires attached to the extremities of the pile, electrical apparatus could be charged. If these wires were touched together and then separated, a spark was produced, etc.

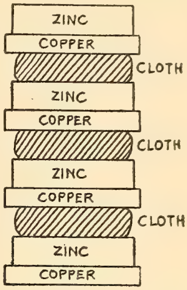
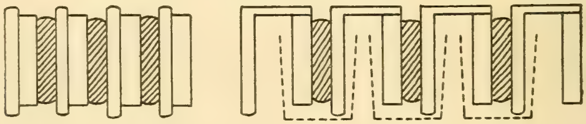


Fig. 94.

The voltaic pile was made known to the scientific world in March of 1800 and has long since been relegated to the museum shelf, but its invention, nevertheless, marks an epoch in the history of electricity. It gave a fresh impetus to the science, which in the next few years advanced by bounds, and it put into the hands of the chemist a new agent which for the first time enabled him to decompose water into its constituent elements and made known to him the metals of the potassium and calcium groups.

191. Volta's Circlet of Cups.—Volta soon noticed that the power of his pile fell off after a short use and he attributed this to the loss of the conducting liquid in the layers of cloth, partly by being squeezed out by the weight of the metal discs and partly by evaporation. To remedy this he devised a plan which will be understood from the following explanation. Let us suppose the pile to be laid on its side, as represented in Fig. 95 *a*. Since he had shown that the electrification by contact was independent of the extent of the surfaces in contact, the same effect would be produced if the copper and zinc pairs were separated and touched only at the top as shown in *b*. Being spread apart in this way,

glass cups, represented by the dotted lines, could be slipped under the pairs which were separated by the moistened cloth, the cloth could then be withdrawn, and the cups filled with the liquid itself. This modified form very quickly displaced the original pile. The individual cups are designated *cells,* and a series of two or more is called a *battery,* the primary meaning of the word battery being



a Fig. 95. b

a number of similar utensils placed side by side. For use with these batteries, the zinc-copper pairs were in the form of a strip joined at the middle and bent into the arc of a circle so as to be inserted into the cups. This is the origin of such terms as "connected in multiple arc" applied to certain groupings of cells to be described later. An arrangement of cells in a circle by which the

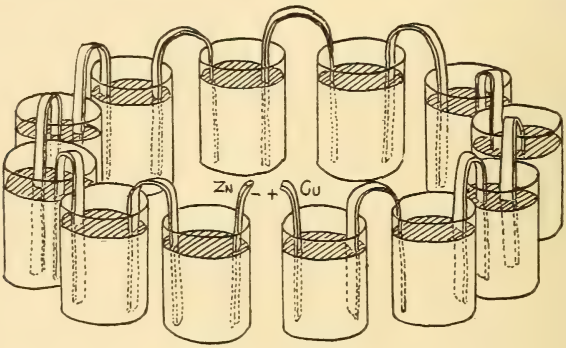


Fig. 96.

positive and negative ends of the battery could, for convenience, be brought close together (Fig. 96), was called by Volta his "*couronne de tasses*" or circlet of cups.

192. Source of Electrical Energy in a Cell.—It will have been noted that in Par. 188 the statement was made that no current could be produced by the contact of dissimilar metals, yet Volta, proceeding on the contrary assumption devised the pile and the battery, both of which produce a continuous supply of electricity.

In Par. 189 we saw that when zinc and copper are brought together in air, the metals, being good conductors, come to a common potential and the air surrounding the zinc is left at a higher potential than that around the copper. When, however, these metals are immersed in a chemically active liquid, a different state of affairs results, for in this case the medium surrounding the metals, instead of being a non-conductor like the air, is a conductor and hence at a uniform potential. We also saw (Par. 187) that when a metal is attacked by a liquid, an electro-motive force is set up from the metal towards the liquid. In this case, the zinc being the more vigorously attacked, the electro-motive force acting from the zinc is greater than that acting from the copper; positive electricity is therefore driven across from the zinc to the copper and the zinc itself is left negatively charged. The copper is, therefore, at a higher potential than the zinc and if it be connected to the zinc by a wire, a current will flow through this wire from the copper to the zinc. The source of the electrical energy in these arrangements is not at the junction of the two metals but at the point of contact of the zinc with the brine or the dilute acid and is due to the chemical action which there takes place. For this reason, the left hand copper strip and the right hand zinc strip in Fig. 95 *b* can be omitted, as shown in Fig. 96, without affecting the strength of the battery. See also Par. 279.

CHAPTER 19.

THE SIMPLE CELL.

193. Simple Voltaic Cell.—A voltaic cell in its simplest form consists (Fig. 97) of a glass cup partly filled with acidulated water, called the *electrolyte,* into which dip a strip of copper and one of zinc, sometimes spoken of as the *elements* of the cell. We shall suppose that, as represented in Fig. 97, to each of these strips there is attached a wire. If the zinc be pure, or if it has been treated as will be explained later (Par. 197), no action will be observed so long as the strips are kept apart. If, however, they are inclined towards each other so as to touch either above or below the surface of the liquid, or if they be brought into contact indirectly by joining the ends of the two wires, then bubbles of gas will immediately appear on the surface of the copper and the zinc will be observed to dissolve away gradually. This corrosion of the zinc and evolution of bubbles will continue only so long as the strips are in contact or the wires are connected, and during this time a current of electricity will flow through the liquid from the zinc to the copper and from the copper through the point of contact of the two strips, or through the connecting wire, back to the zinc. Since, as we shall shortly see (Par. 217), we can not be positive in which direction the current does flow, we, by convention and from analogy with water, agree to consider that it flows from the point of high potential to that of lower, or from positive to negative; therefore, since the current is due to the chemical energy developed on the surface of the zinc and originates there, the zinc plate is

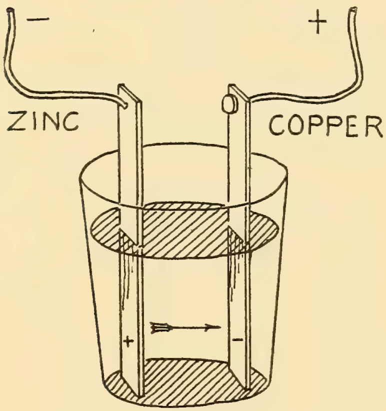


Fig. 97.

called the *positive plate* and consequently the copper is the *negative plate.* The current crosses the liquid to the copper plate, ascends this plate to the attached wire, follows along the wires to the junction with the zinc plate and descends this plate to the point of starting. The points of attachment of the wires to the copper and zinc are called the *poles* of the cell, and since the current flows from the copper out into the connecting wire, the copper pole is called the *positive pole,* the zinc, the *negative pole.* On account of the confusion sometimes resulting from this nomenclature, it is perhaps unfortunate that the copper should be both the positive pole and the negative plate and that the zinc should be the positive plate but the negative pole. As an aid to the beginner it may be remembered that the positive plate is the one which is attacked by the electrolyte and is the point of origin of the current.

194. Material Used for Elements of a Cell.—The elements of a cell are usually metal, or carbon and a metal. The farther apart the elements are on the list as given in Par. 187, the more vigorous will be the chemical action set up in the cell and consequently the greater the electrical energy developed. The positive plate should be of the metal most freely attacked by the electrolyte; the negative plate should be of the metal attacked least. The alkaline and the alkaline-earth metals head the list but decompose water and combine with acids with almost explosive violence; they are, therefore, unfitted for use. The most suitable metal for the positive plate, both from the standpoint of chemical action and of cost, is zinc, while carbon, copper and platinum are the substances most frequently used as negative plates.

195. Chemical Action in a Simple Cell.—If the electrolyte of the simple cell be dilute sulphuric acid, the chemical action when the circuit is closed is in accordance with the following reaction:

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$

The zinc sulphate passes into solution as it is formed and the hydrogen is evolved as bubbles at the surface of the copper plate. Since the chemical action takes place at the surface of the zinc, it would seem that the hydrogen bubbles should be released at that point or else that they should be seen passing through the liquid to reach the copper plate. The reason why neither of these occur is explained in Par. 274.

If the hydrogen be collected under an inverted jar and its weight be determined, and if the zinc plate be weighed at the beginning and conclusion of the experiment, it is found that while two parts of hydrogen are being produced, 65 parts of zinc are eaten away, that is, chemically equivalent amounts of the two are evolved and dissolved respectively, or the action is strictly chemical.

Instead of dilute acid a saline solution is often used as an electrolyte, ammonium chloride being frequently employed. The reaction in this case is

$$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + H_2$$

both the zinc chloride and the ammonia passing into solution.

196. Local Action.—In Par. 193 it was stated that if a plate of *pure* zinc be dipped into dilute sulphuric acid, no effect would be produced. Commercial zinc, however, is far from being pure and contains appreciable amounts of iron, lead and other substances. If such a plate be dipped into the electrolyte, chemical action immediately ensues, bubbles of hydrogen gas are evolved, the plate becomes pitted and may eventually be eaten through, and the acid becomes spent. The explanation is that the minute particles of the foreign metal in contact with the zinc constitute tiny voltaic pairs, local currents set up from the zinc through the electrolyte to the particles and back to the zinc, and cup-shaped depressions are eaten out around these particles until the latter become disengaged and fall. This process is called *local action.* The currents produced are parasitic and wasteful, existing at the expense of the materials of the cell but contributing nothing to its useful energy.

The rapid rusting of a nickel-plated piece of iron, once that the nickel coating is cut through, and the corrosion about the heads of iron nails driven through the copper sheathing of vessels is similar to this local action.

197. Remedy for Local Action.—The logical remedy for local action would be the use of chemically pure zinc but the cost renders this prohibitive. However, in 1830 it was discovered that local action can be almost entirely obviated if the surface of the zinc be amalgamated, that is, covered with a thin layer of mercury. This may be done either by adding about four per cent of mercury to the zinc at the time when it is cast into plates, or by cleaning the surface by dilute acid and then rubbing mercury upon it with a bit of rag. The mercury unites with the zinc forming a sort of

silvery paste but does not dissolve the particles of iron which are either covered up or else float to the surface of the amalgam and drop off. As the zinc in the amalgam is eaten away during use of the cell, the mercury amalgamates new layers of the zinc beneath. The action of the amalgam is not thoroughly understood, for, apparently, by adding the mercury we have brought about the exact condition which we wished to avoid, that is, contact of two dissimilar metals in presence of the acid.

198. Polarization.—If the wires attached to the poles of a simple cell be brought into contact, a current will immediately flow through the circuit, but if it be measured by any of the means to be described later, this current will be found to fall off rapidly. If the copper plate be observed, it will be noted that not all of the hydrogen bubbles released at this plate rise to the top but many remain adhering to it and the surface of the plate rapidly acquires a silvery bloom. The negative plate is then said to be *polarized.* It is this layer of hydrogen which causes the current to dwindle and it does so in two ways, one mechanical, the other electro-chemical. First, the hydrogen being a non-conductor, each bubble in contact with the copper withdraws just so much of the surface of this plate from contact with the liquid and diminishes by just so much the cross-section of the path available for the passage of the current. It therefore cuts down the current by putting resistance in its path. Second, the film of bubbles upon the plate causes it to approximate in behavior to a plate of hydrogen, and since hydrogen has a greater tendency to oxidize than has copper, the effect is to set up a greater electro-motive force opposed in direction to that from the zinc. We have seen (Par. 192) that it was the difference between the electro-motive forces acting from the zinc and from the copper which drove the current through the cell, consequently, when this difference becomes smaller, the current also becomes smaller.

This diminution of the current by polarization may be avoided by surrounding the negative plate by some agent, either solid or liquid, which will oxidize the hydrogen, converting it into water, or will enter into combination with it, releasing in its stead some element which does not increase the resistance of the negative plate. The endeavor to do away with this polarization is largely responsible for the different varieties of cells described in the following chapter.

199. Depolarizers.—Among the many substances which have been employed for this oxidation of the hydrogen are the liquids nitric acid, solutions of nitrate of potassium, of the bichromates of potassium and sodium, of ferric chloride, etc., and the solids black oxide of manganese, peroxide of lead, and oxide of copper. The solid depolarizers may be made into a pasty mass and moulded about the negative plate or may be made into briquettes and fastened to the negative plate by rubber bands. The liquid depolarizers may sometimes be mixed with the electrolyte but in most cases would attack the positive plate, even when the circuit was open, therefore, to prevent their reaching the positive plate but at the same time not to hinder the passage of the current, they are usually put along with the negative plate in an interior unglazed and porous porcelain cup which is placed in the electrolyte. Such cells are sometimes called *two-fluid cells.*

200. Requirements of a Voltaic Cell.—The properties desired in a good primary cell are the following:

(1) It should have a high and constant electro-motive force, preferably greater than one volt.
(2) It should have low internal resistance.
(3) It should give a constant current and should, therefore, be free from polarization.
(4) It should be free from local action, its elements not being consumed except when it is supplying current.
(5) Its elements should be cheap. The cost of plates of gold, platinum or silver is in most cases prohibitive.
(6) Its elements should be durable, not requiring too frequent renewal or too much attention.
(7) It should not emit corrosive or poisonous fumes.
(8) The electrolyte should not freeze readily.

No cell has yet been devised which fulfills all of these conditions, and for different uses they are not equally important. For example, constancy of current, while essential when a small electrical machine, such as a fan, is to be run, is not so where the cells are used intermittently and then only for very brief periods, as is the case with those that operate door and call bells. Again, for telegraphy over a long line of considerable resistance, a moderate internal resistance of the cell is not very objectionable.

The E. M. F. of a cell is independent of the size of its plates or

of the depth to which they are immersed in the electrolyte, that is, of the size of the cell, but depends entirely upon the relative position of its elements in Volta's series (Par. 187). The E. M. F. of a zinc-copper-sulphuric acid cell is the same whether the cell be as large as a barrel or as small as a thimble. Therefore, the elements of a cell having been selected, its E. M. F. is fixed. The quantity of electricity produced varies, however, with the amount of chemical action in the cell and this varies directly with the size of the plates.

CHAPTER 20.

KINDS OF CELLS.

201. Great Variety of Cells.—Any two conducting substances which dip into a vessel containing a liquid which attacks one more than it does the other, constitute a *primary cell,* also called a *voltaic* or a *galvanic cell.* There are, therefore, a great many possible arrangements by which electricity may be generated by chemical means and this number is still further increased when we consider the many expedients adopted for avoiding polarization. It would therefore seem that the number of kinds of cells would be limited only by the ingenuity of the inventor and such would be the case were it not for the required conditions (Par. 200) which not being fulfilled by the majority of the possible combinations cause these combinations to be rejected. Notwithstanding this, the variety is still great and the few described in the following pages must be regarded as types of general classes.

202. Classification of Cells.—Cells may be divided into two general classes, primary and secondary. The primary cell has been defined above (Par. 201); the secondary cell differs from the primary mainly in that when it has become exhausted, an electric current may be passed through it in a contrary direction to the current which it supplied, the chemical changes which have taken place may be undone and the cell can be restored to its primitive condition. It is therefore analogous to a clock which, when run down, can be wound up again. Secondary cells are used in *storage batteries* and will be considered in detail when we reach that subject (Chapter 22).

Primary cells are of two classes, those without depolarizers (such as the simple cell described in Par. 193), and those with depolarizers. This latter class may be subdivided according as the depolarizer is a liquid or a solid. Other subdivisions may be made, as, for example, single-fluid cells, two-fluid cells, dry cells, standard cells, etc., but this classification is not of sufficient importance to be dwelt upon longer.

203. Grove's Cell.—One of the first cells in which a chemical depolarizer was employed was invented by Grove in 1839. This consists (Fig. 98) of a flattened, rectangular outer cell *A* of glass or of vulcanized rubber, containing dilute sulphuric acid into which dips the U-shaped amalgamated zinc plate *B*. Within the loop of this zinc plate there fits a flat porous cell *C* containing concentrated nitric acid and the platinum negative plate *D*. The hydrogen produced by the action in the external cell is attacked by the nitric acid as follows:

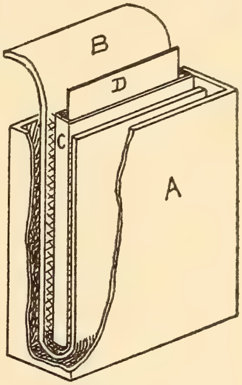


Fig. 98.

$$3H + HNO_3 = 2H_2O + NO$$

The nitric oxide, NO, produces no polarization since it either dissolves in the acid or escapes into the air where, in contact with oxygen, it becomes nitric peroxide, NO_2, a reddish brown, irritating gas. The cell has a high electro-motive force, very nearly two volts, and owing to the great amount of surface of the zinc plate and the short distance from the zinc to the platinum plate, it has small internal resistance. The objections to this cell are the corrosive and poisonous character of the nitric peroxide fumes and the cost of the platinum plates. These last need be no thicker than tin-foil but since the cost of platinum is now (1912) more than that of gold (about $700 per pound avoirdupois), they are necessarily very expensive.

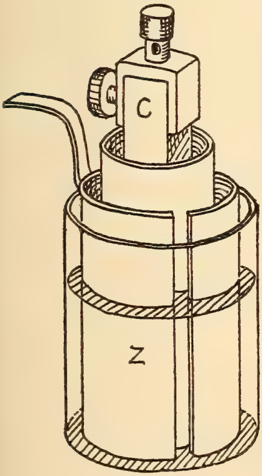


Fig. 99.

204. The Bunsen Cell. — To avoid the expense of the platinum plate, Bunsen, in the year following the invention of the Grove cell, suggested the use in its stead of a plate of hard carbon. These plates are prepared from gas coke or that particular hard and semi-metallic form of carbon resulting from the decomposition by heat of gaseous hydro-carbons and occurring as a deposit in the retorts and flues of gas works. The principle

of the Bunsen cell is precisely the same as that of Grove's cell. The carbon plate (*C*, Fig. 99) is in shape a square prism and dips into nitric acid in an inner porous cup. The zinc plate *Z* is a split cylinder and embraces this inner cup. The cell gives off the same corrosive fumes as the Grove cell but the greatest objection to it is the difficulty of making electrical connection with the carbon plate. This plate being porous, it is difficult to attach wires to it directly. To remedy this, the upper end of the plate is sometimes copper plated, after which the connector is clamped to it as shown in Fig. 99. Also, owing to its porosity, the plate soaks up the nitric acid which, upon rising to the height of the copper plating or of the connecting wires, will corrode the connections. This is partly remedied by soaking the upper end of the plate in melted paraffine which, being impervious to the acid, hinders its rise.

205. The Bichromate Cell.—There are a number of cells which instead of nitric acid employ either chromic acid or the bichromates of potassium or of sodium as depolarizers, but are otherwise the same as the Bunsen cell. It is found that in these the inner porous cell is not necessary and the bichromate solution may be allowed to mingle freely with the sulphuric acid, in fact, they are sold ready mixed under the name *electropoion fluid*. The hydrogen released by the action of the sulphuric acid upon the zinc is oxidized by the bichromate, the products being water and chrome alum thus

$$K_2Cr_2O_7 + 4H_2SO_4 + 6H = 2KCr(SO_4)_2 + 7H_2O$$

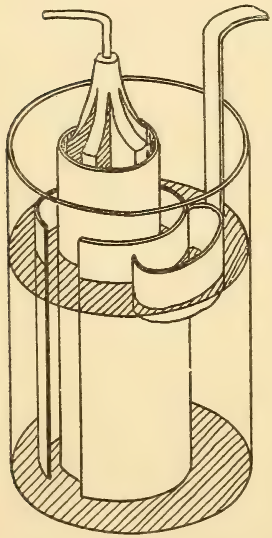


Fig. 100.

206. Daniell's Cell. — The first cell to avoid polarization was invented by Daniell in 1836 and, although using a liquid depolarizer, the principle of its action is quite different from that of the cells described in the preceding paragraphs. Fig. 100 represents one of its many forms. This consists of an inner porous cup, which contains dilute sulphuric acid, and the zinc plate. The zinc is given the corrugated form shown in the figure

in order to expose more surface to the action of the acid. The copper plate, in the form of a split cylinder, surrounds the inner cup and is immersed in a solution of copper sulphate contained in the outer cell. As it is important that this last solution should be kept saturated, there is fastened to the side of the copper plate a little cup or shelf with perforated bottom and this cup is kept filled with crystals of copper sulphate.

The chemical action in the inner cell is the same as already described but the hydrogen on coming in contact with the copper sulphate solution displaces the copper and takes its place and the copper is deposited on the negative plate thus

$$H_2 + CuSO_4 = H_2SO_4 + Cu$$

There is, therefore, no polarization and the copper plate simply grows thicker by the deposition upon its surface of successive films of copper. The copper sulphate solution would, however, become gradually exhausted were it not continually replenished from the crystals on the perforated shelf.

The sulphuric acid in the inner cup is gradually converted to a solution of zinc sulphate but the cell continues to operate, in fact, the inner cup is often filled from the beginning with a solution of zinc sulphate. In this case the following reaction takes place:

$$ZnSO_4 + CuSO_4 = SO_4 + ZnSO_4 + Cu$$

the copper being deposited upon the negative plate as before and the *sulphion*, SO_4, attacking fresh portions of the zinc and again becoming zinc sulphate.

The electro-motive force of a Daniell cell averages about 1.07 volts but fluctuates slightly with the variation in the strength of the two solutions and also with the temperature. Being free from polarization, it is very largely used where constant currents are required, as is especially the case in telegraphy in this country.

207. Gravity Cell.—A saturated solution of copper sulphate has a specific gravity of about 1.20 and if the specific gravity of the zinc sulphate solution be kept below this figure, it is possible to do away with the inner cup of the Daniell cell and to separate the two fluids by the difference in their densities. Such a cell, called a *gravity cell*, is represented in Fig. 101. The copper plate, of the shape shown, is placed upon the bottom of the cell and the copper sulphate solution with extra crystals is poured over it. The wire from this plate is protected by rubber or by a glass

tube up to the top of the cell. The zinc plate, of the shape shown, is hung from the edge of the cell and is covered with a dilute solution of zinc sulphate. As the cell is used the zinc sulphate solution increases in density. It must therefore be tested from time to time by means of a hydrometer (a little graduated glass float which stands higher in the liquid as the latter grows denser, and sinks lower as it grows less dense), and should the density reach 1.15, a portion of the solution must be drawn off by a syringe or a siphon and water added in its place. If the cell be unused for some time, the two fluids will mingle by diffusion and when the copper sulphate solution reaches the zinc plate, metallic copper will be deposited upon this plate with the result that local action will ensue.

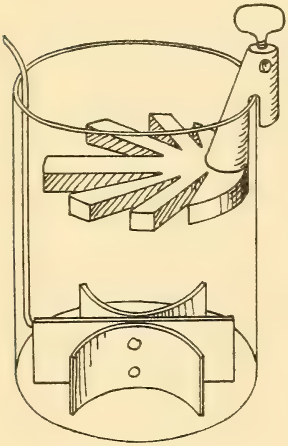


Fig. 101.

From the shape of the zinc plate, these cells are commonly known as *crowfoot batteries.*

208. The Edison-Lalande Cell.—This is an example of a cell employing a solid depolarizer. It has two positive plates of zinc bolted together at the top and arranged one on either side of the negative plate. This last is of cupric oxide compressed into the required shape and size. During the process there is added some cementing material which when heated binds the particles firmly together. The completed plate is inserted in a copper frame by which it is suspended from the lid of the cell. The arrangement is shown in Fig. 102 in which, for the sake of clearness, one of the zinc plates has been omitted. The electrolyte is a solution of caustic potash (potassium hydroxide) which when the circuit is closed attacks the zinc, producing a double oxide of zinc and potassium (potassium zincate) and releasing hydrogen, thus

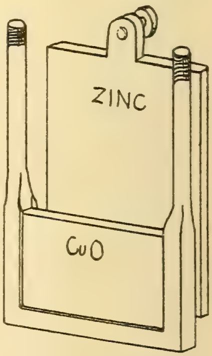


Fig. 102.

$$Zn + 2KOH = K_2ZnO_2 + H_2$$

The hydrogen reduces the copper oxide of the negative plate as follows:

$$H_2 + CuO = H_2O + Cu$$

and there is therefore no polarization.

The electro-motive force of these cells is low (only .7 volt), but the internal resistance is very small and their efficiency is high.

Potassium hydroxide has a great affinity for carbon dioxide and will absorb this gas from the air, becoming potassium carbonate. To prevent this, a layer of heavy paraffine oil must be poured upon the surface of the electrolyte.

209. The Leclanché Cell.—The Leclanché cell, invented in 1868, also uses a solid depolarizer. From its cheapness, simplicity and freedom from dangerous chemicals it is extremely popular and in one form or another is probably more used than all other kinds combined. A common form is shown in Fig. 103. The cell is generally a glass jar, the positive element an amalgamated zinc rod placed in one corner of the jar, and the negative plate is of gas carbon. The depolarizer is manganese dioxide used in the form of a black powder and the many forms of this cell found upon the market are based mainly on differences in the method of applying the depolarizer to the carbon plate. In the original cell the carbon plate was placed in a porous cup which was then packed with the powdered depolarizer. In modern forms the dioxide may be cemented about the carbon plate, or made into briquettes and fastened to this plate by rubber bands, or may even be compounded with the carbon plate itself. Since the dioxide is a poor conductor, when it entirely surrounds the carbon plate it is always mixed with powdered carbon by which its resistance is reduced. The electrolyte is a solution of sal ammoniac (ammonium chloride) and the reaction when the circuit is closed is

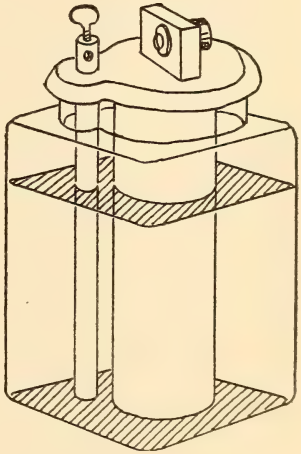


Fig. 103.

$$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + H_2$$

The action of the depolarizer is

$$H_2 + 2MnO_2 = H_2O + Mn_2O_3$$

Since chemical action is much retarded when one of the reagents is in the form of a solid, the depolarization in a Leclanché cell does not take place quickly enough to consume the hydrogen as fast as it forms and the cell polarizes rapidly. However, as the chemical action, oxidizing of the hydrogen, keeps on steadily after the circuit is broken, the cell will recover after a short rest. These cells are, therefore, not fitted to supply a continuous current but are admirably adapted for intermittent use as in telephones, door bells, etc. Their electro-motive force is about 1.4 volts, there is no local action and they require a minimum amount of attention.

210. Dry Cells.—The so-called *dry cells,* in common use in this country, are in principle simply Leclanché cells in which the liquid has been reduced to a minimum. A cross-section of one of these cells is shown in Fig. 104. The cell proper is a zinc can which serves both as the cell and as the positive plate. The negative plate is of gas carbon and may be corrugated or fluted so as to expose more surface. It is placed in the can and packed around with a mixture of manganese dioxide and granular coke. The packing is then saturated with electrolyte, usually a solution of zinc chloride and ammonium chloride, after which the cell is sealed with a layer of pitch or asphalt. This serves a double purpose; it holds the carbon plate rigidly in position and it prevents the evaporation of the electrolyte. To secure the seal more firmly a cannelure or groove is made around the cell near the top. In some of these cells a cementing material is mixed with the depolarizer; in others the can is lined with asbestus or with pasteboard which has been soaked with the electrolyte. For insulation, the cell is usually placed in an outer box of pasteboard.

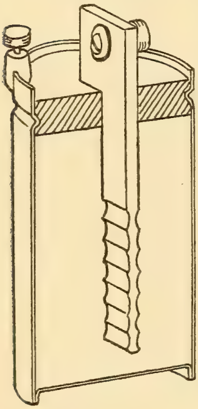


Fig. 104.

For many purposes these dry cells have entirely superseded the wet cells. They are very cheap, costing now less than 20 cents apiece, and for average door-bell use should last from two to three years. If the asphalt seal becomes cracked, they soon dry out and cease to act.

211. Need of Standard Cells.—One of the most important classes of measurements with which the electrician has to deal is that of electro-motive force. In Chapter 11 we examined *electrometers,* a form of apparatus sometimes used for this purpose, and in Chapter 34 we shall describe *voltmeters,* instruments better adapted for practical use since they are arranged so that the electro-motive force which is being measured can be read direct from a printed scale without the necessity of resorting to intermediate calculations. Even the best instruments, however, do occasionally get out of adjustment and it is very desirable that we should possess standards of electro-motive force by which our instruments can be calibrated in the first place and compared and checked in the second. While the average E. M. F. of the cells described in the preceding paragraphs can be stated with considerable accuracy, the actual E. M. F. is dependent upon varying conditions and, between limits, fluctuates too irregularly and with too much uncertainty for these cells to be used as standards. However, there have been devised certain "*standard cells*" in which the variable factors, except that of temperature, have been eliminated and the temperature coefficient, or change of E. M. F. with temperature, determined. These cells are used for their E. M. F. and not to supply current and since the E. M. F. is independent of the size of the cells (Par. 200), they are made very small, some, in fact, being hardly larger than a thimble. Analogous to this would be the use of vertical columns of water as standards of pressure. Since hydrostatic pressure per unit area is dependent upon the height of the column and is independent of its cross-section, and since no current or flow of water is required, these vertical columns could be contained in slender tubes.

212. Clark's Standard Cell.—In 1893 the International Congress of Electricians in session in Chicago passed resolutions defining certain electrical units upon which at that time the scientific world was not universally agreed. These definitions were formally legalized by Act of Congress, approved July 12, 1894. Among others, there was defined the unit of electro-motive force, *the international volt,* and to the definition proper was added that it was "represented sufficiently well for practical use by $\frac{1000}{1434}$ of the electro-motive force between the poles of the voltaic cell, known as Clark's cell, at a temperature of 15° C and prepared in the manner described in the accompanying specification."

There are a number of forms of this cell. The one shown in Fig. 105 is in accordance with the specification referred to. The cell proper is a two-limbed bottle closed with a ground-glass stopper. Through the bottom of each limb there is fused a fine platinum wire, the two serving as the terminals of the cell. In principle, the cell is the same as Daniell's. The positive plate is amalgamated zinc, the negative plate is mercury, the electrolyte is a solution of zinc sulphate and the depolarizer is mercurous sulphate. The zinc amalgam is composed of nine parts of mercury and one of zinc, and is liquid at the temperature of boiling water but sets at ordinary temperatures. It is melted and poured into one of the limbs. Upon this is packed a half-inch layer of crystals of zinc sulphate. In the other limb is poured perfectly pure mercury, then on top of this a layer of mercurous and zinc sulphates worked up together into a paste, and on top of this paste a half-inch layer of the crystals of zinc sulphate. Finally, the bottle is filled to the neck with a saturated solution of zinc sulphate and the stopper is cemented in with shellac, leaving beneath it a small air bubble to allow for expansion of the liquid with changes of temperature. The cell is then placed in a protecting outer case, the wires being brought out to suitable binding posts, and an opening is left in the cover through which a thermometer may be inserted to take the temperature of the cell.

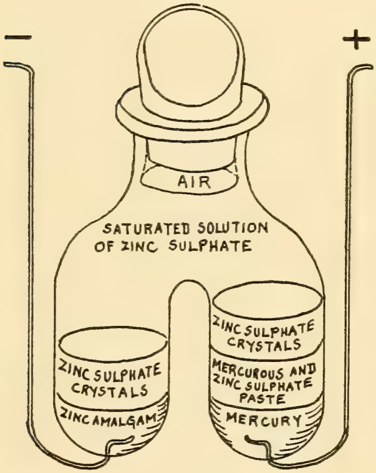


Fig. 105.

The chemical action is similar to that given for Daniell's cell (Par. 206).

$$ZnSO_4 + Hg_2SO_4 = SO_4 + ZnSO_4 + Hg_2$$

the SO_4 attacking the zinc of the positive plate, the Hg_2 coalescing with the mercury of the negative plate and there thus being no polarization.

The E. M. F. of a Clark cell at 15° C (59° F) is 1.434 volts and

its temperature coefficient or change of E. M. F. per degree Centigrade is about .00115. This is negative, that is, the E. M. F. *decreases* as the temperature *increases*. At 50° F it is 1.440; at 80° F it is 1.421. This change in E. M. F. with change in temperature is due to corresponding change in solubility of zinc sulphate and hence variation in the density of the electrolyte. The exact E. M. F. at any temperature t Centigrade is given by the formula

$$E_t = 1.434 - .00119\ (t-15) - .000007\ (t-15)^2$$

213. Weston's Standard Cell.—The Weston standard cell is in principle precisely the same as the Clark cell, cadmium being substituted for zinc, that is, the positive plate being a cadmium amalgam, the electrolyte being a saturated solution of cadmium sulphate, etc., and the mechanical arrangement being similar to that just described. Since the solubility of cadmium sulphate varies but little with temperature, the temperature coefficient is very small, being only .00004 volt per degree Centigrade. For all ordinary purposes, this change may be neglected and the E. M. F. of the cell may be taken as 1.019 volts.

214. Conventional Sign for Cell.—Since in the study of electricity it often becomes necessary to make diagrams in which cells appear, a conventional sign for the same has been adopted. In Fig. 106, *a* represents the plan of two cells connected together and *b* represents the conventional sign for the same two cells. In both, the short heavy line represents the zinc, the long thin line the copper. It will be noted that in the conventional sign the cell itself is omitted as well as the connecting wire between the cells.

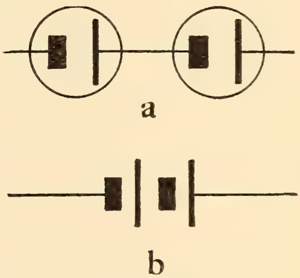


Fig. 106.

CHAPTER 21.

THE ELECTRIC CURRENT AND ITS CHEMICAL ACTION.

215. Electric Current.—In Par. 70 it was stated that when conductors at different potentials are brought into contact (either directly or through a third conducting body), there is a flow of positive electricity from the one of higher potential to that of lower. Again, in Par. 75 it was stated that if new charges were supplied to the body of higher potential as fast as the preceding charges flowed away, then the body would be maintained at a constant potential and the successive charges flowing away would constitute a continuous stream or current. Such is the state of affairs in a voltaic cell. The chemical action at the surface of the zinc plate produces fresh quantities of electricity as fast as those previously produced flow away. These successive charges pass across to the copper plate and raise its potential, and if this copper plate be connected by a wire to the zinc plate a current will flow through the wire. It must, however, be borne in mind that electricity is not matter and that there is no actual movement of material substance. Nevertheless, we do know that when points at different potentials are connected by a conductor, certain perceptible effects are produced along this conductor; among them (a) the temperature of the conductor rises, (b) a magnetic field is established about this conductor and (c) if a part of the conducting path lies through a chemical compound, chemical decomposition will generally ensue. We are agreed then that when these phenomena occur, a current is flowing through the conductor. The terms "current," "flow," etc., are survivors of the time when electricity was spoken of and regarded as a fluid, and being such convenient forms of expression they are retained.

216. No Current Unless Circuit be Complete.—The path over which the current flows is called the *circuit*. There can be no flow unless this circuit be complete, that is, unless there be a continuous conducting path from the surface at which the current originates

back to the other side of this surface. Thus, in a simple cell, considering the surface of the zinc as a layer of appreciable thickness, the current originates at the outer part of this layer where it is in contact with the electrolyte, then traverses the electrolyte to the copper plate, thence out upon the wire and back to the zinc plate and finally down this plate to the inner side of the layer. If the circuit be continuous it is said to be *closed*; if it be not continuous it is said to be *broken* or *open*. Since the current thus returns upon itself, it is analogous to water which entirely fills a pipe bent around into the form of a ring. If this water be put in motion it can be checked by closing a cock in the pipe at any point whatsoever. So the electric current is stopped by breaking the circuit at any point at all.

217. Direction of Flow of Current.—We assume the current to flow from a higher potential to a lower, but (Par. 27) which potential is high and which is low is itself purely a matter of convention, therefore, even admitting that there is a flow, we have no way of determining in which direction it actually takes place. At first sight it seems that we could easily determine this direction. Suppose we have a cell operating an electric bell at some distance; the energy must surely have originated in the cell and moved out along the wire to the bell. But, from the preceding paragraph, there can be no current unless there be a complete circuit, hence there must be two wires or paths from the cell to the bell and we have no way of discovering upon which of the two the current moved out.

Notwithstanding the foregoing, some of the phenomena produced by the current do have direction with respect to the assumed direction of flow. The heating effect of the current in a homogeneous conductor is irrespective of the direction of flow, but the direction of the magnetic field about the conductor and the direction in which the products of electro-chemical decomposition move are dependent upon this flow and are reversed when the direction of the current is reversed.

218. Decomposition of Water.—On March 20, 1800, Volta addressed to Sir Joseph Banks of the Royal Society of London a portion of a letter describing the Voltaic Pile. This letter was not communicated to the Society until some time in June when the remainder had been received, but in the mean time it had been

shown to two of the members, Carlisle and Nicholson. Wishing to test the apparatus, they extemporized one with seventeen silver coins, an equal number of copper discs and pieces of cloth soaked in a weak solution of common salt. In order to make good connection with a metal plate which they were endeavoring to charge, they placed upon it a drop of water and inserted in this drop the end of one of the wires from the pile. At once fine bubbles rose in the liquid. Continuing these investigations, Nicholson within the next few days devised another experiment. He inserted in one end of a glass tube a cork, poured some water into the tube and then corked the other end. Through each of these corks he then thrust a platinum wire so that the ends protruded some distance into the water. When these wires were connected to the extremities of the pile, streams of bubbles were given off from each of the ends, and when tested separately, it was found that oxygen was released at the wire by which the current entered the water and hydrogen at the wire by which it left.

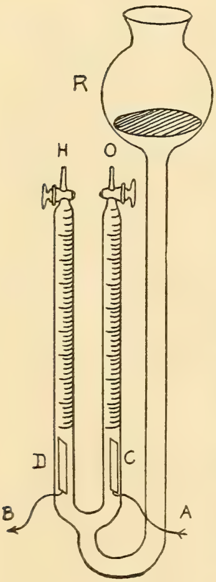


Fig. 107.

219. Electrolysis of Water.—This decomposition produced by the electric current is called "*electrolysis,*" i. e., electric analysis. The electrolysis of water can best be studied by means of the apparatus shown in Fig. 107. This consists of three glass tubes connected as shown. The tubes *H* and *O* are burettes graduated in cubic centimeters, usually to the nearest tenth, the graduations reading from the top downward. Through the bottom of these burettes there are sealed the platinum wires *A* and *B* terminating on the inside in the platinum plates *C* and *D*. The third tube is expanded at the top into the reservoir *R* which is at a higher level than the tips of the burettes. The apparatus is supported on a suitable stand. With the stop cocks *H* and *O* open, water, to which a few drops of sulphuric acid have been added, is poured into *R*. The liquid rises in the burettes and the stop cocks are closed as soon as its level passes them. The addition of the sulphuric acid is usually explained by the statement that it is used merely to improve the conducting power of the water.

Perfectly pure water is a non-conductor, and the acidulated water does conduct, but the true reason for the use of the acid is given below. If a current be now brought in at *A* and out at *B*, bubbles will rise from the plates *C* and *D* and collect in the upper parts of the burettes, pushing down the liquid which will rise in the reservoir. The gas in *O* will be found to be oxygen and that in *H*, hydrogen; furthermore, the amount of gas generated in *H* will be very nearly twice that generated in *O*. The volume of the hydrogen would be exactly twice that of the oxygen were it not for the facts that (a) some of the oxygen is given off in the denser form of ozone, (b) some of each gas, but not proportional amounts, is dissolved in the water, (c) a portion of the gases is occluded by the platinum plates and (d) owing to the difference of the levels of the water in the two burettes, the hydrogen is under greater hydrostatic pressure than is the oxygen.

The chemical action is usually explained by saying that the water is decomposed into its component gases hydrogen and oxygen, and this is correct but it is not the primary reaction which takes place. The sulphuric acid is first separated into H_2 and SO_4, the hydrogen being released and the SO_4 then attacking the water, thus

$$SO_4 + H_2O = H_2SO_4 + O$$

so that the oxygen is released as the result of a secondary reaction.

220. Faraday's Terminology.—The decomposition of chemical compounds by the electric current was investigated by Faraday to whom is due the terminology now employed. As we have already seen, the liquid which undergoes decomposition is called the *electrolyte* and the process itself is *electrolysis*. The vessel in which electrolysis takes place is called an *electrolytic cell*. The plates or wires which dip into the liquid and by which the current is brought in and taken out are termed collectively the *electrodes*; that by which the current enters is the *anode*; that by which it leaves is the *cathode*. The *part molecules* into which the substance being decomposed is split are, in allusion to their movement through the liquid, called *ions* (wanderers); those which appear at the anode are *anions*; those released at the cathode are *cathions* or *kations*.

221. Substances Subject to Electrolysis.—In order that a substance may be electrolyzed it must fulfill the following condi-

tions; it must be a compound substance; it must be a conductor; it must be in a liquid state, either as the result of fusion or of solution. Mercury and the fused metals are conducting liquids but being elementary bodies can not be decomposed. All other conducting liquids undergo electrolysis.

222. Electrolysis of a Fused Compound.—The electrolysis of lead chloride may be taken as an example of the decomposition of a fused compound. The salt is kept in a molten state in a small porcelain crucible placed over a bunsen burner. The electrodes of iron dip into the fused mass. When a current passes, chlorine is liberated at the anode, as may be shown by the bleaching effect upon a piece of litmus paper held just above, and lead is released at the cathode.

223. Electrolysis of a Base.—In many cases of electrolysis the primary reactions are obscured by the secondary. In the electrolysis of water (Par. 219), it is really the sulphuric acid that is electrolyzed, the decomposition of the water being the result of secondary reactions. Similar results follow the electrolysis of the strong bases. For example, a solution of potassium hydroxide electrolyzes as follows:

$$2KOH = K_2 + H_2O + O$$

the oxygen appearing at the anode and the potassium being released at the cathode, but as soon as this metal is released it attacks the water, thus

$$K + 2H_2O = 2KOH + H_2$$

so that the net result is the same as when sulphuric acid is electrolyzed, that is, the water is decomposed. If, however, the cathode be of mercury, the potassium amalgamates with it and by distilling off the mercury from the amalgam the potassium may be separated and collected. In a somewhat similar manner to this, Davy discovered in October, 1807, first potassium and rapidly thereafter sodium and other alkaline and alkaline-earth metals.

224. Electrolysis of a Metallic Salt.—When a metallic salt in solution is electrolyzed, the metal appears at the cathode, the acid radicle at the anode, but, as mentioned above, this primary reaction is frequently obscured by secondary reactions.

In the electrolysis of an alkali oxy-salt, these secondary reactions

occur at both anode and cathode. For example, if sodium sulphate be electrolyzed the sodium is released at the cathode but immediately reacts with the water releasing hydrogen. The SO_4 is released at the anode and, as described above, reacts with the water releasing oxygen. The net result therefore is simply the electrolysis of the water.

If a solution of copper sulphate be electrolyzed the copper is deposited upon the cathode and the SO_4 is released at the anode where one of two effects may be produced according as the anode is or is not attacked by the SO_4. If the anode be of platinum, the SO_4 attacks the water, forming sulphuric acid and releasing oxygen. If, however, the anode be of copper, the SO_4 attacks it, producing copper sulphate which goes into solution. As fast as copper is deposited upon the cathode, an equal amount is dissolved from the anode; the electrolyte therefore remains of constant strength. This is true for other metals than copper. If a salt of a metal be electrolyzed between electrodes of that metal, the anode wastes away, the cathode increases and the electrolyte remains of constant concentration.

The metals, which in the above are said to be released at the cathode, are really deposited upon the cathode in a compact and tightly adhering layer. This is the basis of the important processes of electroplating and electrotyping to be described later. Electrolysis has many other important applications, such as the electrolytic refining of copper, the manufacture of chlorine, of the alkaline hydroxides, of aluminum, etching on metal, photo-engraving, etc.

225. Electro-Chemical Classification of the Elements.—The elements have been classed according to their behaviour under electrolysis. Those which move in the direction of the current and are released at the cathode are called *electro-positive*, this name being given because they move to the *negative* plate. Those which move against the current and appear at the anode or *positive* plate are called *electro-negative.* Hydrogen and the metals are electro-positive; the non-metals are electro-negative. It will be noted that in its electro-chemical behaviour hydrogen conforms to its purely chemical behaviour and arranges itself with the metals. The above classification, which is also extended to compound ions, is not absolute; an element in certain compounds being electro-positive, while in others it may be electro-negative.

226. Faraday's First Law.—It was stated above (Par. 215) that when an electric current is flowing there is no material substance in movement but there is a transfer of energy which manifests itself in the production of heat, of magnetic effects, and of chemical decomposition. It is a known fact that the same amount of chemical action always produces the same amount of energy and, conversely, the same expenditure of energy in the production of chemical decomposition always brings about the same amount. The truth of this was recognized by Faraday, the first to investigate the laws of electrolysis, and was formulated by him to the effect that *the amount of chemical action produced in an electrolytic cell is proportional to the quantity of electricity which flows through the cell.* The amount of chemical action produced by the passage of an electric current may therefore be taken as a measure of the quantity transferred.

227. Voltameters.—An electrolytic cell so made that the chemical action produced by the current can be accurately measured, and hence the current determined, is called a *voltameter.* Voltameters are arranged so that the metal (usually silver or copper), deposited upon the cathode may be weighed, or the amount of gas released may be measured and its weight calculated. The latter class, the gas voltameters, may collect the gases separately, as shown in Fig. 107, or may gather these gases in a common burette thereby obtaining a greater volume for measurement.

We shall shortly see that there is another instrument, a *voltmeter,* used for quite a different purpose, the measurement of electro-motive force. It is unfortunate that these names are so much alike and the beginner must be on his guard not to confound the two.

228. The Coulomb and the Ampere.—To define a current of water, it is not sufficient to state the amount of water which will flow past a certain point but we must also state the rate at which it flows past. So also with the electric current; we must know both the quantity and the rate at which this quantity is delivered.

The practical unit of electrical quantity, the coulomb, is defined as that quantity of electricity which flowing through a gas voltameter liberates .00001035+ of a gram of hydrogen.

Now, a very feeble current must flow a long time to accomplish the same amount of chemical work as a current of greater

strength; on the other hand, the greater the current, the greater the amount of work done in a given time. We can therefore compare currents by comparing the amount of chemical work done in a given time. *The practical unit of current, the ampere,* is defined as *that unvarying current which flowing through a gas voltameter liberates* .00001035+ *of a gram of hydrogen per second.* Why this particular weight was selected will be explained later (Pars. 231, 232, 450). From the foregoing, it is seen that a current of one ampere delivers one coulomb per second, or that if Q be the number of coulombs, I be the current in amperes, and t be the time in seconds, then

$$Q = It$$

This may also be written $I = Q/t$, whence we see that the current in amperes is equal to the rate at which coulombs are delivered, or the number of coulombs per second.

The unit of quantity, the coulomb, must not be confused with the electro-static unit of quantity as defined in Par. 56. The coulomb is equal to very nearly 3×10^9 or three billion of the electrostatic units.

With practical experience in the Laboratory, the student will soon form a conception of the ampere which at first must be to him more or less of an abstraction. The current employed in the 16 candle power 110 volt incandescent lamp is about one-half ampere.

In solving ordinary problems given for practice, it is sufficiently accurate to take the amount of hydrogen released by one coulomb as .00001 (one one-hundred thousandth) of a gram.

229. Equality of Current at Every Cross-Section of a Circuit.—At every cross-section of a circuit through which a current is flowing, the current is the same. This is a simple principle but often confuses the beginner who has a tendency to suppose that a current may start out of a certain strength but may be used up and dwindle away as it progresses around the circuit. The current may be compared to water which completely fills a pipe bent into the shape of a ring. No water can move at any point unless exactly the same amount moves at every other cross-section of the pipe.

A corollary following directly from the above is that *the amount of chemical action at every cross-section of a circuit is the same.*

This may be shown experimentally as follows. In Fig. 108, *A* represents a battery of Daniell cells connected one after the other, or *in series* (three are represented in the diagram but as many as may be necessary are used), and *B*, *C*, *D*, and *E* represent copper voltameters. When the key *K* is closed, completing the circuit, a current flows through the battery, through *B*, then divides, part going through *C* and the rest through *D*, then reunites, passes through *E* and back to the negative pole of the battery. Before

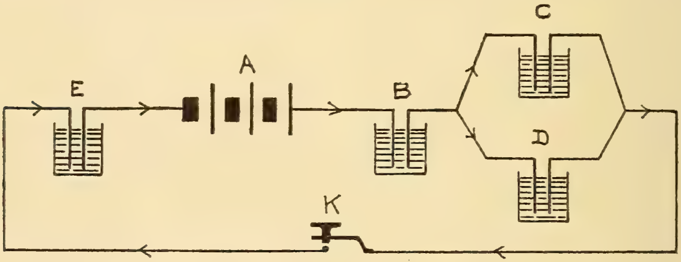


Fig. 108.

closing the key, the cathodes of the voltameters and of each of the Daniell cells are carefully weighed. After the current has flowed for a while, the key is opened, stopping the current, and the cathodes are removed, dried, and carefully reweighed. They are all found to have increased in weight, the increase being exactly the same in all except *C* and *D* and in these their joint increase being equal to the increase in each of the other cathodes. It is to be especially noted that the amount of chemical action is also the same in every one of the battery cells in series.

230. Faraday's Second Law.—Suppose we arrange a similar experiment with a number of voltameters in series but each containing different compounds. Suppose one to be a gas voltameter, one to contain a solution of silver nitrate, one of copper sulphate, one of cuprous chloride and one of tin tetra-chloride. If now the key be closed, the same current will traverse them all. After the current has flowed for a while, open the key, remove and weigh the cathodes and measure and calculate the weight of the hydrogen evolved in the gas voltameter. If we take the weight of this hydrogen as unity we will find that 107.9 parts of silver, 31.8 parts of copper in the copper sulphate solution, 63.6 parts in the cuprous chloride solution and 29.8 parts of tin have been deposited. But

these numbers, 107.9, 31.8, 63.6, and 29.8 are the equivalent weights of the corresponding elements in the respective compounds. (The equivalent weight of an element or of a radicle is defined as that weight of it which combines with or displaces or is chemically equal to one part by weight of hydrogen. It may be obtained by dividing the atomic weight of the element, or the molecular weight of the radicle, by the valency which it has in the compound under consideration.) The foregoing results are expressed in *Faraday's second law* which is to the effect that the weights of the ions of different substances liberated by the same quantity of electricity are to each other as the equivalent weights of these ions.

231. Electro-Chemical Equivalent.—The electro-chemical equivalent of an element is the weight in grams of that element liberated by one coulomb. By definition (Par. 228) one coulomb liberates .00001035+ of a gram of hydrogen, which is therefore its electro-chemical equivalent. The electro-chemical equivalent of any other element is obtained by multiplying its equivalent weight by this electro-chemical equivalent of hydrogen. For example, for silver it is $107.93 \times .00001035+ = .001118$, and for copper it is .000328.

To liberate one gram of hydrogen (about four-tenths of a cubic foot at ordinary temperature) requires $1/.00001035+$, or, in round numbers, 96,540 coulombs. This would require a current of one ampere to flow for nearly 27 hours. This quantity of electricity, 96,540 coulombs, will release one gram-equivalent of any ion, as for example 8 grams of oxygen, 107.93 grams of silver, etc.

From the foregoing, it is seen that to find the weight of any ion released by a given current in a given time, we determine the number of coulombs and multiply this number by the electro-chemical equivalent of the ion.

232. Definition of the Ampere in Terms of Silver.—For practical purposes, because of the difficulty of handling and weighing a gas, it is desirable to have the ampere defined in terms of some solid element instead of hydrogen. Silver is found to be the most suitable and copper the next. In the preceding paragraph we have seen that the electro-chemical equivalent of silver, the weight deposited by one coulomb, or one ampere flowing for one second, is .001118 gram. The International Congress of Electricians of 1893 in the resolutions already referred to (Par. 212), accord-

ingly defined the ampere as that unit "which is represented sufficiently well for practical use by the unvarying current which when passed through a solution of nitrate of silver in water, in accordance with the accompanying specification, deposits silver at the rate of .001118 gramme per second."

233. Applications of Electrolysis, Refining of Copper.—Copper as it comes from the smelter may contain impurities of two kinds, first, objectionable substances such as arsenic, antimony, etc., which injure its ductility and its electrical properties and, second, small amounts of gold and silver which it is desirable to recover if possible. The impure copper is cast into slabs which are used as the anodes in large electrolytic tanks, the electrolyte being a solution of copper sulphate and the cathodes being thin sheets of pure copper. As the current passes, the anode is eaten away, the pure copper being deposited upon the cathode and the impurities settling as a slime to the bottom of the tank whence they are removed from time to time and treated according to their value. If the impure copper contains much gold or silver, the anodes may be enclosed in canvas bags which permit the free passage of the solution but catch the slime which falls. The copper is refined at the rate of about seven pounds per hour per horse-power expended.

234. Electroplating.—The object to be plated is immersed in the electrolyte and serves as the cathode. In gold and silver plating, the anode is a plate of the desired metal and the electrolyte is a double cyanide of potassium and this metal. The deposits from these cyanides are smoother and more compact than those from other salts. There must be a certain relation between the current and the area of the surface to be plated. If the current be too great, the deposit is granular or coarsely crystalline and may not adhere. Portions of the surface which are not to be plated may be covered with a coating of wax or varnish.

235. Electrotyping.—The process of electrotyping is employed to obtain exact reproductions of wood cuts, engraved plates, forms of set type, etc. The need for such reproductions is readily understood. If impressions be taken direct from a wood cut it rapidly wears away and frequently gives out when about 5000 have been struck off. By electrotyping, a reproduction of the cut can be made in copper and this reproduction can be used many thousand times and as many others may be made as desired, the

original cut not suffering in the slightest. Again, a great many million postage stamps are printed annually by the Government and not only must they be struck off several hundred in a sheet but several presses must be running at the same time. If each plate had to be engraved separately the cost would be tremendous and no two stamps on a sheet would be exactly alike. However, the engraver prepares a plate for a single stamp and hundreds of reproductions can be made and these reproductions can then be united in one large plate. Finally, when type have been set for a printed page they are withdrawn from the printer's stock. Should this run low, he must either purchase more or distribute those which have been set up, thus undoing the work. However, by electrotyping he can reproduce the entire page in one piece and the type then become available for other use.

The process consists in pressing the cut or type to be reproduced into a sheet of wax or other plastic material, thus making a mould. The interior of this mould is then dusted with very finely powdered graphite or bronze by which the surface is made a conductor, and using this as the cathode a thin layer of copper is deposited upon it. This thin layer is then backed by pouring into it melted type metal and the resulting plate is fastened to a wooden block.

CHAPTER 22.

THE STORAGE BATTERY.

236. Reversibility of Cells.—Should a simple zinc-carbon cell be connected in closed circuit, a current will be produced and while it is flowing the zinc will waste away and go into solution as zinc sulphate, the electrolyte will grow weaker and hydrogen will be evolved at the carbon plate. Suppose now the circuit to be broken and that there be inserted in it a battery or an electrical machine faced in the opposite direction to the original cell. If this battery or machine produces a greater electro-motive force than the cell, a current will be set up opposite to the original current and will flow through the cell in a reverse direction, that is, the simple cell now becomes an electrolytic cell (Par. 220). The zinc sulphate in solution will be decomposed, the zinc being redeposited upon the zinc plate (Par. 224), the electrolyte increasing in strength and oxygen being released at the carbon plate, in other words, if the current continues to flow for a sufficient length of time the previous chemical action will be undone and, with the exception of the loss of a small amount of water in the form of hydrogen and oxygen, the cell will be restored to its primary condition. Such a cell is said to be *reversible.* It is evident that a primary cell in which the chemical action results in the escape in the form of gas of a portion of the active material can not be entirely reversible.

237. Storage Battery.—A cell which is thus reversible and which when exhausted is regenerated by passing through it from an extraneous source of electrical energy a current opposite in direction to the flow of discharge, is called a *secondary cell,* or an *accumulator,* or, more commonly, a *storage battery,* although strictly the word "battery," as already pointed out, should be applied to a group of two or more cells. When such a battery approaches exhaustion it is said to be *discharged,* and the operation of restoring it is called *charging.* As commonly understood, a storage battery is one whose primary condition is that of exhaus-

tion, that is, one which can not be used until it has first been charged. Reflection will show that the charging current must enter the battery by the same pole from which the discharging current leaves, that is, by the positive pole. The academic distinction between the positive pole and the positive plate of voltaic cells (Par. 193) is not observed in dealing with storage batteries and the *positive plate* is that which carries the *positive pole* and is that plate from which the current issues on discharge and by which it enters on charge. In these storage batteries there is no electricity stored up. The charging current enters at the positive pole, passes through the battery and leaves by the negative pole, but in its passage it performs chemical work or builds up a certain chemical potential which later produces electrical energy when the proper connections are made.

238. Elements of a Secondary Cell.—Experiments have been conducted with many substances to determine their fitness for the elements of a secondary cell but, with the exception of the recently introduced nickel-iron-potassium hydroxide cell of Edison (Par. 250), the great majority of storage batteries employ positive plates of lead peroxide, PbO_2, negative plates of pure lead, and an electrolyte of dilute sulphuric acid of a specific gravity of about 1.20, or about one part of acid by bulk to five of water. There are many objections to lead; it is very heavy, it is soft, and the workmen in it frequently suffer from lead poisoning. There must then be some peculiar qualities of lead which outweigh these disadvantages. Upon examining its chemical properties we are at once struck by the fact that it is the only commercial metal whose sulphate is insoluble. When, therefore, the electrolyte attacks the plates and produces lead sulphate, this salt does not pass off into solution but remains at the precise spot where formed and when the cell is charged the sulphate is reconverted into lead without any change of position. Repeated charging and discharging, therefore, does not materially alter the shape of the plates.

239. Preparation of the Plates.—The peroxide of lead of the positive plate and the pure lead of the negative plate are designated as the *active material* of the cell. Since the chemical action, the source of the electrical energy developed, takes place only at the surface of contact of the active material and the electrolyte.

the object held in view in preparing the plates is to give to this active material the maximum amount of surface. This object is attained in any one or combination of three ways.

(a) *Mechanical.*—The plate may be deeply incised, or grooved, or fluted, or thin tape-like ribbons of lead may be corrugated, coiled up and inserted in apertures in the plate proper, or the active material may be applied to the plate as a paste, or it may be powdered and placed in perforated receptacles which are attached to the plate.

(b) *Chemical.*—The metal may be eaten by acids until it becomes more or less spongy, or it may be cast mixed with a granulated substance which is subsequently dissolved out leaving the plate porous.

(c) *Electrolytic.*—The plate may have attached to it or enclosed in cavities in it a salt of the metal, which salt, as may be desired, is either converted by electrolytic action into the peroxide or else reduced to a finely divided metallic state.

Since neither the peroxide nor the spongy lead possess the requisite mechanical strength for plates, the active material is generally contained or supported in spaces between the ribs of a grid-iron shaped frame of lead. On this account, the plates are frequently called *grids.*

240. The Planté Cell.—The first storage batteries were produced by Planté in 1860. The plates were prepared by placing face to face, and separated by a layer of felt two thin sheets of lead which were rolled up spirally into a cylinder and placed in a cell containing dilute sulphuric acid. On passing a current through the cell the water was decomposed (Par. 219) and the oxygen released at the anode converted the surface of this plate into the peroxide. After a number of hours the current was reversed. The other plate now became the anode and was converted into the peroxide, while the hydrogen released at the cathode reduced the former peroxide to metallic lead, leaving it in a spongy condition. The current was thus reversed several times and each time the chemical action penetrated more deeply into the plates, or the plates were said to be "worked up." It will be seen in the following paragraphs that the principle of the preparation of the plates in more modern storage batteries is the same, although the details are different.

241. The Chloride Accumulator.—A well known form of storage battery is the *chloride accumulator,* so called because in the manufacture of the earlier forms the chlorides of zinc and lead were used. In preparing the *negative* plates, the powdered chlorides of lead and zinc were intimately mixed and melted and the fused mass was then cast into little blocks a quarter of an inch thick and about an inch square. These blocks were then placed in a mould, arranged in regular order and evenly spaced, and melted lead was poured into the mould. The resulting plate can be compared to a window sash, the lead corresponding to the wood

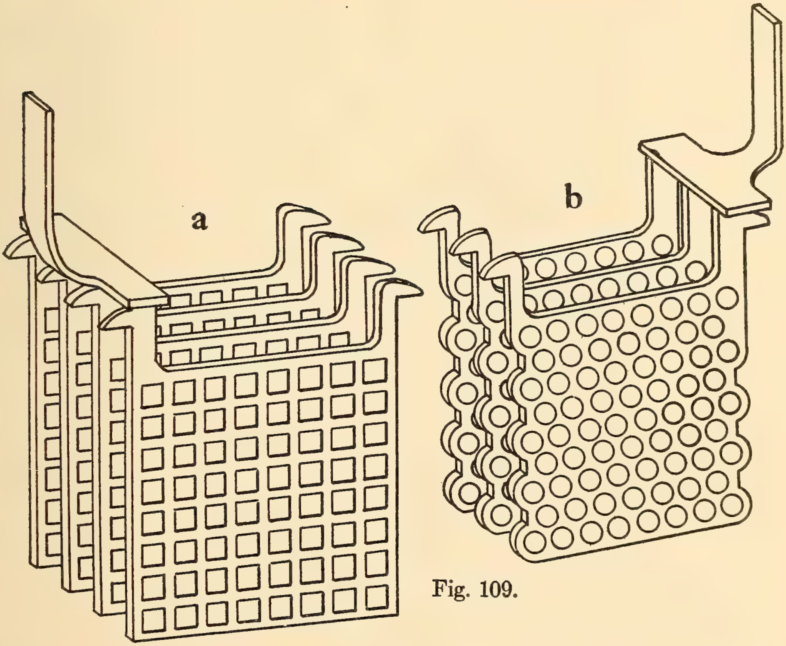


Fig. 109.

work and the chloride blocks to the panes (Fig. 109 *a*). The plate was next soaked in water which dissolved out and removed the zinc chloride and left the lead chloride in a porous condition. Finally, this plate was made the cathode of an electrolytic cell and a current passed through it until the lead chloride was entirely reduced to spongy lead. In more recent forms the negative grid is composed of two faces each containing shallow rectangular cavities, the bottoms of these being finely perforated. They are filled with one of the oxides of lead, the two faces are then pressed

together and rivetted firmly. The perforations permit the electrolyte to reach the lead oxide which by electrolytic action is reduced to spongy lead.

The grid for the *positive* plate was made of lead which, for the sake of hardness, was alloyed with a small amount of antimony. It was cast with rows of circular openings (Fig. 109 *b*) which were not cylindrical but contracted towards the center of the plate. Thin corrugated ribbons of lead were rolled up into cylinders and pressed into these openings, the shape of the openings causing the cylinders to be held firmly. The plate was then made the anode of an electrolytic cell for about 30 hours, the oxygen released by the current converting a part of the lead into lead peroxide. The amount of active material is sometimes increased by filling the crevices in the corrugated tape with a paste of either red lead, Pb_3O_4, or of litharge, PbO, both of which become peroxide in the electrolytic cell.

242. Shape and Size of Plates.—The plates, except the largest sizes, are square. The thickness of the smaller plates is one-quarter of an inch but for the sake of strength this is increased to one-half inch in the larger ones. The size varies with the current which the battery is designed to furnish when discharged at its *normal rate,* that is, at the rate which experience has shown can not be exceeded without more or less injury to the plate. This is generally taken as about six amperes per square foot of positive plate surface. Thus the *E* plate of the chloride accumulator measures 7.75×7.75 inches, or 120 square inches, which is five-sixths of a square foot, and the normal rate is given by its manufacturers as five amperes. If the cell contains three of these plates, its normal rate is 15 amperes, etc. The plates of this battery are designated by the letters of the alphabet, the *B* plate being the smallest and measuring 3×3 inches, and each has twice the *active surface* and twice the normal rate of the next smaller size. Thus the normal rate of an *F* plate is ten amperes.

243. Grouping of Plates.—The plates are cast with three lugs at the top. Two of these rest on the opposite sides of the cell when the plate is in position, support its weight and keep it an inch or so above the bottom of the cell (Fig. 110). The third is used to join the similar plates of one cell to a common terminal or cross strap. They fit into holes mortised in the cross strap and

are "burned" to the strap by a hydrogen flame, the hydrogen reducing any oxide on the surface of the molten metal and thus allowing a perfect joint to be formed.

The number of plates is always odd, there being one more negative plate, so that each positive plate has a negative plate presented to each of its faces. The smallest number of plates is therefore three; on the other hand, cells are made which contain 75 or more. The total number of plates per cell is indicated by a subscript after the letter designating the size, as B_3, C_5, etc.

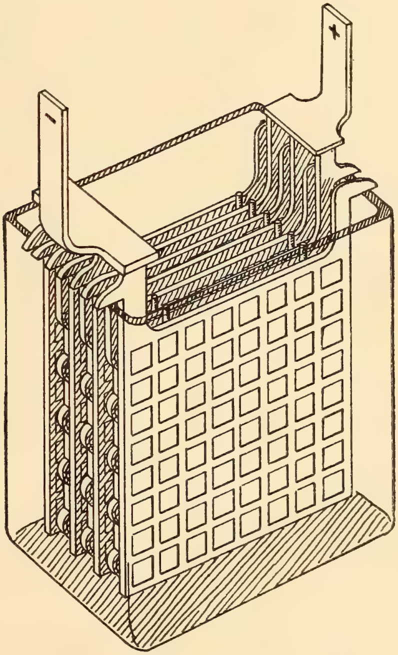


Fig. 110.

The cells, except those of large size and those for use in vehicles, are of glass. They frequently rest in shallow boxes which contain sand so as to distribute the weight evenly over the bottom, the boxes in turn resting on insulating glass supports. The cells for vehicles are of hard rubber and have rubber covers. The larger cells are lead-lined wooden tanks. The largest chloride accumulator cell contains 75 plates, each 15×31 inches, weighs three tons and will furnish 1500 amperes for eight hours.

Should dissimilar plates touch each other directly or be put in contact through any sediment at the bottom of the cell, they will be short circuited (Par. 306). For this reason they are held apart by some form of fender or "separator," and, as stated above, are supported an inch or so above the bottom of the cell. Formerly rods of glass or of hard rubber were used as separators but now preference is given to thin wooden boards of the thickness used in making berry boxes. Owing to this compact arrangement of the plates the internal resistance of a storage cell is very small (Par. 294), usually something less than one-thousandth of an ohm.

244. Reaction on Discharge and Charge.—When the cell has been completely charged, the active material of the positive plate being lead peroxide and that of the negative plate spongy lead, we have the requisite conditions for a simple voltaic cell (Par. 201), that is, two conducting substances immersed in a liquid which attacks one more freely than it does the other. When the circuit is closed the electrolyte attacks the negative plate (Fig. 111 *a*)

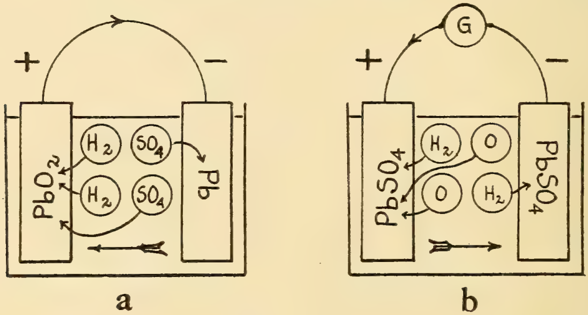


Fig. 111.

producing lead sulphate. Hydrogen released at the positive plate is converted into water at the expense of the oxygen of the peroxide, that is, the peroxide is the depolarizer of the cell. When the peroxide has thus been deoxidized, the remaining lead is attacked by the electrolyte, producing lead sulphate and action ceases. In practice however, the cell is recharged before this limit is reached. The reaction may be written

Positive Plate	Electro-lyte	Negative Plate		Positive Plate	Electro-lyte	Negative Plate
$PbO_2 +$	$2H_2SO_4 +$	Pb	$=$	$PbSO_4 +$	$2H_2O$	$+ PbSO_4$

although actually it is more complicated.

It will be noted that during discharge the acid is withdrawn from the electrolyte and goes into combination with the plates and that water is released in its stead, that is, the E. M. F. of the cell decreases, the resistance of the electrolyte increases and its specific gravity decreases.

The reactions on charge are the reverse of those on discharge. Fig. 111 *b* represents diagrammatically an electric generator sending a current through the cell, both of whose plates are supposed to have become lead sulphate. The water of the electrolyte is decomposed, the hydrogen removing the SO_4 from the plates and forming again H_2SO_4, and the oxygen released at the positive plate reconverting the lead into the peroxide. The reaction is

$$\underset{\text{Positive Plate}}{PbSO_4} + \underset{\text{Electrolyte}}{2H_2O} + \underset{\text{Negative Plate}}{PbSO_4} = \underset{\text{Positive Plate}}{PbO_2} + \underset{\text{Electrolyte}}{2H_2SO_4} + \underset{\text{Negative Plate}}{Pb}$$

As a result of this, the E. M. F. of the cell rises, the resistance of the electrolyte decreases and its specific gravity increases.

245. Charging.—The current for charging a storage battery is generally furnished by a generator, though a battery of a few cells may be charged from a larger battery. This current, as has already been stated, is brought in at the positive pole of the battery. Its E. M. F. should be from 5 to 10 per cent greater than that of the battery and since the E. M. F. of the battery rises as the charging progresses, there must be some arrangement by which the charging E. M. F. may be increased correspondingly. If the E. M. F. of the source of supply be less than that of the battery, the latter during charging must be subdivided into groups which are conveniently charged in parallel (Par. 336). When a battery is discharged it must be recharged at once, for if the discharged plates remain in the acid for even a short time they become injured (Par. 247). The rate at which the battery is charged is fixed by the makers and averages about ten per cent less than the normal rate of discharge. It can not be exceeded without risk. At least as much time is required to charge a battery as to discharge it. When a battery is put into commission for the first time it has to be charged at the normal rate for from 45 to 55 hours continuously but thereafter the normal time is about eight or nine hours.

246. Indications of Charge.—It is important to be able to tell when a battery is properly charged. The indications usually relied upon are the following:—

(a) *Voltage.*—A new cell, when fully charged and while still receiving the charging current, should have a voltage of 2.5 or even slightly more, but this decreases with age. When current is drawn from the cell the voltage almost immediately falls to 2.05 or 2.0 after which it decreases slowly and steadily until the cell approaches exhaustion at which point it begins to drop rapidly (Fig. 112). A cell should never be discharged to a lower voltage than 1.7 and if it reaches this point should be recharged at once. In actual charging the process is continued until three successive readings of the voltmeter at intervals of fifteen minutes show no further rise. Usually some average interior cell of the battery is

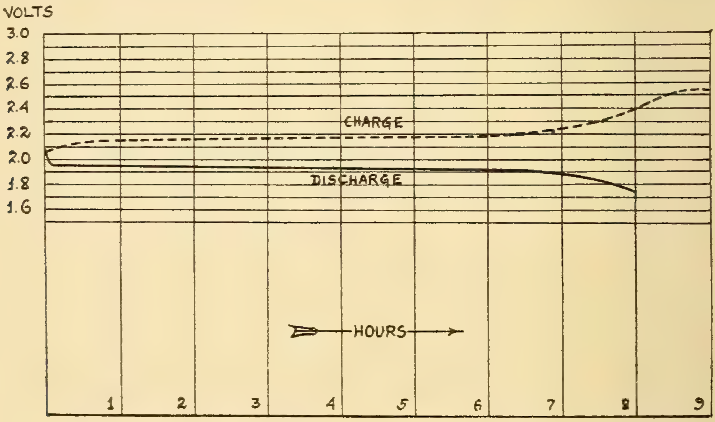


Fig. 112.

selected as a "pilot cell" and its voltage is taken as an indication of that of the others. In order that these observations may be of any value, the voltage must be taken while the battery is either being charged or discharged at the normal rate.

(b) *Specific gravity of the electrolyte.*—Examination of the reactions given in Par. 244 shows that during charge sulphuric acid is driven out from its combination with the plates and is released in the electrolyte. The specific gravity of sulphuric acid (1.834) being nearly twice as great as that of water, that of the electrolyte rises accordingly. When discharged, the specific gravity of the electrolyte may fall as low as 1.175 or even less, and when charged it should lie between 1.200 and 1.210. The specific gravity is read from a *hydrometer*, a little lead-weighted, flattened glass float having a slender graduated stem and look-

ing somewhat like a thermometer (Fig. 113). As the density of the electrolyte decreases the hydrometer sinks deeper into the liquid; as it increases, the hydrometer floats higher and in each case the corresponding specific gravity is indicated by the graduation on the stem of the instrument reached by the surface of the electrolyte.

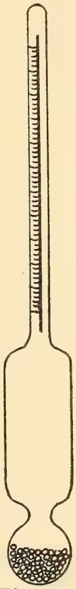


Fig. 113.

(c) *Gassing.*—When bubbles of gas begin to rise freely in the cell, giving the liquid the appearance of boiling, the current has completed its work upon the plates and is decomposing the electrolyte, the charging therefore should not be pushed farther. These mixed gases are explosive, therefore the storage battery room should be well ventilated and no flame should be taken into the room when the cells are gassing.

(d) *Color of the plates.*—When fully charged the positive plates are of a rich chocolate color, the negative plates a lead grey, and these colors afford the expert a means of judging of the state of charge.

247. Troubles of Lead Batteries.—If a lead-sulphuric acid battery be charged or discharged at an excessive rate, or be allowed to stand discharged, the acid attacks the plates and forms a white coating supposed to be the basic lead-sulphate Pb_2SO_5. The plates are then said to be *sulphated.* This coating is insoluble and a non-conductor and practically removes from action the part of the plate which it effects. When not too extensive, it may sometimes be removed by repeated charging and discharging of the cells.

The crystals of sulphate forming within the porous portions of the plate sometimes act as wedges and cause the plate to *buckle,* that is, to bulge out in a dish shape. This usually loosens and causes a loss of the active material of the plate and may produce a short circuit with the adjacent plates of the cell.

248. Care of Lead Batteries.—Lead batteries must be given constant attention. Charging should be done at regular intervals and the battery must never be allowed to stand discharged. Each cell should be numbered; these numbers should be entered in a blank book and a weekly record should be kept of the voltage and the specific gravity of each cell. Inspection of this record will frequently reveal incipient trouble in individual cells and will thus

enable corrections to be applied before serious damage has occurred.

A battery should not long remain idle. If it is not to be used for some months it should be put out of commission. It is charged fully, thus expelling into the electrolyte the acid in combination with the plates. The electrolyte is then syphoned off into carboys, the cells filled with water and allowed to stand for 48 hours, after which the water is drawn off.

249. Objections to Lead Batteries.—The principal objections advanced against lead batteries are—

(a) Poisonous effect of lead upon the workmen engaged in the manufacture of the plates.

(b) Excessive weight of the plates, lead being the heaviest of the commercial metals.

(c) Fragility of the cells and inability to stand shocks and jars.

(d) Need of constant supervision by an expert electrician for proper care of the battery.

(e) Injury resulting to the battery if it remains uncharged for any length of time.

(f) Injury resulting to the battery if it remains long charged and hence necessity of charging and discharging even when use of battery is not required.

(g) Injury produced by short circuits or by charging or discharging at excessive rates.

(h) Injury produced by using the battery if the temperature rises above 100° F.

(i) Loss of active material from the plates.

(j) Production of acid vapors highly irritating to the throat and lungs and corrosive to surrounding objects of metal.

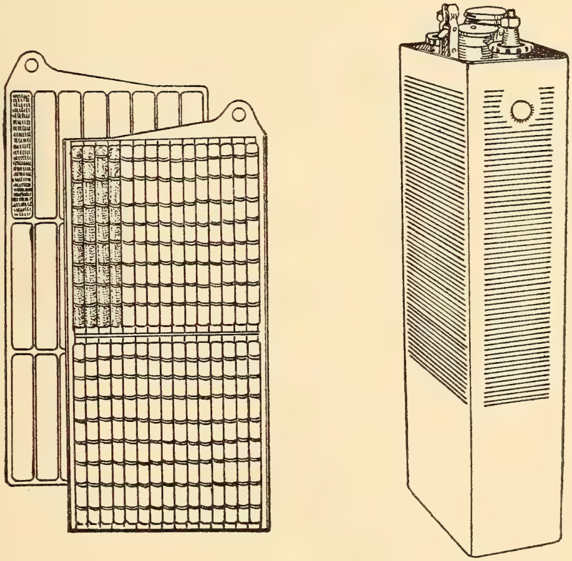
(k) Production of explosive gases.

(l) Loss of charge on standing. This amounts to about 25 per cent per week.

The foregoing indicates that the lead battery is most advantageously employed when it is installed in a suitable building and subjected to constant use under the supervision of a trained electrician, and that it is not well adapted for service in vehicles used roughly and irregularly and cared for by unskilled attendants.

250. The Edison Storage Battery.—The Edison storage battery is designed primarily for use in vehicles and has been developed to avoid as far as possible the objections enumerated in the preceding paragraph. In this battery the active material of the positive plate is nickel peroxide, Ni_2O_3, that of the negative plate is finely divided iron, and the electrolyte is a 21 per cent solution of potassium hydroxide, KOH, to which is added a small amount of lithium hydroxide. The grids are of nickel-plated steel.

The active material of the positive plate, initially in the form of nickel hydroxide, $Ni(OH)_2$, is packed in small pencil-like perforated tubes of nickel-plated steel which are securely fastened to the grid (Fig. 114 *a*). To improve the conductivity of this active material,



b

Fig. 114.

it is interspersed with layers of extremely thin nickel flakes, there being as many as 350 layers in each tube in a length of about four inches. These tubes are banded at intervals by steel hoops which prevent any expansion due to swelling of the material within. The active material of the negative plate, primarily ferrous oxide, FeO, is packed into flat perforated pockets of nickeled steel which are forced into the grid under pressure. A small per cent of mercury is added to the oxide to improve its conductivity.

The plates are held together by nickeled-steel cross bolts which also carry the terminals. Opposite plates are held apart by rubber separators. The cells are of nickel-plated sheet steel, corrugated for rigidity (Fig. 114 *b*). The assembled plates, protected on all sides by rubber fenders, are fitted tightly into the cell which is then closed by a steel lid which is welded on. This lid contains an opening through which electrolyte may be introduced and is arranged with a valve which permits gas to escape from the cell but prevents gas from entering. Potassium hydroxide has a great affinity for carbonic acid gas, CO_2, which, if the cell were left open, would rapidly injure the electrolyte.

There are two regular sizes of plates designated *A* and *B*. The *A* plates are the larger, the rectangular portion being about 5×12 inches. A number following the letter, as *A*-4, indicates not the total number of plates but the number of positive plates in the cell. The normal rate of discharge of an *A* plate is seven and a half amperes. The normal rate of an *A*-4 cell is therefore thirty amperes.

251. Reactions of the Edison Battery.—In Par. 223 it was shown that when a current is passed through a solution of KOH the effect is merely to electrolyze the water. On the first charge the oxygen released at the anode converts the nickel hydroxide into the peroxide, thus—

$$2Ni(OH)_2 + O = Ni_2O_3 + 2H_2O$$

and the hydrogen released at the cathode reduces the iron oxide to metallic iron

$$FeO + H_2 = Fe + H_2O$$

On discharge the reaction is as follows:

$$\underset{\text{Positive Plate}}{Ni_2O_3} + \underset{\text{Electrolyte}}{\begin{Bmatrix} KOH \\ H_2O \end{Bmatrix}} + \underset{\text{Negative Plate}}{Fe} = \underset{\text{Positive Plate}}{2NiO} + \underset{\text{Electrolyte}}{\begin{Bmatrix} KOH \\ H_2O \end{Bmatrix}} + \underset{\text{Negative Plate}}{FeO}$$

On charge this is reversed, or

$$\underset{\text{Positive Plate}}{2NiO} + \underset{\text{Electrolyte}}{\begin{Bmatrix} KOH \\ H_2O \end{Bmatrix}} + \underset{\text{Negative Plate}}{FeO} = \underset{\text{Positive Plate}}{Ni_2O_3} + \underset{\text{Electrolyte}}{\begin{Bmatrix} KOH \\ H_2O \end{Bmatrix}} + \underset{\text{Negative Plate}}{Fe}$$

From the preceding it is seen that the reactions in the cell consist in the transfer of oxygen back and forth and that the electrolyte is unaltered. It may therefore be reduced to a minimum with

a corresponding saving of bulk and weight. It would also seem that it should last indefinitely but, as stated in the preceding paragraph, it absorbs and combines readily with carbon dioxide and on this account should be renewed yearly.

252. Charging the Edison Battery.—Since the electrolyte remains unaltered during charge and discharge and since the plates are enclosed in an hermetically sealed steel case, the only indication of charge of an Edison cell is its voltage taken while charging or discharging. During charge the voltage gradually rises (Fig. 115) until when fully charged and receiving current it

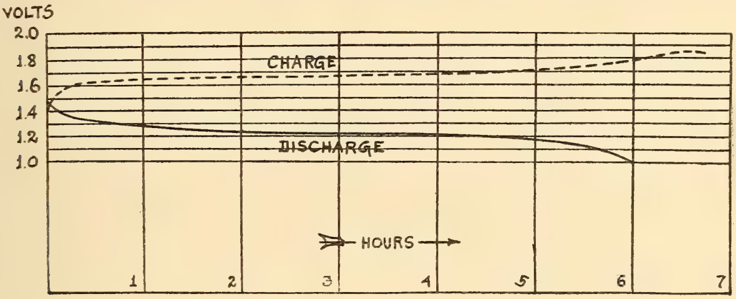


Fig. 115.

reaches a maximum of 1.84. When current is drawn from the cell the voltage drops at once to about 1.4 and then falls gradually, averaging about 1.2 volts until near the end when it drops rapidly to one volt. On the average, a battery is charged at the normal rate in seven hours and discharges in about six.

253. Advantages and Disadvantages of the Edison Battery.—The advantages of the Edison battery are in marked contrast to the disadvantages of the lead battery as enumerated in Par. 249. Thus—

(a) Although the salts of nickel are poisonous, the workmen preparing the plates are free from danger.

(b) The plates are lighter than corresponding lead plates.

(c) The cells could hardly be improved as regards strength. They are uninjured by the most violent jolts and jars to which a vehicle may be exposed.

(d) They require a minimum of attention.

(e) They may be left without injury at any state of charge or discharge.

(f) They may be charged or discharged at excessive rates, may be overcharged, short circuited, or even reversed without permanent injury.

(g) They produce no irritating or corrosive fumes, in fact, by the absorption of carbon dioxide they purify the air. This last renders them especially valuable in submarines.

The disadvantages of the Edison cell are—

(a) Low voltage; only 1.2 as compared to nearly 2.0 of the lead cell, hence a greater number of cells required.

(b) Decrease of activity at temperatures below 40° F.

(c) Greater cost than lead cell.

The efficiency (ratio of energy delivered by the cell to that spent in charging it) of the lead cell is about 75 per cent; that of the Edison cell is only 60 per cent, but weight for weight the efficiency of the Edison cell is the greater.

254. Use of Storage Batteries.—It requires more time to charge a storage battery than it does to discharge it. We have just seen that the efficiency does not exceed 75 per cent. There is therefore a loss of both time and energy and the question arises why should storage batteries be employed. This is best answered by an enumeration of some of the commoner uses of storage batteries. These are—

(a) As a portable source of power and light for vehicles, launches and submarine boats; also for furnishing the ignition spark for automobiles.

(b) As a source of power and light in public buildings, hotels, etc., to run lights, elevators, etc., after the engines have been shut down for the night and thus to save the expense of an extra shift of engineers and firemen.

(c) As a reserve in electrical power plants, supplying power during a temporary stopping of the engines for adjustment, overhauling or repairs.

(d) To light the magazines of a fortification and to operate the mine and the range finding systems.

(e) To carry the "peak loads" of an electric railroad or of a lighting plant. Such a plant must be able to supply the maximum current required during the rush hours. It is also operated most efficiently when the engines are run at a uniform rate. If it supplied constantly the maximum current there would be much

waste during the slack hours. The curve in Fig. 116 may be taken to represent the operation of a trolley line during 24 hours, the horizontal axis being the axis of time, the vertical heights representing the power supplied by the electric plant and consequently

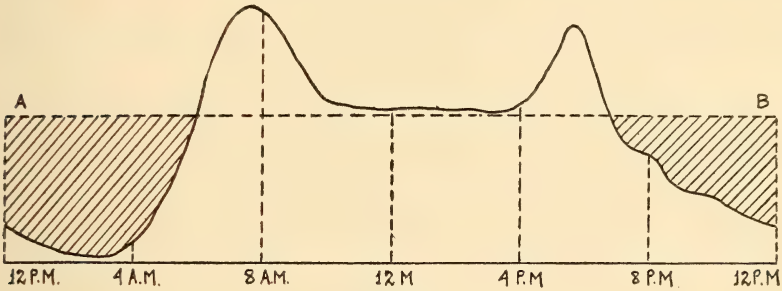


Fig. 116

the area of the curve representing work performed. If the line AB represents the constant output of the engines, the shaded areas represent surplus energy which may be applied to charging a storage battery, the battery in turn being called upon to give back energy when the peak loads occur at 8 A. M. and at 6 P. M.

There are other uses of the storage battery but they can not be explained until our subject has been further developed.

CHAPTER 23.

THEORY OF ELECTROLYTIC DISSOCIATION.

255. Interdependence of the Physical Sciences.—The more our knowledge of the physical sciences is increased, the more we realize their interrelation and their interdependence. The study of no particular one can be successfully pursued if we exclude the help afforded and the side lights thrown upon it by others. This is notably so in the case of electricity. For a proper understanding of the present accepted theory accounting for the phenomena of voltaic electricity, we must turn to physical chemistry and to develop our explanation must begin with certain facts which at first sight appear to have not even a remote connection with our announced subject.

The following outline will assist the student in following the thread of connection between the facts which will now be brought out:

1. Avogadro's law and a derived corollary applicable to gaseous pressure are explained (Par. 256).
2. Exceptions to the law of gaseous pressure are shown to be due to dissociation which is defined (Pars. 257–258).
3. Osmotic pressure is described and its observation and measurement explained (Pars. 259–262).
4. Osmotic pressure is shown to follow the laws of gaseous pressure (Pars. 263–266).
5. Abnormal osmotic pressures are, like excessive gaseous pressures, shown to be capable of explanation under the supposition of dissociation, otherwise called ionization (Pars. 267–268).
6. Ionization is further explained (Pars. 269–274).
7. Electrolytic properties are shown to depend upon ionization (Pars. 275–279).
8. Electricity is shown to be atomic in character (Par. 280).

256. Laws of Variation of Gaseous Pressure.—*Avogadro's Law*, of fundamental importance in Chemistry, is to the effect that under like conditions of temperature and pressure, equal

volumes of all gases, simple or compound, contain the same number of molecules. If we should have a series of cylinders of exactly the same capacity and should fill one with oxygen, one with hydrogen, one with carbon dioxide, one with marsh gas, and so on, each being at the same temperature and exposed to the same pressure, then each would contain exactly the same number of molecules.

Suppose one of these cylinders of the same diameter as the others should be twice as tall. If this one be filled with gas it will, from the above, contain twice as many molecules as the others. Place a piston in the mouth of this cylinder and press it down until the volume of the enclosed gas be reduced one-half, that is, until it becomes the same as that of the other cylinders. The space beneath the piston now contains twice as many molecules as the other cylinders contain. From *Mariotte's Law*, temperature remaining constant, the volume of a gas varies inversely as the pressure. The pressure upon the compressed cylinder is therefore twice that upon the others. Hence we may state, as a corollary to Avogadro's law, that *for a constant temperature and volume, the pressure of a gas varies directly as the number of molecules enclosed.*

From a combination of *Charles'* and *Mariotte's Laws* it is shown that for constant volume, the pressure produced by an enclosed gas varies as the absolute temperature. (The absolute temperature is obtained by adding the constant 273 to the temperature as indicated on the Centigrade scale.) We therefore see that the pressure of a gas confined in a given volume varies (a) with the number of molecules enclosed and, (b) with the absolute temperature.

257. Decomposition and Dissociation.—In general, compound substances if heated to a sufficiently high temperature are resolved into simpler ones. If when these simpler substances are cooled to the primary temperature they remain separate, the original compound body is said to have undergone *decomposition.* On the other hand, if when the temperature falls the simpler substances recombine and reproduce the original substance, this body is said to have undergone *dissociation.* Decomposition is therefore permanent while dissociation is transient and continues only so long as the agency which brought it about is operative.

258. Example of Dissociation by Heat.—Ammonium chloride, NH_4Cl, like other ammonium salts, is volatilized with compar-

ative ease. Its molecular weight being 53.5, the gas produced by the volatilization of 53.5 grams should exert the same pressure as that produced by a *molugram* of any other gas confined in an equal volume and at the same temperature. (A molugram is the molecular weight expressed in grams, as for example 2 grams of hydrogen, 28 grams of nitrogen, 44 grams of carbon dioxide, and so on.) By actual experiment however the pressure is found to be twice as great. From (a) Par. 256 therefore, there must be twice as many molecules present in the gaseous NH_4Cl as there are in the other gases. The explanation is that the NH_4Cl has been dissociated by the heat, each molecule becoming two, one of ammonia, NH_3, the other of hydrochloric acid, HCl. That this is so may be proven in several ways. First, if NH_4Cl became a gas without dissociation, the specific gravity of this gas referred to hydrogen should be 26.7 while it is actually only 13.35 which is the specific gravity of a mixture of equal volumes of NH_3 and HCl. Second, the specific gravity of HCl being 18.2 while that of NH_3 is only 8.5, if the dissociation takes place in a vertical closed tube, the heavier HCl will settle at the bottom, the lighter NH_3 rising to the top. If by means of a stop cock at the middle of the tube the two halves be now cut apart and after cooling be tested separately, the contents of the upper half will be found to be alkaline, that of the lower half acid.

259. Osmosis and Osmotic Pressure.—Suppose the space below the piston of a vertical cylinder to be filled with a gas under normal pressure. If the piston be raised, thereby increasing the space beneath it, the gas will be found to have spread through this new space completely filling it. There is therefore a force or pressure which compels a volume of gas to diffuse or to swell out and occupy a greater space when it has the opportunity to do so.

Again, if in the bottom of a vessel there be placed a concentrated solution of a salt and if then there be poured carefully on top of this solution a layer of pure water, in a short while the dissolved salt, in defiance of gravity, will have spread upward and throughout the liquid until the latter is all of a uniform density. By using a colored salt the progress of the diffusion can be easily observed. There is therefore a force, similar to the gaseous pressure described above, which urges the particles of a dissolved substance to spread equally throughout the solvent.

There are known various membranes, some animal, some vegetable, and some artificial, which will permit the passage through them of certain liquids but will prevent the passage of other substances dissolved in these liquids. On account of this property these membranes are called *semi-permeable.* If a bladder (which is one of these) be filled with an aqueous solution of a salt, tied tightly, and then submerged in a vessel of pure water, it will gradually distend and may finally burst. This is explained by saying that the substance in solution is urged by the pressure described above to spread out into the surrounding solvent but being unable to pass through the membrane it pushes against it and distends it, thus allowing the water on the outside to enter. Although this explanation is admittedly a poor one, the phenomenon does occur and is called *osmosis,* and the force exerted upon the membrane by the dissolved molecules is called *osmotic pressure.*

In the above illustration we have assumed an aqueous solution of a salt but under proper conditions osmosis takes place whatever the nature of the solvent or of the dissolved substance.

260. Demonstration of Osmotic Pressure.—Osmotic pressure may be conveniently shown as follows. A membrane is stretched and tied over the mouth of a glass funnel which is then inverted and filled to the neck with a solution, say of copper sulphate. The inverted funnel is then inserted, as shown in Fig. 117, in a vessel of pure water until the surface of the water and that of the liquid in the neck of the funnel are at the same level. The copper sulphate solution will be observed to rise slowly in the neck of the funnel and may continue to do so for several weeks, attaining its maximum height when the hydrostatic pressure of the liquid in the tube just prevents the passage of additional water through the membrane and the further dilution of the contained solution. The osmotic pressure and the hydrostatic pressure are now in balance and by measuring the latter we determine the former.

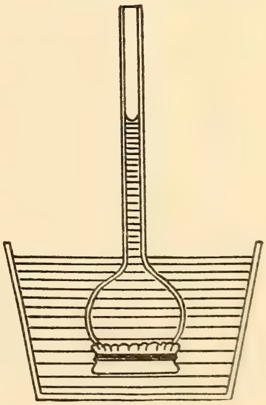


Fig. 117.

261. Measurement of Osmotic Pressure.—The arrangement described in the preceding paragraph is not well suited for the measurement of osmotic pressures. These are relatively great. The osmotic pressure produced by a dilute solution of sugar has driven a column of water to a height of nearly 70 feet, and this pressure is frequently exceeded. The membrane would not stand these pressures and it is impracticable to use tubes of such length. Again, the membrane is not absolutely impermeable to the salt and some escapes into the surrounding solvent. Also, the membrane is distended, thereby increasing the volume of the confined solution and materially altering the degree of concentration. For these reasons accurate determinations of osmotic pressure were not made until within recent years when it was discovered that certain colloidal or gelatinous precipitates, notably the ferrocyanide of copper, act, so far as permeability is concerned, as ideal membranes. The strength of a film of such a precipitate is however very small and in order that it may withstand the pressure to which it is to be subjected it must be supported in some way. This object is now attained by depositing the film within the substance of a finely porous unglazed porcelain cup. These cups, about two inches tall and three-quarters of an inch in diameter, are first filled with a solution of potassium ferrocyanide which slowly soaks into the walls. They are then immersed in a solution of copper sulphate, which soaks in from the outside, and when the two liquids encounter each other the precipitate is formed. The actual process requires several days' time and involves a number of precautions not necessary to mention here. Into the mouth of the prepared cup are cemented the tube up which the liquid is to rise and a second tube with a stop cock by which the solution is introduced. The vertical or pressure tube is sealed at the top and the osmotic pressure may be determined by the amount of compression of the air above the liquid. In practice, the pressure tube is a mercurial manometer. By using these closed tubes to measure the pressure, the amount of the solvent which enters the cup is reduced to a minimum and the concentration of the solution is altered but little.

262. Observations of Pfeffer.—The botanist, Pfeffer, in his investigations in plant physiology, made, with the apparatus just described, a series of observations upon the osmotic pressure produced under various conditions by *dilute solutions of organic*

compounds such as sugars, alcohols, etc. His results were published in 1877 but at that time attracted no special attention and it remained for Arrhenius and Van't Hoff to discover some ten years later the value of his data and its bearing upon the theory which we shall shortly explain.

263. Osmotic Pressure Varies Directly with Number of Molecules Dissolved in Given Volume of Solution.—Pfeffer found that for these dilute solutions the osmotic pressure increased directly with the strength of the solution, that is, if the concentration (and hence the number of molecules in solution) be doubled, the osmotic pressure is likewise doubled, etc. His results for cane sugar were as follows:

Strength of Solution	Osmotic Pressure	Ratio $\frac{P}{S}$
1%	510 mm.	510
2%	1016 mm.	508
4%	2082 mm.	520
6%	3075 mm.	512

In this table, while the pressures do not bear to each other the exact theoretical ratio, the variations therefrom are not greater than are to be expected from experimental errors and from the fact that the observations were not taken under precisely the same conditions of temperature, although they were made within a range of less than three degrees Centigrade.

Comparing different substances, he found that while the osmotic pressure of a one per cent solution of cane sugar at 15.5° C was 520.5 mm., that of a one per cent solution of raffinose at the same temperature was only 299 mm. The relation between these two numbers was not discovered until subsequent investigators worked upon his data. The formula for cane sugar is $C_{12}H_{22}O_{11}$ and its molecular weight is 342; that for raffinose is $C_{18}H_{32}O_{16}5H_2O$ and its molecular weight is 594. Therefore, equal weights of the two do not contain the same number of molecules, a one per cent solution of raffinose containing fewer than a one per cent solution of sugar. Let us see how the pressures compare if we take the same number of molecules of each. Each litre of his cane sugar solution contained $\frac{10}{342}$ of a mologram. The same fraction of a mologram of raffinose would be $\frac{10}{342}$ of 594 or 17.37 grams. If 10 grams produced a pressure of 299 mm., what pressure would 17.37 produce? $10 : 299 :: 17.37 : x$

whence $x = 519.4$ mm. as compared to the 520.5 mm. of the sugar solution.

This and other examples show that substances in solution conform to Avogadro's Law and to its corollaries, that is, equal volumes of solutions which at the same temperature exhibit the same osmotic pressure contain the same number of dissolved molecules, and also, other conditions being constant, the osmotic pressure varies directly with the number of molecules in solution.

264. Osmotic Pressure Follows Mariotte's Law.—An examination of Pfeffer's data will reveal the fact that osmotic pressure also follows the corollary to Mariotte's Law for gaseous pressure, that is, other conditions being constant the osmotic pressure varies directly with the absolute temperature. For example, the osmotic pressure of a one per cent solution of sugar at 14.15° C is 510 mm. and at 32° C is 544 mm. Applying the law to the lower pressure

$$510 : x = 273 + 14.15 : 273 + 32$$

whence $x = 541.7$ mm., agreeing closely with the observed pressure 544 mm.

265. Osmotic Pressure of a Molecule in Solution Equals Pressure of a Gaseous Molecule under Equal Volume and Temperature.—We have seen from Par. 263 above that $\frac{10}{342}$ of a molugram of sugar or of other organic substance dissolved in a litre of water exerts at 15.5° C an osmotic pressure of about 520 mm. Let us see what pressure the same fraction of a molugram (and hence the same number of molecules) of hydrogen confined in the same space and at the same temperature would exert. One gram of hydrogen at 0° C and 760 mm. measures 11.165 litres, therefore, a molugram of hydrogen (2 grams), would under these conditions occupy 22.33 litres, and $\frac{10}{342}$ of a molugram would occupy .6529 litre. At a temperature of 15.5° C this would dilate to .6914 litre and if this be allowed to expand into the space of 1.006 litres (the space occupied by one litre of water to which 10 grams of sugar is added), the pressure would drop according to the proportion

$$760 : x = 1.006 : .6914$$

whence $x = 522.4$ mm.

We see then that the osmotic pressure of the sugar in solution is the same as the pressure exerted by an equal number of mole-

cules of gas confined in the same space and at the same temperature.

266. Van't Hoff's Generalization.—A consideration of the foregoing facts led to the generalization by Van't Hoff which is to the effect that *"the osmotic pressure of a substance in solution is the same as the gas pressure which would be observed if the dissolved substance alone, in gaseous state and at the same temperature, occupied the volume of the solution."* In other words, these substances in solution behave, comparing osmotic pressure to gaseous pressure, precisely as if they had been converted into a gas and filled alone the space occupied by the solution.

Independent theoretical considerations based upon the lowering of the freezing point and the raising of the boiling point by substances in solution lead to the same conclusions and entirely corroborate Van't Hoff.

267. Exceptions to Van't Hoff's Generalization.—Van't Hoff's generalization applies, as we have seen, to dilute solutions of *organic* compounds. If the solutions become concentrated, the laws of osmotic pressure no longer hold strictly. This is thought to be parallel to the failure of gases, as they approach their point of condensation, to follow the laws of Charles and Boyle.

If, now, we turn our attention to solutions of the *inorganic* compounds we find that the majority of them are exceptions and give osmotic pressures *in excess* of those required by theory. *These exceptions embrace solutions of all the acids, all the bases and all the salts.* It might seem therefore that in announcing as general a law to which the exceptions outnumber the agreements, Van't Hoff had overstepped the bounds of prudence.

268. Dissociation Theory of Arrhenius.—In Par. 263 above we saw that osmotic pressure varied directly with the number of molecules in solution. Since in the exceptional cases the pressure is always greater than what it should be in theory, there must be a greater number of molecules present in solution than is indicated by the weights taken. To account for this greater number, Arrhenius advanced the theory that just as the excessive pressure produced by iodine, ammonium chloride, etc., when converted into vapor is explained by the fact that these substances are dissociated by the heat employed, so the excessive osmotic pressures are to be explained by the fact that the substances in solution

undergo dissociation, or *ionization*, that is, split up into a greater number of parts. It is also a part of his theory that these part molecules or ions, whether they be atoms or compound radicles, exert the same osmotic pressure as an undissociated molecule. Some of the consequences following from this theory were so startling and so contrary to the views generally held by chemists that it was at first vigorously combated and reluctantly accepted as one by one the objections advanced against it were explained away. A full exposition of these consequences and replies to the objections would require an extended treatise. We can here do but little more than allude to a few of those most obviously connected with our subject.

269. Why Ionization Takes Place in Solution.—Salts, acids and bases consist of two parts, a metal or hydrogen (or a radicle playing a similar part) combined with an acid radicle or, in the case of the base, with hydroxyl. The metal or hydrogen portion carries a positive charge of electricity; the remaining radicle carries an equal negative charge. These two parts may therefore be regarded as held together by the attraction of these opposite charges. The charges being relatively great (Par. 278) and the distance separating the parts being infinitely small, the attraction is very great (Par. 53). In Par. 90 we saw that if two charged bodies which in air attract or repel each other with a certain force were placed in some other medium whose dielectric coefficient is K, then the force exerted between the two bodies would be only $\frac{1}{K}$th of what it was in air. The dielectric coefficient of water is given in Par. 92 as 80, or with the exceptions of hydrogen peroxide and hydrocyanic acid, greater than that yet determined for any other substance. The force which held the ions together is therefore reduced to $\frac{1}{80}$th of itself when the substance is brought into solution, and the ions drift apart. This view is corroborated by the variation in dissociation produced by using solvents of different dielectric coefficients.

270. How Ionization Takes Place.—Ionization takes place differently from the dissociation by heat. The metallic salts split into the metal and the acid radicle; the acids split into hydrogen and the acid radicle; the bases split into the metal and the hydroxyl radicle. Now such radicles as NH_4, OH, SO_4, etc., which this requires, are unknown as separate entities. The ionization of

KCl supposes the presence in the water of atoms of potassium and of chlorine. If this be so, some of the chlorine should reveal itself by its color and odor. Further, it is well known that potassium placed upon water decomposes it with such violence as to produce flame and forms potassium hydroxide. None of these effects are produced and this was once regarded as a grave objection to the theory. This objection is answered by the statement that ions carrying electrical charges differ from those that do not. A metallic ion can go into solution only when it has a positive charge, and once in solution it can not be withdrawn until this charge is removed or neutralized. This can be shown experimentally thus. A plate of zinc dipped into hydrochloric acid is attacked vigorously and goes into solution. If, however, this plate be charged negatively, the action of the acid immediately ceases. So long as the potassium ion carries a positive charge it remains in solution, but when this charge is withdrawn by contact with the negatively-charged cathode the potassium regains its usual properties and decomposes the water. It is interesting to note that over one hundred years ago Davy conjectured that "in this state of transition or electrical progression the chemical elements are deprived of their wonted properties, their affinities being rendered dormant or counteracted by the predominating influence of the electrical attraction."

271. Ionization Incomplete.—Should NaCl in solution be completely ionized, the osmotic pressure produced would be twice that produced by an equal number of molecules of sugar. Barium chloride, $BaCl_2$, since it ionizes into Ba, Cl, Cl, should produce three times this pressure. Were this the case, doubts about Arrhenius' theory would disappear, but it is not the case. The osmotic pressure of NaCl is not twice that of a sugar solution of the same molecular concentration. The explanation is that these salts do not completely ionize. At ordinary temperatures moderately dilute solutions of salts, strong acids and strong bases ionize from 80 to 90 per cent. However, as the dilution increases so does the dissociation and it approaches the theoretical figure when the dilution reaches one molugram per 1000 litres.

272. Experimental Demonstration of Free Ions.—The presence of free ions was shown by Ostwald in the following experiment. A horizontal glass tube (Fig. 118) about one-half inch in diameter

and some 20 inches long is bent up at right angles at the ends, these terminal portions being expanded to the size of a test tube and a piece of platinum wire *C* being fused through the bottom of the end *B*. The tube is filled with dilute sulphuric acid. In the end *A* is inserted a rubber stopper through which passes an

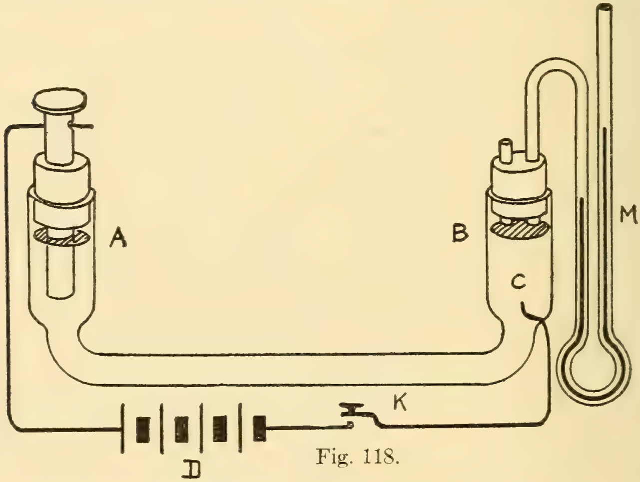


Fig. 118.

amalgamated rod of pure zinc. In the end *B* is inserted a stopper carrying a slender glass manometer *M* which is filled with water, colored for ease of observation. The zinc rod is connected to the positive pole of a battery of five or six cells, *D*; the platinum wire *C* is connected through the key *K* to the negative pole. The instant the key is closed, the manometer indicates an increase of pressure in *B* due to the hydrogen released at *C*.

Just before the key was closed this hydrogen must have existed in the immediate vicinity of *C* in the form of free ions. From Par. 270 they must have carried positive charges. But the cathode *C* was also positively charged and these ions were therefore repelled. As soon, however, as the key was closed, the charge on *C* was withdrawn, the hydrogen ions moved up to *C*, gave up their charges and then recovered their status as free hydrogen atoms.

273. Ions Not from Same Molecule.—According to the older theories, when the circuit was closed the zinc and sulphuric acid in *A* reacted, producing zinc sulphate and hydrogen and this hydrogen travelled from *A* to *B* and appeared at *C*.

The following considerations will show that it is impossible that

the hydrogen atoms released in A should be instantly shot across the 15 or 20 inches of electrolyte to C. By moderate exertion a small lead ball may be thrown several hundred feet. If this ball be cut up into fine shot the force required to throw it to this distance would be very much greater. If it be reduced to dust we could not command sufficient force, and a particle of dust might contain several million atoms. Finally, the hydrogen atom is over 200 times lighter than the lead atom and instead of moving through air moves through the liquid. It is thus seen that the force required would be beyond all reason.

As a matter of fact, the ions move from both electrodes in opposite directions and at different rates of speed. These rates have been accurately measured. The swiftest of the ions, the hydrogen, moves under ordinary conditions a little faster than one-thousandth of an inch per second.

274. Grotthus' Theory.—We have already mentioned (Par. 195) that no signs of the moving ions can be seen between the electrodes. Grotthus in 1805 attempted to explain this by the theory that there was an exchange of hydrogen atoms from molecule to molecule of the acid between the electrodes, just as each individual in a bucket chain at a fire passes a bucket to the person on one side of him and receives a bucket from the person on the other side. The correct explanation is that so long as these ions carry charges they do not possess their ordinary properties and do not aggregate into visible masses.

275. Electrolytes and Non-Electrolytes.—In Par. 267 we saw that *solutions of all the acids, all the bases, and all the salts, and only these, produce osmotic pressures in excess of those called for by theory.* From what has been brought out in the preceding pages, the student will now be prepared for our final and most startling generalization, namely, *those and only those solutions which produce abnormal osmotic pressure conduct electricity or are electrolytes.* All other solutions are non-conductors or non-electrolytes.

276. Electrolytic Properties Depend Upon Ionization.—Since the common property of these solutions, excessive osmotic pressures, has been shown to result from ionization, it is but natural to assume that their electrolytic property has the same cause. A vast accumulation of facts points to this same conclusion.

Sulphuric acid when free from water is a non-conductor. Perfectly pure water is also a non-conductor. Such water never exists in nature and perhaps may never be prepared, but by a special treatment to remove dissolved gases, and a final distillation in vacuo, water has been prepared of such purity that a column of it one millimeter (one twenty-fifth of an inch) long had the same resistance as a copper wire of the same diameter but encircling the earth at the equator 300 times. A solution of sulphuric acid in water is, however, a very good conductor.

Again, since we have seen (Par. 271) that ionization increases with dilution, a dilute solution, the amount of dissolved substance being kept constant, should conduct better than a strong one, and this is found to be the case.

A solution of hydrochloric acid in water is a very good conductor; a solution of the same in chloroform, no ionization taking place, is a non-conductor. Such examples may be multiplied indefinitely.

277. Vapor Tension.—In Par. 259 illustrations were given of the force or pressure which causes gases to diffuse through space, and dissolved substances to spread through unoccupied solvent. This tendency to diffuse is general. If a liquid be introduced beneath a bell jar, a portion of the liquid passes into a state of vapor and fills the jar and the evaporation continues until the pressure of the vapor above the liquid balances the force which tends to throw off the liquid into space. To this force the name *vapor tension* has been applied. It is to be noted that in order to pass from a liquid to a vapor a certain amount of heat must be taken in by the vapor. The vapor passes off accompanied by this latent heat which is necessarily lost by the liquid left behind.

278. Solution Tension.—Nernst advanced the theory that a similar state of affairs obtains for solids immersed in liquids, that is, there is a force, designated by him *solution tension,* which tends to drive particles of the solids off into solution in the liquid. We have seen (Par. 270) that a metallic ion can go into solution only when it carries with it a positive charge. Therefore, parallel to the heat in the case of the vapor, the liquid about a metallic plate becomes positively charged and the plate becomes correspondingly negatively charged. Ions continue to be thrown off from the metal until the force throwing them off, or the solution tension,

is just counterbalanced by the contrary force of attraction which tends to pull the positively charged ions back to the negatively charged plate. To this theory the objection was advanced that if a metal plate threw off ions it would lose weight but in many cases no such loss can be detected by even the most delicate measurements. The reply to this is that the quantity of electricity carried by the ions is so great that equilibrium is reached long before there passes into solution enough ions to be detected by our most refined methods of weighing. For example, to carry into solution 31.8 grams of copper would require 96,540 coulombs (Par. 231) and to carry in only one-thousandth of a gram (the smallest amount that can be weighed in an ordinary analytical balance) would require over three coulombs, each of which is about three billion electrostatic units (Par. 228).

279. Theory Applied to the Simple Cell.—Consider the case of the simple cell (Par. 193). Both the zinc and the copper throw off ions into the electrolyte but the zinc has the greater tendency to pass into solution therefore more zinc ions go into solution and the zinc plate becomes more negatively charged than the copper plate. The result is that, as compared to the zinc plate, the copper plate is positively charged. When these plates are connected through the external circuit, the current flows from the copper to the zinc, the negative charge on the zinc is partly neutralized and the zinc plate can therefore throw more ions into solution, and so on.

280. Atomic Character of Electricity.—We have seen above that the passage of a given quantity of electricity through an electrolyte always releases equivalent weights of ions. Since 96,540 coulombs liberate one gram of hydrogen and 107.9 grams of silver, and since this ratio is constant no matter how many coulombs flow through the electrolyte, the quantity of electricity that would release one microcrith of hydrogen would also release 107.9 microcriths of silver, that is, the quantity that releases one atom of hydrogen releases one atom of silver and one atom of any other univalent element. Since the quantity of electricity which releases an ion is equal to the charge which the ion carries, we see that all univalent ions carry equal charges, either positive or negative. Bivalent ions carry twice the charge of univalent ions, and trivalent ions carry three times this charge, and so on. Every

unit of valency therefore is accompanied by the same definite quantity of electricity, either positive or negative, and since there are no fractions of these charges and they vary by whole numbers, or in simple ratio, Helmholtz concluded that electricity was divided into elementary portions or atoms. These electrical atoms which accompany ions have been named *electrons.* Assuming that an ion and an atom of hydrogen are the same, the electron has been calculated as 2.4×10^{-10} electrostatic units.

281. Extensive Scope of Theory of Electrolytic Dissociation.— The scope of the theory of electrolytic dissociation is extensive. Its applications to pure chemistry are even more wonderful than those that we have just considered. It explains why water is one of the products of most chemical reactions; why the majority do not take place unless water be present; why, for example, dry sulphuric acid has no effect upon blue litmus; why dry hydrochloric acid does not react with dry ammonia; why dry sulphuric acid does not attack dry sodium. It also explains such facts as why silver chloride is precipitated by the soluble chlorides yet not by the chlorates; why KOH precipitates metallic hydroxides yet CH_3OH does not, etc., etc. The statement is even made, though not yet universally accepted, that no metathetical reaction is possible unless preceded by ionization either by solution, by fusion, or by vaporization. It is being developed by many investigators and there is every reason to believe that remaining objections which may be advanced against it will shortly be explained away.

CHAPTER 24.

RESISTANCE.

282. Resistance.—For the beginner it is helpful in forming a physical conception of certain electrical phenomena to think of electro-motive force as a pressure which drives or pushes, or tends to drive or push, electric charges. If two points between which there exists a difference of potential be connected by a conductor, the electro-motive force will cause a flow of electricity from the point of higher potential to that of lower, and the greater the difference in potential between the two points the greater the pressure and the greater the quantity of electricity that will flow across in a given time. This movement is also affected by the nature of the conductor between the two points. For example, it takes a longer time for a given quantity of electricity to flow through a long thin wire than it does through a short thick one. We have seen (Par. 228) that the current is measured by the quantity of electricity flowing past a given point in a unit of time, hence the current in the long thin wire is smaller than that in the short thick wire. The long thin wire therefore cuts down or reduces the current by obstructing its flow. This hindrance which the wire offers to the flow is called its *resistance.*

283. Example of Effect of Resistance.—The following experiment will show the effect of resistance. Fig. 119 represents diagrammatically a cell or battery A and in the external circuit

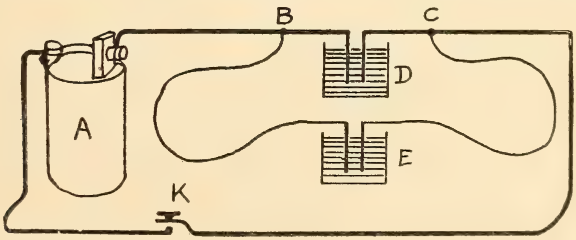


Fig. 119.

two copper voltameters D and E. When the key K is closed the current from the cell divides at B, a part going through the upper

voltameter D, and the remainder through the lower voltameter E. The electro-motive force which drives the current through the two voltameters is precisely the same, since it is due to the difference of potential between B and C, but in the upper voltameter it has to drive it through the short stout wire and in the lower voltameter it has to drive it through the longer and thinner wire. If the key be kept closed for a convenient time and then opened and the cathodes be weighed, it will be found that the cathode of D has increased considerably more in weight than that of E, hence a greater quantity of electricity has passed through D in the given time, that is, the current through D has been greater than that through E.

284. The Practical Unit of Resistance, the Ohm.—This subject was investigated first by Ohm who showed that the resistance of a given conductor of uniform cross-section varies directly as its length and inversely as the area of its cross-section. At the time when he carried on his researches there were no units of resistance and he therefore extemporized standards by means of definite lengths of wire of a given size which, for the sake of compactness, he wrapped up into coils. He used these *resistance coils* himself and distributed others among those of his scientific friends who wished to verify his results.

The practical unit of resistance, the *ohm*, is named in his honor and will be defined later (Par. 291); for the present we must content ourselves with the statement that it is about the resistance of a piece of ordinary iron telegraph wire, one-sixth of an inch in diameter and one hundred yards long; or about the resistance of ten feet of annealed copper wire one-hundredth of an inch in diameter.

285. Laws of Resistance.—We saw above that Ohm showed that the resistance of a conductor of uniform cross-section varies directly as its length and inversely as the area of its cross-section. He also showed that it depends upon the material of which the conductor is composed. If R represent the resistance of such a conductor, this law may be expressed

$$R = \frac{l}{s}\rho$$

in which l is the length of the conductor, s is the area of its cross-section, and ρ is a factor

depending upon the material and called its *specific resistance.* Resistance also varies with the temperature of the conductor.

In addition to the foregoing, there are a few substances whose resistance varies under certain conditions in an anomalous manner. For example, when bismuth is placed in a magnetic field its resistance increases; when selenium is exposed to light its resistance decreases. The resistance of some substances, notably carbon, decreases with pressure. The prime factors of the resistance of a conductor, however, are length, area of cross-section, material and temperature and these we shall now consider in detail.

286. Resistance Varies Directly with Length of Conductor.—This statement requires no amplification. The principle has numberless applications. By measuring the resistance of a foot of a given wire we can easily calculate the resistance of any specified length of it. To determine the length of a submarine cable coiled upon a reel, it is not necessary to unwind it. We measure its total resistance, obtain by measurement or from a table the resistance of the wire per foot, whence we get at once the total number of feet.

If conductors of different lengths, cross-sections or materials be connected one after the other, or *in series,* the total resistance of the resulting conductor is the sum of the separate resistances.

287. Resistance Varies Inversely as Area of Cross-Section of Conductor.—The resistance of a conductor varies inversely as the area of its cross-section, that is, the greater this area, the less the resistance and the less this area, the greater the resistance. For the usual current electricity it is unaffected by the geometrical shape of the cross-section, and whether this be circular or square or irregular or tube like, if the area be the same the resistance is the same. The resistance of a wire cable of many strands is the same as that of a single conductor whose cross-section is equal to the sum of the cross-sections of the separate strands. Since wires are usually circular in cross-section, the resistances of equal lengths of wire of the same material are to each other inversely as the squares of the diameters of the wires.

288. Specific Resistance.—If in the expression (Par. 285) for the resistance of a conductor

$$R = \frac{l}{s}\rho$$

we make l = one centimeter and s = one square centimeter, we have

$$R = \rho$$

But ρ is the specific resistance of the material of which the conductor is composed, whence we see that this specific resistance is measured by the resistance of a centimeter cube of the substance or of a prism or cylinder whose cross-section is one square centimeter and whose length is one centimeter. The resistance of a piece of metal of this size is so small that it is usually expressed in millionths of an ohm, or *microhms*. For example, the specific resistance of silver, which is the least, is about 1.5 microhms, that of copper about 1.6, that of brass about 7, that of wrought iron 10 to 15, that of lead about 20, that of mercury about 95, that of cast iron over 100. On the other hand, the specific resistance of the ordinary electrolytes runs from 1 to 30 ohms while the specific resistance of lead glass is given as 84 *trillion* ohms and that of flint glass is two hundred thousand times greater.

289. Variation of Resistance with Temperature.—The resistance of all substances changes as their temperature varies. The resistance of the metals increases as their temperature rises; on the other hand, the resistance of electrolytes and of most non-metals decreases with increase in temperature. This is of especial importance in the case of carbon. The resistance of the carbon filament in an incandescent lamp when hot and giving light is very nearly, if not quite, fifty per cent less than when cold.

The amount of change in resistance per ohm per degree is called the *temperature coefficient*. The metals therefore have a positive temperature coefficient; the non-metals and electrolytes have a negative coefficient. Starting at 0° C, the resistance of many metals decreases about $\frac{1}{273}$d for every drop of 1° C. At this rate their resistance would entirely vanish at −273° C, which is the absolute zero of temperature as deduced from Charles' law of gaseous pressure. It is interesting to find this significant temperature thus indicated by an independent deduction. It must be noted however, that recent experiments show that at the temperature of liquid air the resistances no longer decrease at the same rate.

It is highly desirable that we should be able to prepare standards of resistance which would be independent of temperature, and certain alloys have been discovered whose temperature coefficient

is so small that for most purposes it may be neglected. Typical of these is *manganin*, composed of 84 parts copper, 4 parts nickel and 12 parts manganese.

290. The Platinum Thermometer.—This change of resistance with temperature is utilized in the construction of certain forms of pyrometers, thermometers for the measurement of temperatures beyond the range of the mercurial thermometer or extending up to 1000° C. In most of these a platinum wire is wrapped around a slender tube of mica which is then slipped into an outer tube of fire-resisting porcelain closed at one end. The free ends of the wire are brought out of the other end and arranged for attachment to a resistance-measuring instrument which may be at some distance. The porcelain tube is then inserted into an opening in the walls of the furnace or dipped into the molten metal whose temperature is to be determined. When the coil has attained the temperature of the surrounding medium, the resistance of the wire is measured by means to be described later (Chap. 26) and the corresponding temperature is given by reference to a table or is sometimes read directly from a scale which is a component part of the apparatus.

291. The Ohm Defined in Terms of a Column of Mercury.—The comparisons in Par. 284 are only crude approximations and can hardly be made anything more, for the resistance of iron and of copper varies greatly with even slight traces of impurities and with the temper and annealing. Mercury is a metal which by simple distillation and washing is readily obtained in a high state of purity; it is also free from the troubles of tempering and annealing and finally its resistance is nearly sixty times greater than that of copper. The apparent disadvantage of not being able, on account of its liquid state, to obtain it in wires is easily overcome by pouring it into glass tubes of the required size, and electric connection with it is made by simply dipping into it the conducting wires. The International Congress of Electricians in Chicago in 1893 (Pars. 212, 232) defined and recommended that there be adopted "as a unit of resistance, the *International Ohm* . . . represented by the resistance offered to an unvarying electric current by a column of mercury at a temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area and of the length of 106.3 centimeters." This corresponds to a cross-section

of one square millimeter but the weight of the mercury is given instead of the diameter of the tube since, of the two, the weight is the more easily and accurately measured.

292. Resistance and Conductance.—The terms *resistance* and *conductance* are reciprocals. The less the resistance of a conductor, the greater its conductance; the greater its resistance, the less its conductance. The unit of resistance is the ohm. There is no need for a unit of conductance yet it has been given a name, the *mho* (the word ohm backwards). A body whose resistance is three ohms has a conductance of one-third mho.

There is no conductor devoid of resistance; so also there is no absolute non-conductor. Substances may be arranged in order of their relative conductance or, as it is frequently called, their *conductivity,* this being the reciprocal of specific resistance, also called *resistivity.* Silver is the best conductor and copper comes next. Conductivity is expressed in percentage, that of annealed copper being taken as 100 since copper and not silver is the standard material for electric wiring. The following table gives the conductivity of the commoner metals as determined by Fleming and others.

Metal	Conductivity
Silver, pure	108.60
Copper, annealed	100.00
Gold	97.80
Aluminum	63.00
Zinc	27.72
Brass	22.15
Iron, wrought, average	15.00
Steel	11.60
Lead	7.82
German Silver	5.32
Mercury	1.69

293. Resistance of Conductors in Parallel.—If an electric circuit splits into two or more portions which again unite, it is called a *divided circuit.* Such a circuit of three branches is represented in Fig. 120. The three branches are said to be *in parallel.* A turnout which enables cars travelling at different speeds, or in opposite directions on a single track, to pass each other is sometimes called a shunt. From analogy, any branch of a divided

circuit may be called a *shunt* for the remaining branch or branches.

It frequently becomes necessary to determine the resistance of a divided circuit, that is, the *joint resistance* of two or more conductors in parallel. Suppose we have in parallel two wires, one of

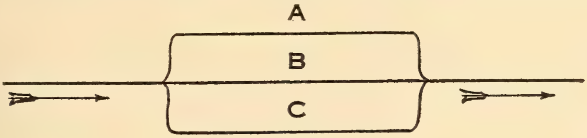


Fig. 120.

ten ohms and the other of one ohm resistance; what is their joint resistance? The tendency for a beginner is to say the average of the two, but reflection will show that the two wires side by side are equivalent to a single wire of greater cross-section and hence of less resistance than either. In other words, the joint resistance of any number of resistances in parallel is always less than that of the least.

Joint resistance may be determined as follows: If A, B and C be the resistances of the branches in Fig. 120, their conductance is $\frac{1}{A}$, $\frac{1}{B}$ and $\frac{1}{C}$. Their joint conductance is the sum of the separate conductances or

$$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}=\frac{AB+AC+BC}{ABC}$$

Their joint resistance is the reciprocal of this or

$$R=\frac{ABC}{AB+AC+BC}$$

and in general *the joint resistance of any number of resistances in parallel is the reciprocal of the sum of the reciprocals of the separate resistances.*

If there be but two resistances, the formula becomes

$$R=\frac{AB}{A+B}$$

or the joint resistance is the product of the two divided by their sum.

Should A, B and C be equal, the expression becomes

$$R=\frac{A}{3}$$

and in general the joint

resistance of any number of equal resistances in parallel is equal to that of a single resistance divided by the number in parallel.

294. Internal Resistance of Cells.—In the employment of voltaic cells as a source of electrical energy, the question of their resistance is of great importance. In Par. 288 we saw that while the specific resistance of copper is about 1.6 microhms (millionths of an ohm), that of the usual electrolytes runs from 1 to 30 ohms, that is, the resistance of the electrolyte is on an average 10,000,000 times greater than that of the copper. This resistance, spoken of as the *internal resistance* of the cell, follows the same laws as other resistances (Par. 285). With a given electrolyte, we may reduce the internal resistance of a cell in two ways. First, by bringing the plates of the cell closer together we may shorten the path which the current has to follow. Second, by increasing the area of the plates we increase the number of available paths for the current, or increase the cross-section of the total path. A thin sheet of copper parallel and close to the zinc plate offers far less resistance than the same mass of copper in a more compact form. As the zinc and copper plates are flattened out and increased in size the glass cell must keep pace, but as it gets larger it increases rapidly in cost. Reflection will show that two cells in parallel are electrically equal to a single cell with plates twice as large. Therefore, the usual method of increasing the cross-sectional area of a battery is to join cells in parallel.

295. Wire Tables.—As the practical electrician has to deal largely with wires, it is important that he should possess information as to the different sizes, their dimension, weight, resistance, etc. Such data is embodied in *wire tables* which are issued by the wire manufacturers and are also found in the various electrical hand-books. The sizes of wire are designated by numbers corresponding to certain *wire gauges*. It is unfortunate that there are in existence four or five of these gauges and that their numbers do not correspond nor do their sizes of wire vary in accordance with any fixed rule. In this country the gauge in most common use is the American wire gauge of the Brown and Sharpe Company. The Birmingham wire gauge is also in use. The No. 1 wire on the Brown and Sharpe gauge is very nearly .3 of an inch in diameter, and the smallest wire, or No. 40, is about .003 of an inch. There are four sizes larger than No. 1 and they are designated single 0,

double 0, treble 0, etc. The No. 10 wire on the B. & S. gauge is just about .1 of an inch in diameter and if of copper its resistance is about one ohm per 1000 feet. As a rule of thumb, by subtracting three from the gauge number of any wire we get the number of the wire whose cross-sectional area is twice as great. The cross-sectional area of No. 7 is twice that of No. 10.

COPPER WIRE TABLE, BROWN AND SHARPE GAUGE.

Resistance at 20° C.

Size of wire	Diameter, inches	Ohms per foot	Feet per ohm	Pounds per foot
0000	0.460	0.00004893	20,440	0.6405
000	0.4096	0.00006170	16,210	0.5080
00	0.3648	0.00007780	12,850	0.4028
0	0.3249	0.00009811	10,190	0.3195
1	0.2893	0.0001237	8,083	0.2533
2	0.2576	0.0001560	6,410	0.2009
3	0.2294	0.0001967	5,084	0.1593
4	0.2043	0.0002480	4,031	0.1264
5	0.1819	0.0003128	3,197	0.1002
6	0.1620	0.0003944	2,535	0.07946
7	0.1443	0.0004973	2,011	0.06302
8	0.1285	0.0006271	1,595	0.04998
9	0.1144	0.0007908	1,265	0.03963
10	0.1019	0.0009972	1,003	0.03143
11	0.09074	0.001257	795.3	0.02493
12	0.08081	0.001586	630.7	0.01977
13	0.07196	0.001999	500.1	0.01568
14	0.06408	0.002521	396.6	0.01243
15	0.05707	0.003179	314.5	0.009858
16	0.05082	0.004009	249.4	0.007818
17	0.04526	0.005055	197.8	0.006200
18	0.04030	0.006374	156.9	0.004917
19	0.03589	0.008038	124.4	0.003899
20	0.03196	0.01014	98.66	0.003092
21	0.02846	0.01278	78.24	0.002452
22	0.02535	0.01612	62.05	0.001945
23	0.02257	0.02032	49.21	0.001542
24	0.02010	0.02563	39.02	0.001223
25	0.01790	0.03231	30.95	0.0009699
26	0.01594	0.04075	24.54	0.0007692
27	0.01420	0.05138	19.46	0.0006100
28	0.01264	0.06479	15.43	0.0004837
29	0.01126	0.08170	12.24	0.0003836
30	0.01003	0.1030	9.71	0.0003042

296. Circular Measure of Wires.—Owing to the errors likely to occur from lack of agreement in the sizes of the various wire gauges, it is becoming more and more the custom among electricians to designate wires by their diameters expressed in thousandths of an inch or *mils*, indeed, by recent orders of the War Department this has been made mandatory for our army. If we compare the area of cross-section of a wire whose diameter is one mil with that of one whose diameter is n mils we see, since the areas of circles are to each other as the squares of their diameters, that the cross-section of the larger wire is n^2 times greater than that of the smaller. Because of this very simple relation, the area of cross-section of a wire of one mil diameter is taken as the unit of area and called a *circular mil.* To find the area in circular mils of the cross-section of any other wire we simply square its diameter expressed in thousandths of an inch. This method of comparison is very much simpler than expressing the cross-sections in square inches. A piece of wire one foot long and one mil in diameter is called a *mil foot.* The resistance of a mil foot of annealed copper is 9.59 ohms at 32° F and 10.505 ohms at 75° F. With this data we may, by applying the law given in Par. 287, determine the resistance of a copper wire of any size and length.

CHAPTER 25.

OHM'S LAW.

297. Ohm's Law.—As a result of his investigations, Ohm announced in 1827 the law which bears his name and which is to the effect that *in any electric circuit the current varies directly as the electro-motive force and inversely as the resistance of the circuit.* Expressed in symbols this becomes

$$I = \frac{E}{R}$$

in which, if E be the E. M. F. in *volts* and R the resistance in *ohms*, I is the current in *amperes.*

In its determination Ohm employed the rather crude appliances which he extemporized for the purpose (Par. 284). Since his time the delicacy and accuracy of electrical apparatus have been immensely increased, yet the most careful and refined observations serve merely to afford stronger confirmation of his conclusions.

The importance of this law can not be over-estimated. In the study and application of electricity it is fundamental and in one form or another it is met at every turn. On account of its very simplicity there is sometimes a failure to recognize that it is unique, and occasionally it is spoken of as "self evident." Such is far from being the case. There is no material substance which follows such a law. Pressure causes liquids and gases to flow through pipes, yet if this pressure be doubled the flow is by no means doubled.

When applying the law to a more or less complex circuit, E represents the *total* E. M. F. and R the *total* resistance. Thus there may be several cells or batteries or electrical machines contributing to the E. M. F., in which case the sum of the E. M. F.s must be taken. Again, through error or by design a cell or battery may be reversed so as to oppose the remaining E. M. F. Such opposing E. M. F. is spoken of as *counter E. M. F.* or *back E. M. F.* Back E. M. F. is also produced by polarization (Par. 198) and, as we shall see later, by the operation of motors in the circuit. In

summing up the total E. M. F. of the circuit, back E. M. F. is to be considered as negative. The resistance R includes not only the resistance of the line but also that of the contacts, joints and connections and of the electrolyte and elements of the cells. The law can therefore be given

$$I = \frac{E' + E'' + E''' + E'''' + \&c.}{R' + R'' + R''' + R'''' + \&c.}$$

or the current in the circuit is equal to the algebraic sum of the separate E. M. F.s divided by the sum of the separate resistances.

298. Drop of Potential.—The three quantities, current, electromotive force and resistance are bound together by Ohm's law so that any two being given, the third may be determined. It may at first sight appear unnecessary to state such a self-evident truth but it is desirable to lay especial emphasis upon the fact for, until the student has become familiar with the law, the tendency is rather to restrict its use to the determination of current only.

The law may be put in the form

$$E = IR$$

and it is helpful to the beginner if he will accustom himself to interpret this as meaning that E is the electro-motive force necessary to drive a current of strength I through a resistance R.

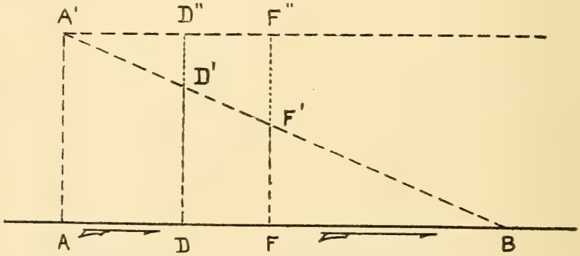


Fig. 121.

Suppose AB (Fig. 121) to represent a portion of an electric circuit, the point A being of higher potential than B, and suppose that by means of one of the instruments to be described later (Chapter 34) we measure the difference in potential between A and B. Lay off on some convenient scale AA' proportional to this difference of potential. If we move along AB to some point D and measure the difference of potential between D and B

we will find it to be less than that at A, or represented by DD'. Likewise, at F this difference of potential is still smaller and is proportional to FF', that is, as we move from A towards B the difference of potential between the successive points and B steadily grows less, or there is a falling off from the difference of potential represented by AA'. At D, for example, this *drop of potential* is $D''D'$ and at F it is $F''F'$.

The drop of potential between any two points is always equal to the product of the current into the resistance between the points. Certain elementary applications of this principle will be shown in the following paragraphs.

299. Ohm's Law Applies to Any Portion of the Circuit.—In Par. 297 we saw that Ohm's law was applicable to the entire circuit even though this be made complex by including heterogeneous resistances and sources of E. M. F. It also applies to any portion of a circuit, that is, the current flowing between any two points in a circuit is equal to the difference of potential between these two points divided by the resistance between them. We have seen (Par. 229) that the current at every cross-section of a circuit is the same; if, therefore, we determine it at one point we have it for any other point. Knowing the current, if we have the resistance between two points we can, by what we have shown in the preceding paragraph, determine the difference of potential, or *drop*, between the two points. These principles enable us to solve a variety of problems. For example, let $ABCD$, Fig. 122, represent part of an electric circuit. The resistance of the portion AB is 12 ohms, that of the incandescent lamp BC is 220 ohms, that of CD is 8 ohms. The difference of potential between A and B is 6 volts. What current is flowing in the circuit and what is the potential of the points A, B and C if that of D be taken as zero?

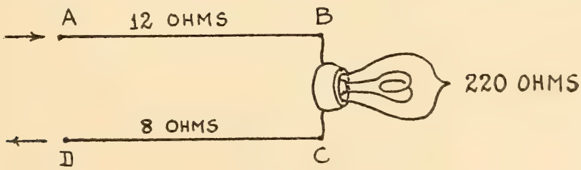


Fig. 122.

The current between A and $B = \frac{E}{R} = \frac{6}{12} = \frac{1}{2}$ ampere, which is also the current for the rest of the circuit. The drop from B to $C =$

$IR = \frac{1}{2} \times 220 = 110$ volts; that from C to $D = \frac{1}{2} \times 8 = 4$ volts. The potential of C is therefore 4 volts, that of B is 114, and that of A is 120.

300. Division of Current in Divided Circuit.—This principle of drop of potential furnishes a simple determination of the division of an electric current in a divided circuit.

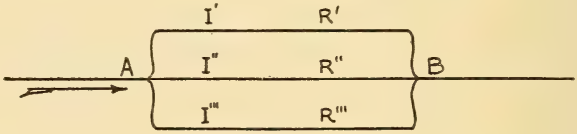


Fig. 123.

Let Fig. 123 represent a divided circuit of three branches whose resistances are respectively R', R'' and R'''. Call the corresponding currents I', I'' and I'''. The current in the main branch upon arriving at A divides into these three portions which reunite at B. The drop from A to B is the same over each of the three routes, therefore

$$I'R' = I''R'' = I'''R'''$$

which may be written

$$I' : I'' : I''' = \frac{1}{R'} : \frac{1}{R''} : \frac{1}{R'''}$$

that is, *the current in the branches of a divided circuit are to each other inversely as the resistances of the respective branches.*

In making an actual calculation, if the fractions in the second member of this proportion be brought to a common denominator, their numerators indicate at once the relation between the several currents.

If there be but two branches, the above becomes

$$I' : I'' = \frac{1}{R'} : \frac{1}{R''}$$

which may be written

$$I' : I'' = R'' : R'$$

a somewhat simple form for calculations.

301. Shunts.—In practical electricity it frequently becomes necessary to employ a divided circuit of two branches which must

be so proportioned that the main current divides between them in accordance with some desired ratio. For example, suppose that we wish to measure a current which is much larger than can be measured directly by the instruments at our disposal. If we can arrange a divided circuit so that exactly one-hundredth of the total current flows through one branch, we can measure this small current and always know that the entire current is one hundred times greater. This division is brought about by *shunts* (Par. 293).

In Fig. 124 we desire to measure the current flowing in AD. G is our measuring instrument which with its connecting wires BG and GC has a resistance of R ohms. BC is the shunt. What must be the resistance of the shunt so that one-hundredth of the total current will flow through G?

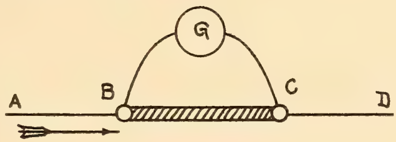


Fig. 124.

Call the current through the instrument I; that through BC will be $99I$. If x be the resistance of BC, then, as shown in the preceding paragraph

$$I \times R = 99I \times x$$

whence

$$x = \frac{R}{99}$$

or the resistance of the shunt must be one-ninety-ninth of the resistance of G and its connecting wires and leads. In a similar manner we can determine the resistance of shunts to bring about division of the total current in any desired ratio. It is to be noted that these shunts are constructed for use with a particular instrument and cannot be used with another of different resistance.

302. Rheostats.—A consideration of Ohm's law, $I = E/R$, will show that by varying R we can vary the current inversely and suggests that by introducing or removing resistance from a circuit we may regulate the current at will. Instruments for this purpose are called *rheostats*. The principle of their use will be

understood from the diagram (Fig. 125). A series of metal contacts are arranged upon the arc of a circle *DE* and connected between these contacts are resistance coils. Pivoted at the center

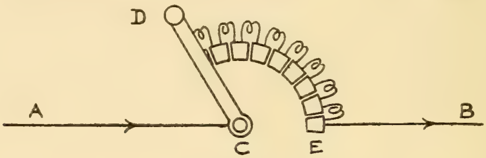


Fig. 125.

of the arc is a metal arm *CD* which can be moved about over the contacts. Suppose the current to come in by *A*. As represented in the figure, it must now traverse all the coils from *D* to *E* before it can leave by *B*, and it is therefore cut down. Had the arm *CD* been still farther to the left, the circuit would have been broken entirely, *R* would have been infinite and the current zero. As the arm is slid around to *E* the coils are successively cut out, the resistance correspondingly reduced, and the current correspondingly increased, reaching its maximum when the arm reaches *E*. The controller by which the motorman starts and stops a trolley car is similar in principle.

It will be shown later that regulation of current by rheostat is a wasteful method and except for temporary purposes, such as for starting and stopping motors, should not be employed.

303. Kirchoff's Laws.—Where an electric circuit is composed of interlacing branches and especially where there are in it several seats of electro-motive force, confusion and uncertainty may arise as to the correct way of applying Ohm's law in the determination of the separate currents and potentials. To obviate this, Kirchoff has formulated a set of rules which render this application almost mechanical. These are:

I. *If several conductors meet at a common point, the algebraic sum of the currents in these conductors is zero.*

This is but a statement of the fact that electricity does not accumulate at a point and that therefore as much flows away as flows to the point. If currents flowing to the point be considered positive, those flowing away must be regarded as negative.

II. *If two or more conductors form a closed figure, or a mesh in a network of conductors, the sum of the products of each current of this*

mesh into the resistance through which it passes is equal to the algebraic sum of the electro-motive forces acting around this same mesh.

This is another statement of the fact that the total drop of potential in going around a closed circuit is equal to the sum of the partial drops. The convention must be adopted that in going around a closed circuit, if the E. M. F. acting in a clockwise direction be considered positive, that acting in the opposite direction is negative.

304. Example of Application of Kirchoff's Laws.—By combining these laws it is always possible to obtain as many independent equations as there are unknown quantities and hence these unknown quantities may be determined. The following concrete example will make the matter clear. Fig. 126 represents a network of conductors in two of the branches of which there are batteries *E* and *F*, sources of E. M. F. The currents in the separate

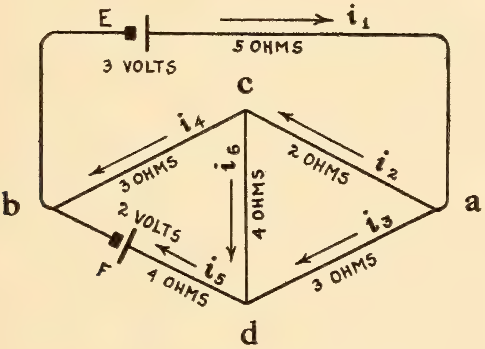


Fig. 126.

branches are designated i_1, i_2, i_3, etc., and their assumed direction is indicated by the arrows. In the final solution, a negative value of a current indicates that the actual direction is opposite to that assumed. The E. M. F. of the batteries and the resistances of the branches are indicated on the diagram. We are required to determine the currents in the separate branches.

From Kirchoff's first law we obtain the following "point equations":

point *a*	$i_1 - i_2 - i_3 = 0$
point *b*	$i_4 + i_5 - i_1 = 0$
point *c*	$i_2 - i_4 - i_6 = 0$
point *d*	$i_3 + i_6 - i_5 = 0$

From the second law we obtain the following "voltage equations":

mesh $i_1i_2i_4$	$5i_1 + 2i_2 + 3i_4 = 3$
mesh $i_2i_3i_6$	$3i_3 - 4i_6 - 2i_2 = 0$
mesh $i_4i_6i_5$	$4i_6 + 4i_5 - 3i_4 = -2$
mesh $i_1i_3i_5$	$5i_1 + 3i_3 + 4i_5 = 3 - 2$

We now have eight independent equations from which to determine six unknown quantities, and the remainder of the process is but a matter of combination and elimination.

305. Lost Volts and Useful Volts.—Should there be connected up to a circuit of resistance R a cell whose E. M. F. is E and internal resistance r, the resulting current would be given by the expression

$$I = \frac{E}{R + r}$$

which may be written

$$E = IR + Ir$$

Interpreting this as explained in Par. 298, we see that a part of the E. M. F. of the cell is spent in driving the current through the external resistance R and the remainder in driving this current through the internal resistance of the cell. The volts, Ir, consumed on the interior of the cell are called the *lost volts* and we profit only by those upon the external circuit, or IR, which are therefore called the *available* or *useful volts*. Since the less the lost volts, the more the useful volts, it is of importance to keep the former at a minimum. Ir may be reduced in two ways; by reducing the current or by decreasing the internal resistance. If there be no current, there is of course no wastage. The internal resistance may be reduced by selecting an electrolyte of low resistance (though usually choice is restricted), by bringing the plates closer together, and by increasing the size of the plates (Par. 294). Lost volts have also to be considered in the operation of electrical machinery.

306. Short Circuit.—The commonest source of injury to electrical machinery is a *short circuit*, which may be defined as the removal, usually accidental, of the greater part of the resistance from a "live" circuit.

Suppose B (Fig. 127) to represent a battery supplying current for the incandescent lamp L. The internal resistance of the battery is almost negligible, the resistance of the wires should be very

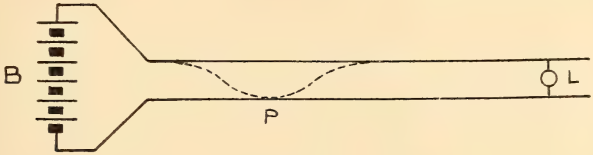


Fig. 127.

small. Suppose the E. M. F. of the battery to be 111 volts, the resistance of the lamp to be 220 ohms and that of the battery and wires to be 2 ohms. The current is

$$I = \frac{111}{220 + 2} = \frac{1}{2} \text{ ampere}$$

If by some accident the wire should sag, as shown by the dotted line, and touch the lower wire at P, at that instant the current would be *short circuited* through the point P, the resistance of the lamp and of the wire beyond P being eliminated. The current is now

$$I = \frac{111}{1} = 111 \text{ amperes}$$

or it has suddenly increased over two hundred times. If the wires had been designed to carry only ten or fifteen amperes they will be fused, apparatus in the circuit will be "burned out," insulation will be charred and possibly fires started. To avoid the injury resulting from such accidents, use is made of *fuses*, pieces of soft, easily-fusible wire inserted in the circuit which is to be protected. If the current exceeds that which the fuses are intended to carry, they melt before damage is done to the rest of the circuit. This same protection is also afforded by certain automatic apparatus called *overload switches* (Par. 414).

307. Definitions Based Upon Ohm's Law.—Since the three quantities I, E and R are bound together by Ohm's law, any one may be defined in terms of the other two. Thus the *ampere* is sometimes defined as the current produced by an E. M. F. of one volt applied to a conductor whose resistance is one ohm. So also the *volt* is defined as that E. M. F. which applied to a resistance of one ohm will produce in it a current of one ampere. The *ohm* may

be similarly defined but such definition adds but little to our knowledge.

Since Ohm's law may be written

$$R = \frac{E}{I}$$

and since E and I fluctuate together so that R remains always constant, the resistance of a conductor is defined by some writers as the ratio of the difference of potential of the ends of the conductor to the current produced in it. To define a *property* as a *ratio* is not altogether satisfactory. It is perhaps better to say that this ratio affords a measure of the resistance.

CHAPTER 26.

MEASUREMENT OF RESISTANCE.

308. Measurement of Resistance.—One of the most important classes of measurements with which the electrician has to deal is that of resistance. Logically, this subject should have been taken up in connection with that of resistance in Chapter 24, but the methods employed could not be clearly presented until after the consideration of Ohm's law and the explanation, as given in Chapter 25, of the principles of the drop of potential and the division of current in a divided circuit. Even now we shall have to anticipate certain principles which can not be fully developed until later.

In these measurements, the methods to be employed vary with the amount and character of the resistance. Thus, very high and very low resistances are measured in a different way from those covering a moderate range. Again, the measurement of the internal resistance of cells and of the resistance of electrolytes must be undertaken in an entirely different manner from that of a metallic conductor. These facts will be brought out in the following pages.

309. Drop of Potential Proportional to Resistance Passed Over.—If there exists between AB (Fig. 128), two points of a cir-

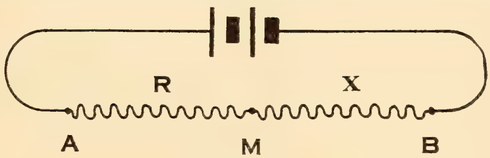


Fig. 128.

cuit, a difference of potential E, there will be a flow of electricity from the point of higher potential to that of lower. The value of this current as given by Ohm's law is $I = E/R$, whence $E = IR$, which last expression, as we have already seen (Par. 298), may be interpreted as expressing the fact that E is the electro-motive force required to drive a current of strength I through the conductor of resistance R.

To drive the same current through a resistance only one-half as great requires only one-half as much E. M. F., or, if the resistance of AM be one-half of the total resistance between A and B, then one-half of the total E. M. F. will be expended in driving the current from A to M, and the difference of potential between M and B is only one-half of that between A and B. In more general terms, for a constant current, the expenditure of E. M. F., or the drop of potential, is directly proportional to the resistance passed over.

310. Measurement of Resistance by Drop of Potential.—Should we have at our disposal a known resistance and an instrument for measuring difference of potential (Chapter 34), the foregoing affords us a means of measuring the resistance between any two points in a circuit. For example, suppose that the resistance R between A and M, Fig. 128, be known and that we desire to determine the resistance x between M and B. We have simply to measure with our instrument the drop E' between A and M, and the drop E'' between M and B. From the preceding paragraph

$$E' : E'' = R : x$$

whence

$$x = \frac{E''}{E'} R$$

This method supposes the current to be constant during the two observations; the battery should therefore be one of constant E. M. F. and the observations should be taken in quick succession so as to avoid change in the current due to the increase of the resistance of the circuit caused by the heating effect of the current.

311. Resistance Coils.—The known resistances used as described in the preceding paragraph are usually in the form of coils. These *resistance coils*, especially those used as standards of resistance, are made with great care and accuracy and embody many refinements. They range from .001 of an ohm to 10,000 ohms. A section of one is shown in Fig. 129. From the ebonite lid there extends downwards a hollow metal cylinder which has an insulating covering of shellac-coated silk. Around this cylinder is wrapped the coil proper which is of silk-insulated manganin wire (Par. 289). For reasons which are explained later (Par. 315), the wire is doubled upon itself at its middle point and the winding is begun at this loop. The ends of the coil are attached to heavy copper

terminals bent downward as shown. The coil is connected up in the circuit by inserting these turned-down ends into mercury cups which in turn are connected to the lead wires. The whole is pro-

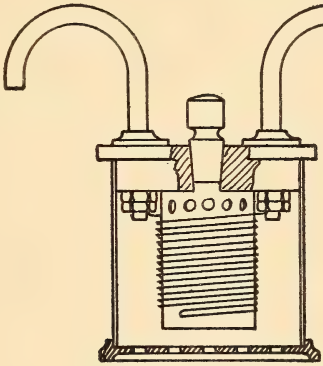


Fig. 129.

tected by a brass case which is perforated by many small openings. The object of the interior metal cylinder is to conduct away heat developed in the wire and at the same time to afford a large surface for radiation. The object of the openings is to allow the enclosed coil to cool off more rapidly and also to permit the temperature to be kept down by submerging the entire coil in oil. The plug in the center of the lid is to permit the insertion of a thermometer for reading the temperature of the coil so that the proper correction for temperature may be applied.

312. Drop in Divided Circuit.—The usual way of measuring ordinary resistances is by means of the *Wheatstone bridge,* a piece of apparatus whose principle will be understood from the following explanation. Consider a divided circuit of two branches and let A (Fig. 130) be the point of high potential. The current at B divides into two parts inversely proportional to the resistances of the two branches, i. e., the greater part goes along the branch of least resistance, the lesser part along the branch of greater resistance. There is a continuous drop of potential along each branch of the circuit from B to D, in other words, the drop of potential over the two branches is exactly the same. Suppose following the right hand branch we reach a point M at which we have passed over one-half of the resistance in that branch; the difference of potential between M and D is only one-half of that between B and D.

Similarly, following the left hand branch and reaching a point N at which we have passed over one-half of the resistance in that branch, the difference of potential between N and D is only one-half of that between B and D. Hence, the points M and N are at the same potential. This can be shown by connecting between these points a sensitive galvanometer G. (Galvanometers are described in Chapter 30. For the present it is sufficient for us to know that a galvanometer (more strictly a *galvanoscope*) is an instrument which indicates by the movement of its needle that a current is flowing in the circuit of which it forms a part, and by the direction of the motion of the needle indicates the direction of the current.) Should there be a difference of potential between M and N, a current would be produced and would be revealed by a deflection of the galvanometer needle, but the needle will be found to remain at rest.

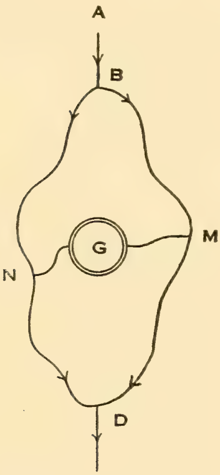


Fig. 130.

The foregoing illustration is based on the supposition that the resistance of BM and of BN are each one-half of the resistance of the respective branches, but the principle is equally true for $1/n$th, that is, if the resistance of BM be $1/n$th of that of the right hand branch and the resistance of BN be $1/n$th of that of the left hand branch, the points M and N will be at the same potential and there will be no flow of current between them if they be connected through a galvanometer.

313. Principle of the Wheatstone Bridge.—Let us now consider a divided circuit of two branches (Fig. 131), each branch subdivided into two parts as shown, and suppose that in the left hand branch we know the resistance of A and of R and further can vary that of R at pleasure, and that in the right hand branch we know the resistance of the

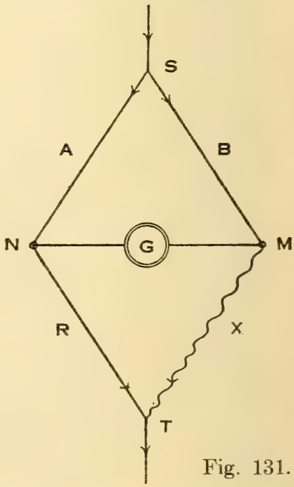


Fig. 131.

portion B but do not know that of the remainder X and wish to determine it. Of the total resistance of the right hand branch, X is some definite fraction, say $1/n$th. Since R may be varied at pleasure, it can be adjusted so that it is $1/n$th of the total resistance in the left hand branch. When such a state of affairs is reached, the points M and N will, from what has been shown above, be at the same potential and the galvanometer connected between M and N will reveal no current. The system is now said to be "balanced."

Since X is $1/n$th of the total resistance in the right hand branch, B is $n-1/n$ths, and since R has been made $1/n$th of that in the left hand branch, A is $n-1/n$ths.

Hence
$$A : B :: R : X$$

Whence
$$X = \frac{BR}{A}$$

or, when the system has been brought to a balance, *the resistance in X is equal to the product of the resistances in the adjacent arms divided by that of the opposite arm.*

314. A Second Demonstration.—The same thing can be readily shown by applying the principle of drop directly. Call the current in the left hand branch I', that in the right hand branch I'', and the resistance in the four arms A, B, R, and X, respectively. The drop from S to N is equal to the current times the resistance or $I'A$; that from S to M is equal to $I''B$. But M and N being at the same potential these drops are equal. Similarly, the drop from N to T, or $I'R$, is equal to the drop from M to T, or $I''X$.

We then have the two equations,

$$\text{(I)} \qquad I'A = I''B$$
$$\text{(II)} \qquad I'R = I''X$$

Dividing (II) by (I) and striking out common factors

$$\frac{R}{A} = \frac{X}{B}$$

Whence as above

$$X = \frac{BR}{A}$$

The foregoing is the principle upon which the Wheatstone bridge is constructed.

The expression $X = \frac{BR}{A}$ can be written $X = \frac{B}{A}R$, whence it is seen that if B and A be so selected that B/A is some multiple or submultiple of ten, calculations will be simplified since all that will then be necessary will be to point off decimal places or add zeros to the value of the known resistance R.

315. Arrangement of Resistances.—In the actual apparatus the resistance in the arms A, B, and R is usually varied by removing or changing the position of certain plugs. For example,

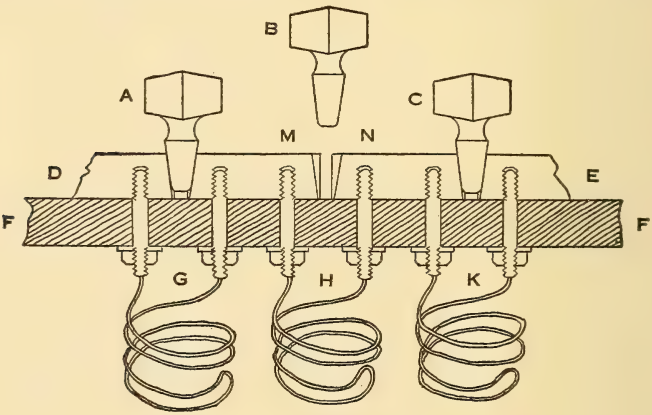


Fig. 132.

the arm A, a portion of which is represented in Fig. 132, consists of a heavy brass bar DE secured to the ebonite plate FF and cut entirely through at regular intervals by tapering openings into which fit the corresponding ebonite-handled brass plugs A, B, C. The separate sections into which the bar is divided are connected beneath the plate FF by the resistance coils G, H, K. These are wound as described in Par. 311 so as to avoid self-induction. For the present we may explain this by stating that when a circuit through a coil of wire is completed there is produced through induction an opposing E. M. F. which causes the current to lag and prevents it from rising to its full strength at once. When a coil is made by winding it from a loop at its middle point, each turn of the coil carrying a current is paralleled by an equal turn in which the current flows in the opposite direction and the inductive effects of the two turns exactly neutralize each other. These coils have a resistance of 1 ohm, 10 ohms, 100 ohms, etc.,

and therefore bear to each other the ratio of 1 : 10 : 100, etc. With the plugs in position the current passes from *D* to *E* through the bar and coils, the combined resistance of which is so small as to be negligible. With the plug *B* removed, the current must follow the path *D M H N E*, that is, the resistance of the coil *H* has been introduced into the circuit.

In the arm *R* the arrangement is similar but there is a much greater number of coils whose resistances are in ohms 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1000, 2000, 3000, 4000, etc., thus enabling any combination from 1 to 11110 to be obtained. This arm is usually called the "rheostat" and is consequently designated in diagrams by letter *R*.

316. Evolution in Form.—The theory of the Wheatstone bridge is best explained as above from a diagrammatic diamond-

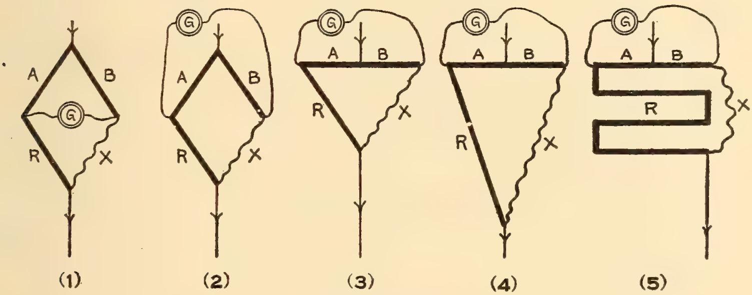


Fig. 133.

shaped figure as in (1), Fig. 133. The commercial form of this apparatus bears no superficial resemblance to the figure but has been evolved directly from it as the following will show.

1st step. The galvanometer need not be placed in the diamond but may be connected outside as shown in (2).

2d step. *A* and *B* need not make an angle with each other but may be flattened down as shown in (3).

3d step. *R* being the arm which carries the greatest number of resistance coils should, relatively to *A* and *B*, be elongated as shown in (4).

4th step. Finally, for the sake of compactness, the arm *R* may be folded back upon itself as shown in (5).

Other minor changes consist in the arrangement of the terminals to facilitate connections, and in the insertion in the battery and

galvanometer circuits of keys permanently attached to the instrument. Sometimes a galvanometer is included in the case. The final result is an instrument of which Fig. 134 shows a form made by the Leeds & Northrup Co.

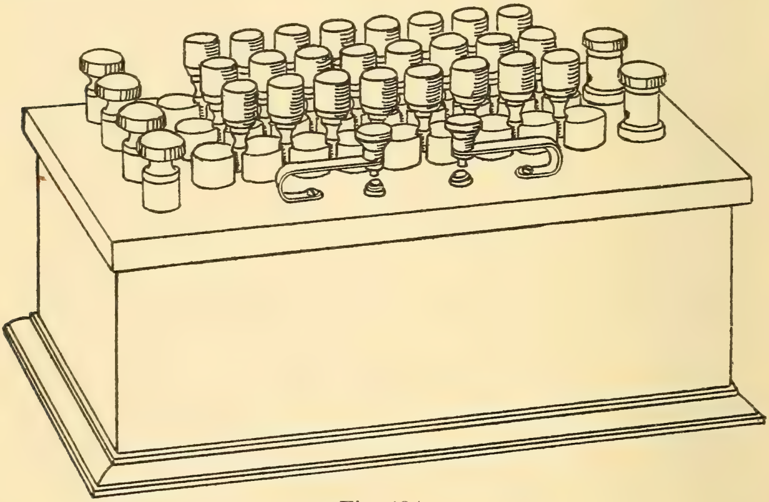


Fig. 134.

The various circuits between the keys and other parts of the bridge are inside the case but are usually indicated by white lines marked on the cover.

317. Connections for a Measurement.—Whatever be the form of the bridge it is well to bear in mind the following:—first, the current enters (or leaves) at the junction of A and B and leaves (or enters) at the junction of R and X; second, the galvanometer is connected between the junction of A and R and that of B and X. (It should however be observed that it may readily be shown that the battery and the galvanometer may be interchanged, the resistance of their respective leads altered at will, and the E. M. F. of the battery varied, all this without affecting the balance.) Finally, in the factor by which R is to be multiplied, the resistance of B, the arm connected to X, is the numerator and that of A, the arm opposite X, is the denominator.

318. Operation of Measurement.—To measure the resistance, say of a wire X, the apparatus is brushed free from dust, and plugs brightened, being especially careful to remove all grease

or oil so as to insure perfect contacts. Connections are then made as shown in Fig. 135. A plug is removed from coil of the same resistance in both *A* and *B*, their ratio therefore being unity. Various plugs are then removed from *R* until with both battery

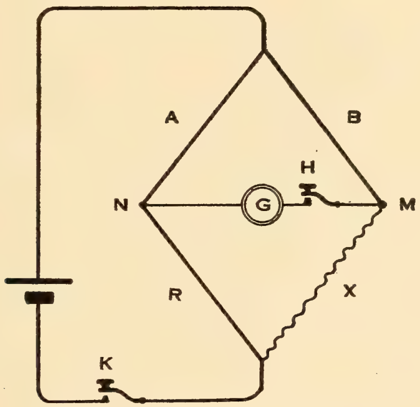


Fig. 135.

and galvanometer keys closed the apparatus is as nearly balanced as possible. At this point the sum of the unplugged resistances in *R* is as near the unknown resistance *X* as it is possible to get with the ratio of unity in B/A.

319. Bracketing.—The plugs in *R* are not removed at haphazard but preferably the resistance should be arrived at by a system of "bracketing." For example, the first plug to be removed should be selected so that the resistance thrown in is certainly greater or less than the one to be measured. Suppose it to be less. The battery key *K* is closed and then the galvanometer key *H*. Suppose the galvanometer needle to be deflected to the left. Replace the plug and remove a second one so as to throw in a resistance certainly greater than that to be measured. Upon closing the keys, if the needle is now deflected to the right the unknown resistance lies between the two. Replace the plug and remove a third which will throw in a resistance as near half way of the interval between the first two as possible. If upon closing the keys the needle is deflected to the left, the third resistance is too small, if to the right it is too great. Proceed in this way keeping the unknown resistance between limits and halving the interval at each successive attempt.

With a little experience the bracketing can be materially shortened by observing the amount of swing produced in the needle by the trial resistances. This decreases rapidly as the correct resistance is approached and indicates which of two is the nearer.

320. Order of Closing Keys.—The order in which the battery and galvanometer keys are closed is not a matter of indifference. It is essential that the battery key be closed first. For consider Fig. 135. The coils in R are wound so as to avoid self-induction but this object may not be completely attained and with a number of coils unplugged the inductance may not be negligible. Again, if the resistance X be that of a coil, especially if it be wrapped around an iron core, its self-induction will be large. Finally, if X be a cable it may have considerable capacity as a condenser. In any of these cases, when the battery key K is closed the current will not rise at once to its full strength in the branch affected. Suppose the bridge to be balanced accurately and the galvanometer key closed first; when K is closed the current in one branch or the other not rising at once to its full strength, M and N will be momentarily at different potentials and there will be an instantaneous rush of current through G causing a deflection of the needle and incorrectly indicating a lack of balance. On the other hand, if K be closed first there will still be this retardation but its effect will disappear in a fraction of a second, M and N will reach the same potential and when H is closed there will be no deflection of the galvanometer needle.

There may be used a special key which by making successive contacts as it is pressed down will insure the proper sequence of closing.

To avoid violent swings of the needle, the galvanometer key at first should be given a mere tap.

321. Proper Ratio to Use.—The first determination gives the resistance of X to the nearest unit or ohm. If it be desired to measure it to the first, second, or third place of decimals, the plugs in A and B must be so adjusted that the ratio B/A is .1, .01, or .001 and the corresponding decimal places are pointed off in the final reading of R. If the resistance to be measured be large, the ratio B/A must be 10, or 100, or 1000.

It will be noted that some of the ratios can be obtained by several

different combinations, thus $\frac{10}{10}$, $\frac{100}{100}$, $\frac{1000}{1000}$, all give the ratio unity. It can be shown that other things being equal, the greatest sensibility is obtained when the resistances in the four arms of the bridge are as nearly equal as possible. For example, if the resistance to be measured is about 100 ohms and this is to be measured to the nearest unit, the ratio should be $\frac{100}{100}$, or if to the nearest tenth then $\frac{100}{1000}$.

The instrumental sensibility depends directly upon the sensitiveness of the galvanometer, or its ability to indicate very minute currents when the bridge is nearly balanced.

322. Bridge with Reversible Ratios.—There is sometimes used, instead of the bridge described above, a variation by which a coil is saved in each of the arms *A* and *B*, making six instead of eight, and yet the same ratios are preserved.

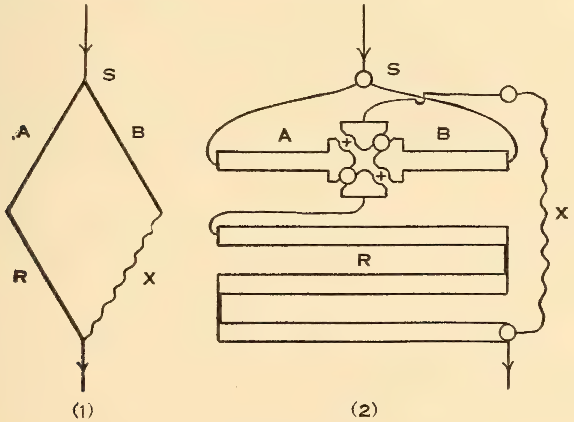


Fig. 136.

Its arrangement is shown in (2) in Fig. 136 and is as if the arms *A* and *B* of (1) had been separated at *S* and each rotated outward from the center. These outer ends (2) are then connected by a heavy wire with *S* which must now be regarded as the junction of *A* and *B* and, according to Par. 317, is the point at which the battery current enters. The inner ends of *A* and *B* are connected to the *R* and the *X* arms by movable plugs. With the plugs in the positions shown by the small circles in (2), *A* is connected to *R* and *B* to *X*. If these plugs be shifted to the positions marked by the crosses, *A* becomes connected to *X* and *B* to *R*, in other words (see Par. 317), *A* and *B* interchange.

The A arm contains the coils 1, 10, 100, the B arm 10, 100, 1000. The smallest B/A ratio obtainable with the plugs in the first position is $\frac{10}{100}$ or .1. If it be desired to use a smaller one, shift the two plugs, A becomes B and B becomes A, and the ratios $\frac{1}{100}$ and $\frac{1}{1000}$ become available.

323. The Dial Bridge.—In the bridges described above, resistances are thrown in the rheostat by *removing* plugs. There are other forms, such as the dial bridge and the decade bridge, in which resistances are introduced by *inserting* plugs. The connections of a dial bridge are shown diagrammatically in Fig. 137.

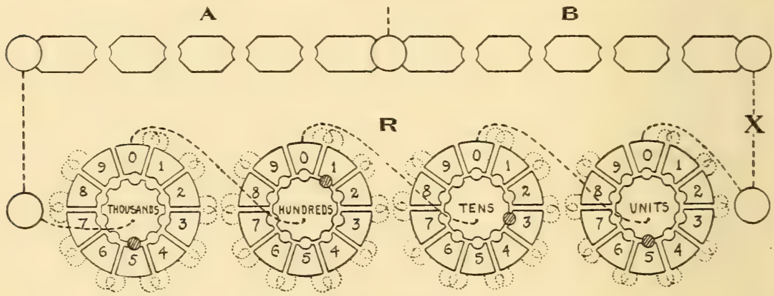


Fig. 137.

The A and B arms are like those of the ordinary bridge but the rheostat is composed of dials, usually four, which are marked units, tens, hundreds, and thousands, respectively. Each dial consists of a heavy center piece of brass surrounded by ten keystone shaped pieces, these being numbered 0, 1, 2, etc., to 9. Between the successive keystone pieces, except numbers 9 and 0, are resistance coils, those at each dial being all of the same resistance. Thus, at the unit dial each coil has a resistance of one ohm; at the ten dial each has a resistance of ten ohms, and so on. The current entering by A goes to the center of the first dial, then through the plug to the corresponding keystone piece, thence through the coils in series to the 0 keystone and thence to the second dial, etc. The diagram represents a resistance plugged in of 5135 ohms.

This form is more expensive than the first but has a number of advantages, among them, the smaller number of plugs to be handled and consequent smaller number of contacts (four as compared to fifteen or more) and the much less danger of error in reading off resistances.

324. Resistances that may be Measured by Bridge.—The bridge is not suited to the measurement of very high or of very low resistances. Theory requires that with the plugs inserted the resistances in the arms should be zero while, as a matter of fact, they have a resistance which may affect the fourth place of decimals. The resistance in the contacts of the plugs themselves may affect the third place. Therefore, in measuring very small resistances these neglected resistances may cause a considerable error, and in the case of a very large resistance any error in the balance is multiplied a hundred or a thousandfold by applying the ratio B/A. In general, the measurements should lie between .01 and 100,000 ohms.

325. The Slide Wire Bridge.—A simplified form of bridge, used especially in the measurement of low resistances, is the so-called *slide wire bridge*. This consists (Fig. 138) of a wire WW of uniform

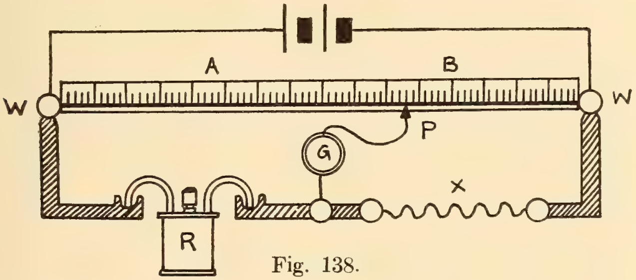


Fig. 138.

cross-section stretched between heavy copper terminals and above a graduated scale. Since this scale is usually a meter subdivided into millimeters, the instrument is often called a "meter bridge." Connections are made as shown in the figure, R being a standard resistance coil (Par. 311) whose resistance is preferably as near as possible to that of X, the resistance to be measured. The terminal P of the galvanometer is slid backwards and forwards along the wire WW until balance is attained, at which point, if A and B be the resistances of the corresponding portions of the wire, we have, as in any other bridge, $X = BR/A$. Since the wire is of uniform cross-section, the resistance of the portions is directly proportional to their lengths, hence in the above expression the lengths of A and B, which may be read directly from the printed scale, can be and are used instead of the actual resistances, which last need not be known at all.

326. Measurement of High Resistance.—The principle of the measurement of high resistance is simple. We measure accurately the current driven through the resistance by a known E. M. F., whence, by Ohm's law, the resistance is obtained at once. For example, to measure the resistance of the rubber insulation of a reel of submarine cable, the entire cable, except the two free ends, is submerged in a tank of water (Fig. 139). To one of the ends of

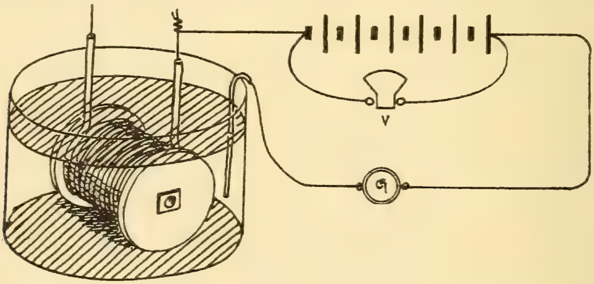


Fig. 139.

the cable core is attached a terminal of a battery. The other terminal is connected to *G*, a very delicate current-measuring instrument (a reflecting galvanometer, Par. 378), and the circuit is completed by a wire extending from *G* and dipping into the water in the tank. The E. M. F. of the battery is measured by the instrument *V*, and the resulting current by *G*, whence *R* follows from Ohm's law. Reflection will show that should the total length of the cable be *n* yards, the average resistance per yard is *n times the total resistance.* In actually carrying out this measurement, many refinements and precautions are observed, not necessary to mention here.

327. Measurement of Resistance of Electrolytes.—The resistance of an electrolyte can not be measured by the means described above. We have seen (Par. 215) that the passage of a current through an electrolyte produces chemical decomposition; the current used in balancing a bridge would therefore bring about this electrolysis. If gas be released at either anode or cathode, the resistance which we are trying to measure would be very greatly increased. Also the products of electrolysis will still set up a back E.M.F. which by cutting down the current through the electrolyte would lessen the drop in the corresponding branch and render valueless observations based on movements of the galvanometer needle.

We may, however, make these measurements by employing a rapidly alternating current, that is, a current which many times a second reverses its direction of flow. In this case, a galvanometer can not be used to indicate a balance but in its stead a telephone receiver is employed, taking the place of G in Fig. 138. So long as an alternating current flows through the receiver a buzzing sound is produced, but when the bridge is balanced the sound dies out. Explanation of these facts will be given later.

328. Measurement of Internal Resistance of Cells.—In measuring the internal resistance of a cell the same difficulties are encountered as in the case of electrolytes and in addition the current produced by the cell itself prevents the use of the bridge. There are, however, several methods by which this internal resistance may be measured. The simplest is by using the instruments for measuring E. M. F. and current, which instruments will be described in Chapter 34. We first measure the E. M. F. of the cell when no current is flowing. We then cause a moderate current to flow from the cell, measure this current and the external or useful volts (Par. 305). The difference between the E. M. F. of the cell and the useful volts is the lost volts or Ir, and knowing I we determine r.

CHAPTER 27.

THE POTENTIOMETER.

329. Measurement of Electro-Motive Force of Cells.—The simplest and usual way of measuring the electro-motive force of a cell is by means of a *voltmeter*, an instrument described in Chapter 34. It will be shown, however, that in order to obtain a reading from the voltmeter, there must be a flow of current through the instrument. It is true that this current is so small that for all ordinary cases it is entirely negligible, but if there be a current there will also be lost volts (Par. 305) and since a voltmeter reads only the useful volts, its indications are always some slight amount less than the true E. M. F. Therefore, to obtain strictly accurate results, the E. M. F. of a cell should be measured when no current is flowing. This may be done with an electrometer, as explained in Chapter 11, but preferably by a *potentiometer*, an instrument which we shall now describe.

330. Preliminary Arrangement of a Potentiometer.—Let us suppose that we start with one or two cells giving us a constant

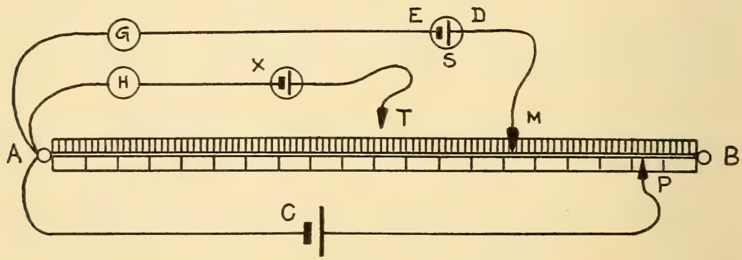


Fig. 140.

E. M. F. of about two volts, seven or eight feet of rather thin wire of uniform cross-section, and a graduated paper scale. Provided the scale be graduated uniformly, the unit is immaterial, but a millimeter scale running up to 2000 is very convenient. We will tack the paper scale upon a board AB (Fig. 140), and stretch the wire above it. To the end A of the wire we connect the negative terminal of our cell C; the other terminal makes sliding contact at P.

If the E. M. F. of the cell be two volts, the difference of potential between P and A before P is touched to the wire will be two volts and after contact is made it will still be *about* two volts. If it were exactly two volts when P is at the 2000 division on the scale, there would be a drop of two volts from P to A and each division of the scale would correspond to a drop of one-thousandth of a volt. If P be slid in towards A, these two volts will be spread over a shorter length of the wire and each division on the scale would correspond to a drop of more than one-thousandth of a volt. On the other hand, if P be slid out from A, the scale divisions can be made to correspond to less than one-thousandth of a volt, therefore, by sliding P backwards and forwards we can vary the drop over the scale and at one particular point this drop will be exactly one-thousandth of a volt per millimeter. This point is located as follows:

331. Calibration of Potentiometer.—To the same end A of our stretched wire we connect through a galvanometer G the negative terminal of a standard cell S. If this be a Clark's cell whose E. M. F. is 1.434 volts (Par. 212), we connect its positive terminal to the wire at M, a point 1434 millimeters from A. If M be at a higher potential than 1.434 volts a current will flow from M to D, while if it be at a lower potential a current will flow from D to M. In either case this flow will be indicated by a deflection of the needle of the galvanometer G. If there be a flow, we slide the contact P backwards or forwards until a point is found where G indicates no current and we then know that the potential of M is the same as that of D, that is, 1.434 volts, and that consequently each division of the scale corresponds to a drop of one-thousandth of a volt. The contact P is left at this point. The instrument is now in adjustment so that the printed figures on its scale read thousandths of a volt, in other words, it has been calibrated.

332. Measurement with Potentiometer.—To measure the E. M. F. of a cell X, its negative terminal is connected through the galvanometer H with A and its positive terminal is connected to a contact T which is moved back and forth along the wire until H indicates no current. Suppose this point to be the 925th millimeter from A, then the E. M. F. of X is .925 volt.

Instead of using a second galvanometer H, the negative terminal of X could have been attached to G, that is, S and X can use G in common.

333. Forms of Potentiometer.—As in the case of the Wheatstone bridge, the actual instrument bears no resemblance at all to the diagrammatic representation in Fig. 140. For example, for the sake of compactness the long wire is wound in a helical coil around an ebonite cylinder, etc., etc. There are numerous forms of potentiometers but the principle of all is the same, that is, they measure an unknown E. M. F. by balancing against it an equal and opposite E. M. F. which latter is known.

CHAPTER 28.

GROUPING OF CELLS IN BATTERIES.

334. Grouping of Cells.—The cells composing a battery may be connected up in several ways. If they are connected one after the other they are said to be *in series.* If all of the positive poles are connected to one common wire and all of the negative poles to another, they are said to be *in parallel.* If they are divided into groups, the cells in each group being connected in series and these separate groups being then connected in parallel, the battery is said to be grouped *in multiple,* or better, so many in parallel and so many in series. For example, if we have ten cells we might group them all in series, or all in parallel, or two abreast and five deep, that is, two in parallel and five in series, or finally, five abreast and two deep, that is, five in parallel and two in series. Each of these arrangements is quite proper under certain conditions but it will be shown in the following paragraphs that it is not a matter of indifference which shall be employed.

335. Cells in Series.—In Par. 192 we saw that in a voltaic cell the copper or positive pole is at a higher potential than the zinc or negative pole. Suppose that we have a number of simple cells, each of an E. M. F. of one volt, and that we should arrange them

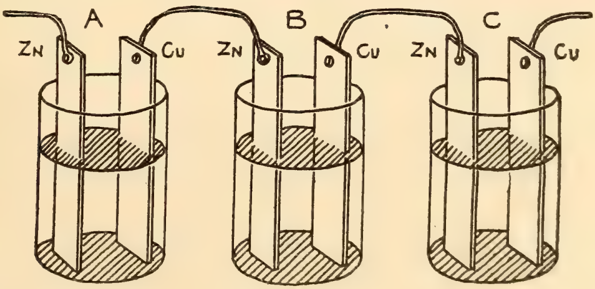


Fig. 141.

in series, the copper plate of each, as shown in Fig. 141, being connected to the zinc plate of the adjoining one. The copper plate of *A* and the zinc plate of *B* being connected are at a com-

mon potential, therefore the zinc plate of B is one volt higher than that of A. The copper plate of B being one volt higher than its zinc plate is consequently *two* volts higher than the zinc plate of A. Similarly, the copper plate of C is *three* volts higher than the zinc plate of A, and in general *the total E. M. F. of a number of similar cells connected in series is equal to the E. M. F. of one cell multiplied by the number in series.* This principle applies even though the circuit includes cells of different kinds, electrical machines, etc., and the most general statement is that in any electric circuit containing several sources of E. M. F. in series the total E. M. F. is the sum of the separate E. M. F.s.

336. Cells in Parallel.—Fig. 142 represents three cells in parallel. The three positive poles being brought together at a common point A are all at the same potential, that is, one volt higher than the three negative poles which are brought together

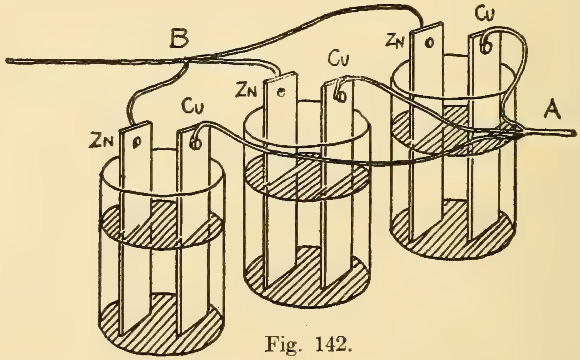


Fig. 142.

at B. This combination therefore has no greater E. M. F. than has a single cell and it is in fact, as we have already seen (Par. 294), equivalent to a single cell whose copper and zinc plates are three times as large as those of the original cells.

337. Comparison of Series and Parallel Groupings.—We may by a concrete example best illustrate the different effect of the two kinds of groupings. Suppose that we have a number of cells, each of an E. M. F. of 2 volts and an internal resistance of .25 ohm. From a single cell in a circuit of negligible external resistance the current obtainable is

$$I = \frac{E}{R + r} = \frac{2}{0 + .25} = 8 \text{ amperes}$$

With two in series, the E. M. F. is twice as great (Par. 335), but also the resistance is twice as great (Par. 286), therefore the current is the same, and so on for any number, that is, with a circuit of negligible external resistance the effect of grouping cells in series is to increase the voltage but not the current. Should, however, the external resistance be great, a different state of affairs results. For example, let $R = 100$ ohms (the resistance of about 6 miles of iron telegraph wire), then for one cell

$$I = \frac{2}{100 + .25}$$

and for two cells

$$I = \frac{4}{100 + .50}$$

The difference in these denominators being negligible, we see that in this second case we have doubled the current.

If, starting again with negligible external resistance, we arrange two of these cells in parallel, the E. M. F. is no greater than for one cell (Par. 336) but the resistances of the two cells being in parallel, the total resistance is only one-half that of one cell (Par. 293), hence the current is doubled. For three cells it is trebled, and so on, that is, with negligible external resistance, the effect of grouping cells in parallel is to increase the current but not the voltage.

With a large external resistance, the grouping of cells in parallel, since it does not increase the E. M. F. nor change the total resistance to any significant extent, does not alter the current.

We may sum up by saying that with a large external resistance we increase the current by grouping the cells in series; with a small external resistance, we increase it by grouping them in parallel.

338. Analogy Between Voltaic Cells and Pumps.—Difference of potential has been compared to difference of water level (Par. 70). Since a difference of potential is produced in a cell, we may continue the comparison by drawing an analogy between a cell and a pump. In Fig. 143 the pumps *A* and *B* are analogous to two cells in parallel; the two lift the water no higher than a single pump but they lift twice the quantity. The pumps *C*, *D* and *E*

are analogous to cells in series; they lift no more water than a single pump but they raise it three times as high.

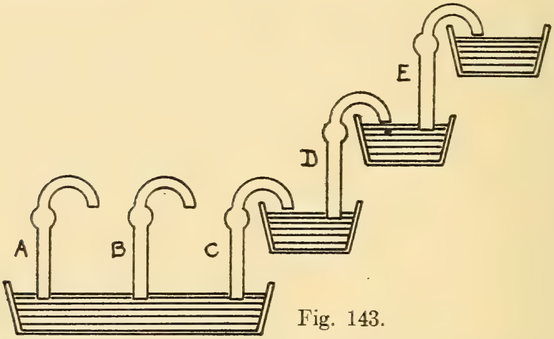


Fig. 143.

339. Parallel-Series Grouping.—Suppose we have N cells, each of an E. M. F. of e volts and an internal resistance of r ohms, and suppose that they are arranged (Fig. 144) s in series and p in

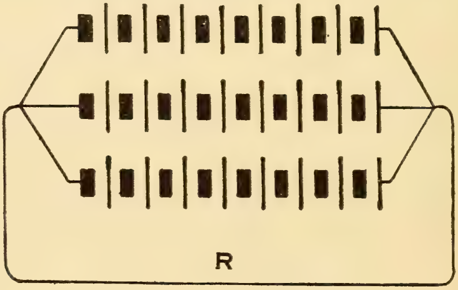


Fig. 144.

parallel in a circuit of external resistance R. The resulting E. M. F. is equal to the E. M. F. of one cell multiplied by the number in series (Par. 335) or se. The resistance of one of the series is rs, but since there are p rows in parallel, the total internal resistance is $\frac{rs}{p}$.

The current produced by this arrangement is

$$I = \frac{se}{R + r\frac{s}{p}}$$

340. Maximum Current.—The question may arise, given N cells, how should they be grouped to obtain the maximum current?

The expression for the current is given in the preceding paragraph and, since $N = sp$, can be written

$$I = \frac{Ne}{\frac{NR}{s} + rs}$$

If this be differentiated and the first differential coefficient be placed equal to zero, the resulting values of s will correspond to maximum or minimum values of I. This differentiation is tedious. However, since Ne is a positive constant, I will be a maximum when $\frac{NR}{s} + rs$, the denominator of the expression, is a minimum.

Place
$$x = \frac{NR}{s} + rs = NRs^{-1} + rs$$

Differentiating

$$\frac{dx}{ds} = -NRs^{-2} + r = r - \frac{NR}{s^2}$$

Placing this equal to zero, we have

$$s^2 = \frac{NR}{r}, \text{ whence } s = \pm\sqrt{\frac{NR}{r}}$$

which is the value sought. This will in general only approximate to the desired arrangement since the *mathematical* supposition is that s and p are continuous variables, while actually they are both discontinuous or positive whole numbers. For example, if N be 10, the only possible values of s are 1, 2, 5 and 10, yet the actual solution will generally produce some mixed number. In such a case we should make the calculation of the current from the two groupings which come nearest to the one indicated by the solution and select accordingly.

If in the above equation of condition for maximum current

$$s^2 = \frac{NR}{r}$$

we substitute for N its value sp, we get

$$s^2 = \frac{spR}{r}$$

whence

$$R = r\frac{s}{p}$$

But (Par. 339) R is the external resistance and $r\frac{s}{p}$ is the internal resistance, whence we arrive at the important conclusion that *the current is a maximum when the battery is so grouped that the internal and the external resistances are equal.*

We saw (Par. 305) that the useful volts of a cell or battery are given by IR, the lost volts by Ir. Since in the case of maximum current $R = r$, the lost volts amount to one-half of the total E. M. F.

341. In Multiple Arrangements Equal E. M. F. is Required of Groups in Series.—In a parallel-series arrangement the series groups must all have the same E. M. F. This requires that where the cells are all of one kind there should be the same number in each series group. The arrangement shown in Fig. 145 is not

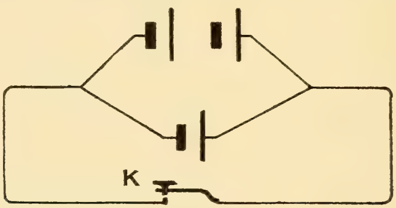


Fig. 145.

permissible. The battery should be quiescent when the key K is open but the three cells now constitute a closed circuit in which the E. M. F. of the two upper cells acting in a clockwise direction is not counterbalanced by the opposing E. M. F. of the single cell. If the E. M. F. of each cell be e and its resistance r, there will flow through the single cell a reverse current whose strength is $I = \frac{e}{3r}$. The elements of the two upper cells will therefore consume away and the zinc plate of the single cell will have copper deposited upon it which will cause local action. With K closed, the loss is not so great and it will diminish as the external resistance decreases, but even in this case the elements of the single cell will consume away much more rapidly than those of the two in series.

342. Diagrams of Parallel-Series Grouping.—A parallel-series grouping represented as in Fig. 144 doubtless aids the beginner, but in actual practice cells are seldom arranged in this geometrical order. Especially is this the case with storage batteries in which

the cells are very heavy and are placed in rows on shelves or benches. Reflection will show that after all it is not necessary to move the cells themselves but rather to shift the connecting wires. Thus in Fig. 146 eight cells are represented in four different

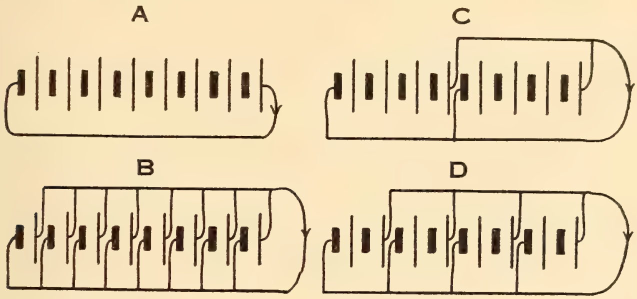


Fig. 146.

groupings, the cells themselves not being disturbed. In *A* they are all in series, in *B* all in parallel, in *C* four in series and two in parallel, and in *D* two in series and four in parallel.

343. Cost of Power from Primary Cells.—For the small and irregular currents required in telegraphy and in operating telephones, call bells, annunciators, alarms, etc., a battery of primary cells is the most suitable and economical source of electrical energy, but where the current is required to furnish appreciable mechanical power through suitable machines, the cost is prohibitive. The chemicals consumed in the cell correspond to the fuel consumed in the boiler of a steam engine, and while one pound of carbon burned in air evolves enough heat to raise 8080 pounds of water 1° C, in round numbers four pounds of zinc must combine with six pounds of sulphuric acid to produce the same amount of heat. With modern machines electrical energy may be produced as cheaply as one cent per horse-power per hour but the same energy supplied from primary cells costs from 30 to 50 times as much. Where many telephones or telegraph lines are operated from a central station it is now the practice to use storage batteries instead of the batteries of Daniell or gravity cells formerly employed.

PART IV.

ELECTRO-MAGNETICS.

CHAPTER 29.

MAGNETIC FIELD ABOUT A WIRE CARRYING A CURRENT.

344. Oerstedt's Discovery.—In 1819, in the course of a lecture on electricity, Oerstedt, Professor of Physics at Copenhagen, observed that when a wire carrying a current was brought near a magnetic needle a deflection of the needle was produced. He recognized at once the importance of this discovery as demonstrating what up to that time had been merely a conjecture, that is, that there existed some connection between electricity and magnetism. He set to work immediately to investigate the matter and soon discovered not only that an electric current produced a deflection of a magnetic needle near it but that the direction of this deflection depended both upon the direction in which the current was flowing and upon the position of the conductor with reference to the needle. His results were announced in 1820. The news reached the French electrician Ampere on September 11 and was received by him with eagerness. Within one week thereafter he had repeated Oerstedt's experiments and had added to the latter's discoveries; had confirmed by specially devised experiments and had presented in a paper to the Academy a complete theory of the new science of electro-dynamics (Par. 360).

345. Right Hand Rule for Deflection of Needle.—It is helpful to the electrician, whether he be an advanced student or only a beginner, to have some easy rule for determining, or some mechanical way of remembering, in which direction certain phenomena take place. Thus Ampere gave the rule of the "swimming man" by which, the relative positions of the conductor and of the needle and the direction of the current being given, the direction in

which the north end of the needle would move could be predicted. Other rules have been given by subsequent writers. Of these, the following is thought to be the most useful, both because of its simplicity, it being a true "rule of thumb," and because, as will be shown later, of its applicability to a number of varied conditions. It should be committed to memory.

Place the palm of the right hand upon the wire, the extended fingers pointing in the direction of the flow of the current, the palm turned towards the needle; the extended thumb will indicate the direction in which the north pole of the needle will move.

Fig. 147 represents the application of this rule. The current flowing in the wire in the direction indicated by the arrow will

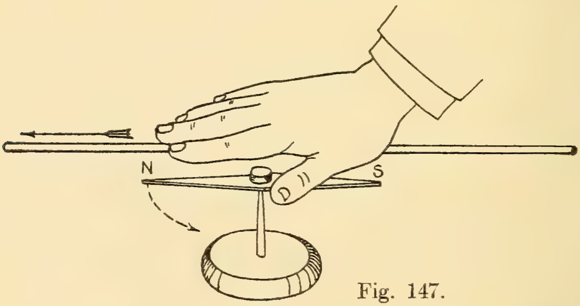


Fig. 147.

cause the north pole of the needle to move out in the direction in which the thumb is pointing.

If the wire be below, in order that the palm should be turned towards the needle, the hand must be held back down, in which case the thumb will point away from the observer and this is the direction in which the north pole will actually move. In fact, the rule is perfectly general and applies if the wire be vertical and in front of either pole or if it be to either side of the needle, only in this last case the needle must be capable of movement in a vertical plane.

346. Magnetic Field About a Wire Carrying a Current.—In Pars. 143 and 144 it was shown that a needle in a magnetic field tends to turn so as to place its longer axis and its own lines of force parallel to the lines of force of the field. The needle in Oerstedt's experiment turns for the same reason, that is, the current flowing through the wire establishes about this wire a magnetic field with which the needle tends to coincide in direction.

This field may be studied in a similar manner to the other magnetic fields already described. If a vertical wire, a portion of an electric circuit, be passed through a hole in the center of a horizontal sheet of cardboard or of glass which has been sprinkled with iron filings (Fig. 148) and if the circuit be then closed and the horizontal sheet be tapped while the current is flowing, the filings will be seen to gather and form in more or less distinct circles around the wire as a center. The lines of force of the field are circles, and it was shown first by Ampere that these circles lie in planes perpendicular to the wire. In Oerstedt's experiment as described in Par. 344, the needle can never place itself at right angles to the wire, for the controlling force, the horizontal component of the earth's magnetism, is always effective. However, if a perfectly balanced needle be mounted so that its axis of rotation is parallel to the earth's field, then this field has no influence upon its rotation and if Oerstedt's experiment be now performed, the needle will always set itself at right angles to the wire.

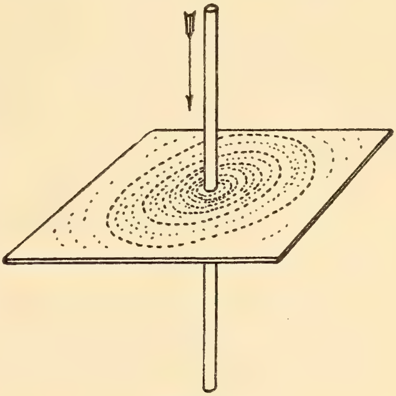


Fig. 148.

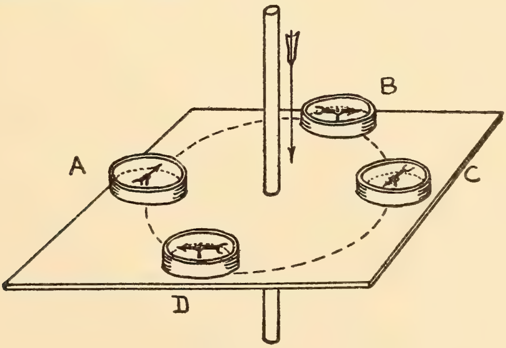


Fig. 149.

347. Direction of Field.—The experiment with the iron filings shows the lines of force of the field to be circles but does not indicate their direction. This latter may be determined as follows.

Using the same horizontal sheet and vertical wire as in the preceding experiment, distribute at equal distances apart on the circumference of a circle whose center lies on the wire a number of small compasses, *A*, *B*, *C*, *D* (Fig. 149). Before the circuit is closed, these all point in the same direction. Let us assume that this is the direction indicated by the needle *A*. Suppose now the circuit to be closed and the current to flow *down* the wire. The needle at *A* will not change its position, *C* will be entirely reversed, *B* will point to the right and *D* to the left, that is, if we look at the needles from above, they point around the circle in the direction of the motion of the hands of a clock. Had the current flowed *up*, the needles would all have pointed in a counter-clockwise direction around the circle.

348. Clock Rule for Direction of Field.—The foregoing experiment suggests another simple rule for determining the direction of the field about a wire carrying a current.

Suppose the eye placed so as to look along the wire in the direction in which the current is flowing; the positive direction of the field about the wire is the same as the direction of motion of the hands of a clock.

Of course, if the current is flowing *towards* the eye, the field is counter-clockwise. This rule should not supplant the right hand rule given in Par. 345. Either one could be used to the exclusion of the other but it is better to have both at command.

349. Wire Carrying a Current is not Itself a Magnet.—Although surrounded by a magnetic field, a wire carrying a current is not itself a magnet. If a clean copper wire through which a current is flowing be dipped into iron filings and then lifted, the filings will cluster around the wire but will drop off when the current is broken. At first sight this seems to indicate that the wire has become magnetized but it can be shown that such is not the case. When the wire is thrust into the filings they become magnetized, since magnetic bodies placed in a magnetic field become magnets (Par. 120), and if they surround the wire, or if any of them adhere to it through stickiness, they cling together like the links of a chain and really adhere to each other instead of to the wire. If an elongated filing be placed at right angles to the wire and with its ends lying upon one of the circular lines of force surrounding the wire, one of these ends will be urged in one direction around

the circle, the other end in the opposite direction; the result is that the filing will move broadside towards the wire. There is, however, no radial component between a wire carrying a current and a magnetic pole in its field. In this respect the field about a conductor is unique. While all other forces exerted between bodies act along the line joining the bodies, the force upon a pole in a field about a wire acts *at right angles* to the line joining the wire and the pole.

350. Rotation of a Magnetic Pole by a Current.—In Par. 135 the positive direction of a magnetic field was defined as that direction in which a free north pole would move. Such a pole released near the north end of a magnet would move off along a line of force, curving around until it came to rest against the south face. The statement was made (Par. 142) that a magnetic line of force is a closed curve, but the moving pole can not travel around a complete orbit for its progress is arrested by the material substance of the magnet. In the field about a wire carrying a current the case is different. Here the lines of force are circles, return upon themselves and do not necessarily pass through any solid body. A pole released in such a field should therefore rotate as long as the field is maintained. Although we can not obtain a free pole, we can approximate to the theoretical condition by arranging a circuit so that only one pole of the magnet lies in the field and we can thus produce mechanical rotation.

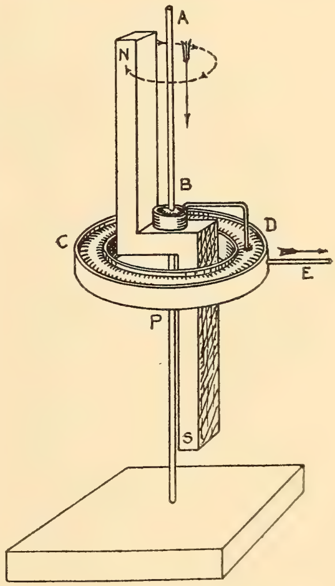


Fig. 150.

Fig. 150 represents diagrammatically such an arrangement. *NS* is a magnet bent in the center at an angle and placed upon the pivot *P* about which it is free to rotate. *B* is a little cup of mercury into which dips the conductor *AB*, thereby securing movable electric contact with a minimum of friction. *CD* is an annular cup of mercury surrounding but not touching the magnet. From *B* a wire *BD* is carried over and bent down so as just to

touch the surface of the mercury at D and to sweep along this surface as the magnet rotates. DE is a conductor leading away from the annular cup. If the current enters at A, it goes to B, thence to D and out by E. It therefore passes by the pole N but not by the pole S. According to the rule given in the preceding paragraph, the field about AB, viewed from A, is clockwise. The pole N will therefore spin around in the direction shown by the dotted line. If the current be reversed the direction of rotation is also reversed; so also if the magnet be inverted, the direction of rotation is reversed.

351. Rotation of a Current by a Magnetic Pole.—The reaction between the pole and the field being mutual, it follows that if the pole be fixed and the conductor be free to move, the latter may be made to rotate about the former. This may be shown by the apparatus represented in Fig. 151. NS is a magnet run through a cork which is inserted in the lower end of a short and broad glass tube. The annular space around the projecting pole N is filled with mercury. A current is led down by the wire A, through the flexible joint and B into the mercury cup and out by C. While the current flows B is surrounded by lines of force which viewed from A are clockwise. If B were stationary and N were free to move, N would travel around B in a clockwise direction, that is, N would move out towards the observer. However, N being fixed, B moves back from the observer and travels around N in a clockwise direction.

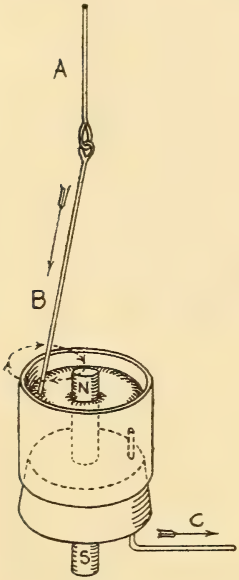


Fig. 151.

352. Left Hand Rule for Direction of Motion.—The conductor described in the preceding paragraph is in the field of the magnet and owes its motion to the interaction of this field with its own. Any conductor carrying a current and placed in a magnetic field will move if it be free to do so. It is useful to have a rule by which the direction of this motion can be foretold. The following is such a rule. *Place the palm*

of the left hand upon the wire, the extended fingers pointing in the direction of the flow of the current (*Fig. 152*) *and the palm turned to receive the lines of force of the field; the extended thumb will point in the direction of the motion of the conductor.*

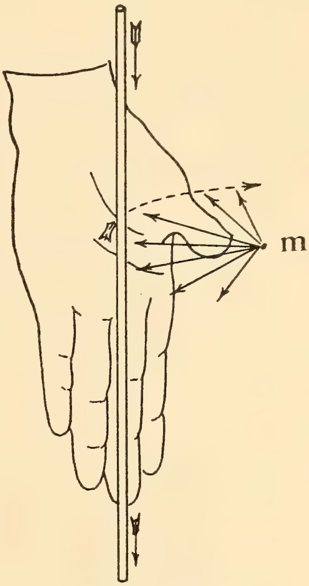


Fig. 152.

353. Intensity of Field About a Straight Conductor.—A magnetic field is known when we have determined its direction and intensity. We have shown above (Par. 347) how to determine the direction of the field about a conductor carrying a current. The intensity may be measured as explained in Pars. 148–150. In two simple cases (which fortunately are the ones most frequently encountered), it may be calculated. These are, first, when the conductor is straight, and second, when it is bent into the arc of a circle.

In Fig. 153 let AB represent a portion of a straight wire of indefinite length carrying a current of strength I. (The unit in which I is measured is for the moment held in abeyance; see Par. 355.) Let m represent a unit pole at a distance r from the wire. The force exerted upon m will measure the intensity of the field at that point (Par. 136). Let A represent an infinitely small section of the wire, its length being dy. It has been shown by Laplace that the force exerted upon a magnet pole by an infinitely short element of a conductor carrying a current is directly proportional to the strength of the pole, to the strength of the current, to the length of the element, and to the sine of the angle which this element makes with the line joining its center and the pole; it is also inversely proportional to the square of the length of this line. In the case

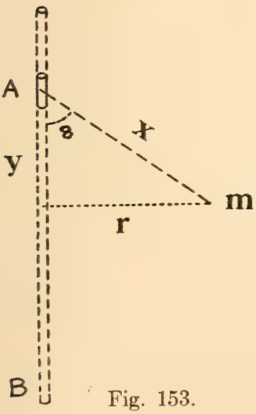


Fig. 153.

represented, the force exerted by A upon the unit pole at m is therefore

$$df = \frac{I.dy}{x^2} \cdot \sin \alpha \qquad \text{(I)}$$

This expression integrated between proper limits will give the intensity of the field produced at m by the corresponding lengths of AB.

From the figure $x = \dfrac{r}{\sin \alpha}$

$$\text{also } y = \frac{r}{\tan \alpha}, \text{ hence } dy = r\,(-\operatorname{cosec}^2 \alpha\, d\alpha)$$

Substituting these values in (I) and remembering that $\operatorname{cosec} \alpha = \dfrac{1}{\sin \alpha}$, we obtain

$$df = \frac{I}{r} \cdot \sin \alpha\, d\alpha$$

Integrating $\qquad f = \dfrac{I}{r} \cdot \cos \alpha + \text{a constant.}$

Taking this between the limits $\alpha = 0°$ and $\alpha = 180°$, we have

$$f = \frac{2\,I}{r}$$

or the field at any point about an indefinitely long straight wire is directly proportional to the current and inversely proportional to the *simple* distance from the wire.

354. Field on the Axis of a Circular Coil.—The field produced at a point on the axis of a circular coil may be determined as

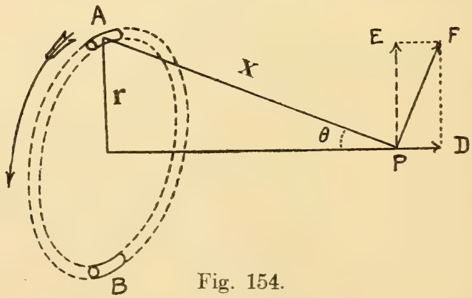


Fig. 154.

follows: With a current of strength I flowing as indicated by the arrow (Fig. 154), the infinitely small portion of the coil at A exerts

upon a unit north pole at P a force in the direction PF which is $\frac{I.dl}{x^2}$, dl being the length of A. This may be divided into two components, one $PD = \frac{I.dl}{x^2} . \sin\theta$, and the other PE. The diametrically opposite element of the coil at B likewise exerts a force upon P which may be divided into two components, one in the direction PD, the other opposite and equal to PE and hence counterbalancing it. Every element of the coil therefore exerts in the direction PD a force upon P equal to

$$\frac{I.dl}{x^2} . \sin\theta$$

The sum of these elementary forces is

$$f = \frac{I.2\pi r}{x^2} . \sin\theta$$

Substituting for $\sin\theta$ its value r/x

$$f = \frac{I.2\pi r^2}{x^3}$$

or, the field at any point on the axis of a circular coil varies directly with the current and inversely as the cube of the slant distance.

If the point P be moved to the center of the coil, x becomes equal to r and the above expression becomes

$$f = \frac{I.2\pi}{r}$$

Should the coil consist of n turns, the field produced is n times as strong as that produced by one turn, therefore, the above expressions for the field must be multiplied by n.

An important consequence follows from the foregoing. Since the field at the center of a circular coil varies directly with the current, the measure of the field may be used as a measure of the current. This will be shown in the following paragraph.

355. Absolute Unit of Current.—Since, as has just been seen, we obtain the intensity of the field at the center of the coil by adding up the effects produced by each infinitesimal section of the coil, the field produced by a *portion* of the coil must be

directly proportional to the length of this portion, or, if this length be l

$$f = \frac{I.l}{r^2}$$

If in this expression we make r and l each one centimeter, we have

$$I = f$$

and if f be one dyne, I is unity, whence we derive at once the definition of *the absolute unit of current as that current, which flowing through one centimeter of a conductor bent into the arc of a circle whose radius is one centimeter, exerts a force of one dyne upon a unit pole placed at the center of the circle.*

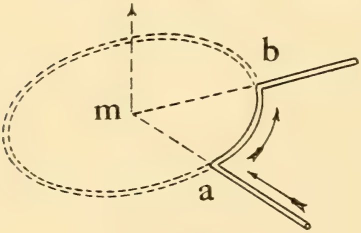


Fig. 155.

If in Fig. 155 the length of the conductor from a to b be one centimeter and if it be bent into the arc of a circle of one centimeter radius, the current which flowing through this conductor exerts a force of one dyne upon the unit pole at m, has a strength of one absolute unit.

The absolute unit of current, as will be explained later (Chap. 39), is ten times as great as the practical unit, the ampere, or, one absolute unit equals ten amperes. Therefore, in applying the expressions in Pars. 353 and 354, if I be given in amperes, it must be reduced to absolute units or divided by ten in order that f should be in dynes.

356. Force Exerted by a Magnetic Field upon a Conductor Carrying a Current.—The force exerted upon a unit pole at m by the field of ab (Fig. 155) is shown to be

$$f = \frac{I.l}{r^2}$$

If the strength of the pole be m instead of unity, the force is

$$f = \frac{m.I.l}{r^2}$$

If the current flows as shown in the figure, and if m be a north pole, this force acts upward. An equal downward force acts upon ab. In the above expression $\frac{m}{r^2}$ is the field along ab due to the pole m (Par. 136) and is uniform. Calling this H, we have

$$f = I.H.l$$

or, the force exerted by a magnetic field upon a conductor carrying a current and at right angles to the field is proportional to the current, to the intensity of the field and to the length of the conductor. This force is at right angles to the field and to the conductor and, as explained in Par. 355, is expressed in dynes when I is in absolute units.

Fig. 156 represents a cross-section of such a conductor lying in a field NS. If the current is flowing away from the observer,

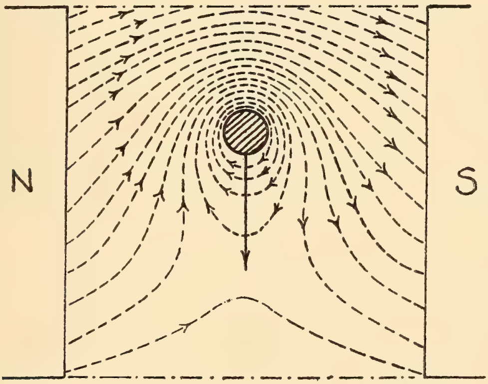


Fig. 156.

the lines of force about the wire are clockwise, therefore, on the upper side of the wire they coincide in direction with those of the field but on the lower side they are opposite in direction. The field is therefore distorted as shown, the lines thickening up above the wire and thinning out below. Since lines of force have a tension in the direction of their length, or a tendency to shorten, the result is that the wire is urged downward. Application of the left hand rule (Par. 352) indicates this downward motion.

357. Work Done in Moving Across a Magnetic Field a Conductor Carrying a Current.—From the preceding paragraph, the force exerted upon a conductor carrying a current and lying at right angles to the field is $I.H.l$ dynes.

If the conductor be moved at right angles to the field and to its own length, it will move either against this force or with it. In the first case, work must be done upon the conductor; in the second case, work is done by the conductor. In either case, if the distance moved be x centimeters the work, being force $\times$ path, is

$$W = I \,.\, H \,.\, l \,.\, x \text{ ergs}$$

But $l.x$ is the area in square centimeters swept over by the conductor in its movement, H is the number of lines of force per square centimeter (Par. 145), therefore $H.l.x$ is the total number of lines of force cut by the moving conductor. Placing this equal to N we have

$$W = I.N \text{ ergs}$$

or the work done in moving across a magnetic field a conductor carrying a current of I absolute units is equal to the product of the current into the number of lines of force cut.

358. Work Done in Moving Across a Magnetic Field a Coil Carrying a Current.—This is merely a particular case of the foregoing but furnishes conceptions which facilitate the application of the principle in certain deductions which we shall make later on. Suppose the moving conductor to be in the form of a closed coil and, for the sake of simplicity, suppose this to be rectangular and to be moved so that while two sides cross the field at right angles to the lines of force the other two sides move lengthwise through the field. Since these latter cut no lines of force they perform no work. If the field be uniform each of the other two sides performs an equal amount of work, but the current in them flows in opposite directions so that in one IN is positive while in the other it is negative. The net result is zero, or, no matter how it may be moved, if in its successive positions in a uniform field a coil remains parallel to its original position, no work is done.

If, however, the field be not uniform, the work done by one of these sides will be IN ergs, that by the other $-IN'$ ergs, and the total work is $I(N-N')$ ergs, or the work done in moving a coil in a magnetic field is equal to the product of the current in the coil into the *change in the number of lines of force embraced by the coil.* This is general, that is, it is true whatever the shape of the coil and whether its motion be one of translation or of rotation. It also follows that the same amount of energy is expended if

the coil be kept motionless and the field embraced be moved or varied.

If two separate and similar coils be moved in succession across the field, the work done by each is, from the foregoing, IN ergs, in which N is the change in the number of lines of force embraced by the coil, the total work being $2IN$ ergs. If they be moved simultaneously the work will be the same. Finally, they need not be separate coils but may be two turns of the same coil and still the work will be $2IN$ ergs. In general, therefore, if the field within a coil of n turns carrying a current I be increased or decreased by N lines of force, the work done will be nIN ergs.

359. Energy Expended upon an Electro-Magnetic Field.—The conclusions in the preceding paragraph are irrespective of the origin of the field. It may therefore be produced in any way, even by the current itself. When a current is sent around a coil, N lines of force are produced in the coil. By a similar method to that followed in Par. 96, or by an application of the integral calculus, it may be shown that if the current starts at zero and increases to a value I, the energy expended in establishing the field is $\frac{1}{2}IN$ ergs over and above that spent in the mere heating of the conductor. This energy is absorbed in the field and restored when the circuit is broken. This fact explains why the current never rises instantly to its full strength when the circuit is closed and also why the current always lingers after the circuit is broken, revealing itself as a spark. This subject will be referred to again when the discussion of induction is reached.

360. Electro-Dynamics.—In Par. 356 it was shown that a conductor carrying a current and placed in a magnetic field is acted upon by a force at right angles to the field and to the conductor. Since conductors carrying currents are surrounded by magnetic fields (Par. 346), it follows that if two such conductors be placed near together, each will lie in the magnetic field of the other and each will be subjected to a force. Ampere, who made this discovery in 1820, applied the term *electro-dynamics* to that branch of electricity which treats of the forces exerted between currents, and formulated the laws given in the following paragraphs.

361. Force Exerted Between Conductors Carrying Currents.—Two parallel conductors attract one another if the currents in them

flow in the same direction but repel each other if the currents flow in opposite directions.

A and *B*, Fig. 157, are two such conductors. Considering *B* as lying in the field about *A*, application of the left hand rule (Par. 352) will show that *B* is urged at right angles to its length and

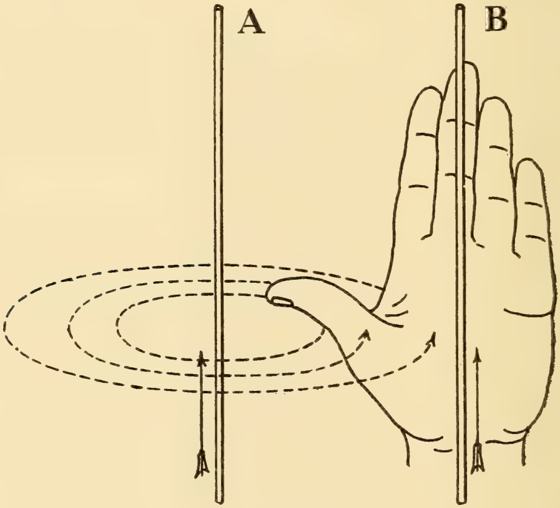


Fig. 157.

towards *A*. Similarly, *A* is urged towards *B*. Had the currents flowed in opposite directions the wires would have repelled each other.

It may be shown by Laplace's law (Par. 353) that if the two wires be not parallel, the electro-magnetic effect of either current can be resolved into two components, one parallel to the remaining current, the other perpendicular to it and contributing nothing to the forces between the two wires. In the most general case, therefore, if two conductors cross, those portions in which both currents flow either towards or from the point of crossing attract each other while those portions in which one current flows towards and the other from the point of crossing repel each other.

It is not necessary that the two conductors be parts of different circuits. The same law applies to portions of a single circuit. If, for example, a current be passed through a helical coil, the adjacent turns attract each other and the coil tends to shorten.

362. Intensity of Force Between Parallel Conductors Carrying Currents.—If the wire *A*, Fig. 157, be of indefinite length and if there be flowing in it a current of strength I', the intensity of the field produced by it at any point along *B* is (Par. 353)

$$H = \frac{2I'}{r}$$

r being the distance between the two wires. But we have seen (Par. 356) that the force exerted by a field H upon a wire of length l carrying a current of strength I is $f = I.H.l.$ Substituting in this the value of H from above, we have

$$f = \frac{2II'l}{r}$$

or the force exerted upon the second wire *B* is directly proportional to the product of the currents in the two wires and to the length of *B* and inversely proportional to the simple distance between the wires.

CHAPTER 30.

GALVANOSCOPES AND GALVANOMETERS.

363. Galvanoscopes.—Oerstedt's discovery affords us a means of determining whether or not a current is flowing in a conductor, and if flowing, in what direction. For example, in the case of an electric wire crossing the ceiling of a room, it is only necessary to hold a magnetic needle an inch or so below the wire when, if a current is flowing, the needle will be deflected and the direction of this deflection, in conjunction with the right hand rule, will reveal the direction of flow of the current. Instruments designed to give information of this character are called *galvanoscopes*.

364. Increase of Sensitiveness.—We frequently have to deal with currents so small that the deflection they produce in an ordinary needle is imperceptible. In such cases the only remedy is to increase the sensitiveness of the instrument. A needle when in use is acted upon by two forces; first, the deflecting force which causes it to move and, second, the controlling force which resists deflection. We therefore have two expedients; we may multiply the effect of the deflecting force or we may weaken the controlling force. The highest degree of sensitiveness is attained by combining these two. We shall now examine these in detail.

365. Schweigger's Multiplier.—Suppose a needle to be placed at the center and to lie in the plane of a vertical coil. Application of the right hand rule will reveal the fact (shown already in Par. 354), that when the circuit is closed, the top, the sides, the bottom of the coil, all contribute to produce a deflection of the needle in one and the same direction. As the palm of the right hand is slid along the coil, the thumb points constantly in the same direction which is that of every line of force enclosed by the coil. If, therefore, instead of simply passing the wire by the needle, as in Oerstedt's experiment, we take a turn entirely around it, the deflecting force is very much increased. Finally, we need not stop at one loop. Every succeeding turn adds its lines of force to those already in the field and we may, therefore, use a coil of a great

many turns and multiply by just so much the effects of the current. Such is the principle of Schweigger's multiplier (Fig. 158) which consists of a suitable frame which may be rectangular, oval or circular and around which are wrapped many turns of insulated

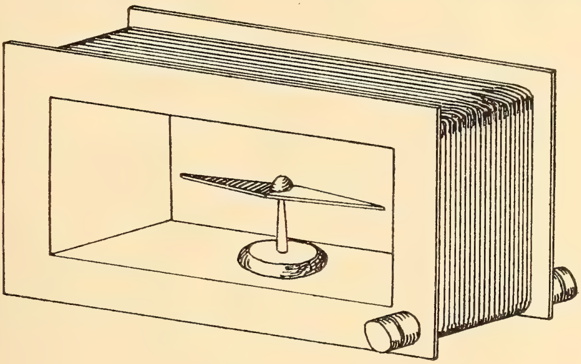


Fig. 158.

wire. The frame must be of some non-magnetic material such as wood, ebonite, brass, etc., otherwise it would acquire magnetism from the current. In the center of the coil is pivoted the needle whose deflection is to be observed.

A multiplier should be used with feeble currents only. With a strong current it is not necessary; moreover, the resistance of the many turns of wire would cut down a large current. It is true that it also reduces a small current but not so much proportionally. The general rule is that a multiplier is used when the circuit already contains great resistance but should not be used if the resistance be small.

366. Methods of Weakening Controlling Force.—The second method of rendering a needle more sensitive, the weakening of the controlling force, may be applied in two ways:

(a) Haüy's method. The earth's controlling force may be very nearly neutralized, there being left a small excess just sufficient to control the needle.

(b) Astatic combinations. The earth's controlling force may be entirely neutralized, and some very feeble force, such as the torsion of a silk fibre, substituted therefor.

367. Haüy's Method.—Haüy's method of weakening the earth's control is shown diagrammatically in Fig. 159 as being applied to the needle of Schweigger's multiplier. The needle is suspended in

the center of the multiplier, the plane of the coil being in the magnetic meridian. A brass rod AB projects upward from the top of the coil and upon this rod there slides a bar magnet NS, usually curved as shown. Now, as this bar magnet is slid down towards the coil, its north pole repels with an increasing force the

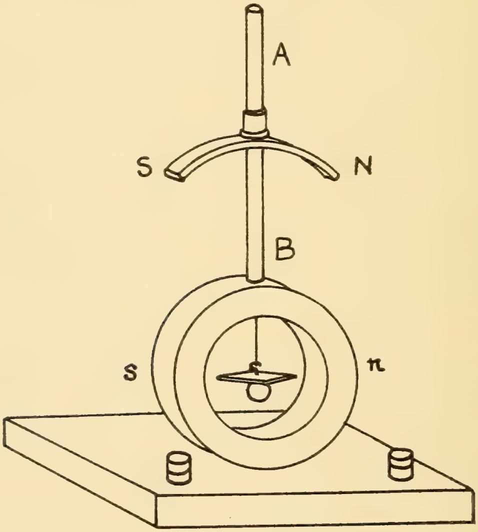


Fig. 159.

north pole of the needle. A point is finally reached where NS exactly counterbalances the earth's controlling force upon the needle and if this point be passed the needle is reversed. By stopping the bar magnet just above this critical point, the effect of the earth's control may be reduced to a minimum. This method is employed in Thompson's mirror galvanometer (Par. 377).

368. Astatic Combinations.—Two needles of equal size and strength fastened rigidly together in reversed positions and with their axes parallel constitute an *astatic pair* (Fig. 160). This combination is independent of the earth's control and the controlling force is generally the torsion of a fine silk fibre by which the needles are suspended. They are usually mounted so that the lower needle swings in the center of a multiplier, the upper needle travelling over a graduated scale on the top of the coil and thus serving as a pointer. Application of the right hand rule will show that the

current in the portion of the coil between the two needles will cause them both to rotate in the same direction. By using a coil wrapped like a figure eight, both needles may be surrounded by coils.

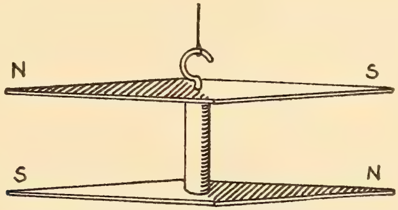


Fig. 160.

There are a number of other astatic combinations. A suspended horseshoe magnet is astatic and may be used as an astatic pair.

369. Magnetic Shells.—Should we be able to cut from the end of a bar magnet a thin slice, and should this slice preserve its original polarity, we would have a *magnetic shell,* a thin piece of metal, one face of which would be of north polarity, the other south.

Another conception of a magnetic shell is to suppose that we had a great number of very small magnets, like minute type, and that we should arrange them over the area of a circle side by side

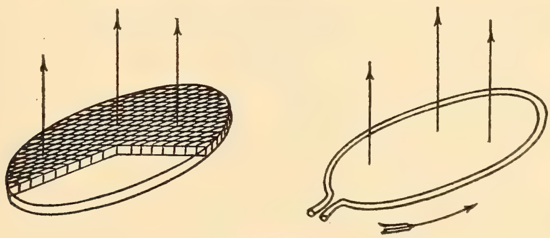


Fig. 161.

like the cells of a honeycomb (Fig. 161), the north poles all pointing up, the result would be a magnetic shell. If a coil of wire be bent into a circle of the same size as the shell, a current could be sent through the wire which would produce inside the coil as many lines of force as emerged from the shell. Since we have shown (Par. 365) that these lines all emerge from one face of the coil and in the same direction, the coil and the shell are magnetically equivalent to each other. Coils carrying a current behave in many ways as

if they were magnets. They have polarity and will attract or repel a magnet, depending upon the pole of the magnet and the face of the coil to which it is presented. They also attract or repel each other.

This conception of a magnetic shell is used in mathematical discussions of electricity. We will not have occasion to use it further but there follows from it a very important principle which we shall now develop.

370. De La Rive's Floating Battery.—One form of De La Rive's floating battery is represented in Fig. 162. It consists of a turnip-shaped glass cell with a constricted lower part containing dilute acid. Upon a cork in the mouth of the cell is mounted a vertical

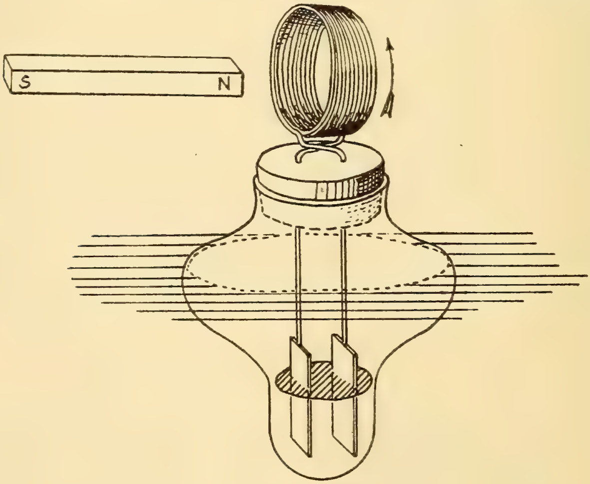


Fig. 162.

coil of wire of a number of turns, the ends of this coil extending below the cork and terminating, one in a copper, the other in a zinc plate which dip into the acid. The arrangement is therefore seen to be only a simple cell, the coil constituting the external circuit. The cell is placed in a basin of water so that it floats freely. If the current flows around the coil in the direction indicated by the arrow, the lines of force of the coil pass through from right to left, or, from what we have just seen, the coil is equivalent to a magnetic shell whose north face is to the left.

Suppose that, as represented in the figure, the north pole of a bar magnet be presented to the north face of the coil. The float-

ing cell, as was to be expected, will back away or recede, but, more than this, it will turn around until its south face is presented and will then approach the magnet and, instead of stopping when it has reached the pole, will continue to advance and will thread itself upon the magnet until it has reached the middle point. Its lines of force and those of the magnet now coincide in direction. It is now in a position of stable equilibrium for if it be pushed towards either end of the magnet and released it will immediately return to its median position. On the other hand, suppose the coil to be held and the magnet thrust into it in reversed direction, that is, with its lines of force opposite in direction to those of the coil. If the coil be released when exactly at the center of the magnet it will remain, but it is in unstable equilibrium, for if displaced ever so slightly in either direction from this central position it will slip off the magnet, turn around and return.

One of these cells floating freely in a vessel of water will finally come to rest with the axis of the coil in a north and south position, that is, with its field coinciding in direction with that of the earth. Two such cells will move about until the fields of their coils coincide in direction.

371. Maxwell's Law.—The principle in accordance with which the movements described in the preceding paragraph take place

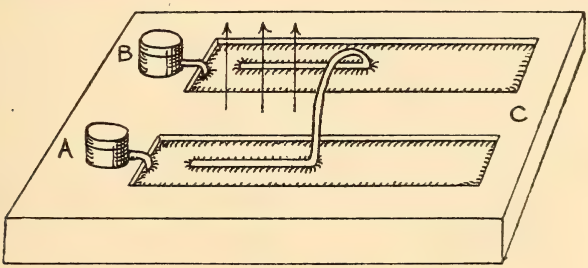


Fig. 163.

has been formulated by Maxwell to the effect that *every electro-magnetic system tends to change its configuration so that the exciting circuit will embrace in a positive direction the maximum number of lines of force.* This law applies to all combinations of closed circuits and magnetic fields, whether these fields be produced by magnets, by other circuits, or even by the circuit itself. This last is shown by an experiment devised by Ampere. In a wooden block (Fig. 163) there are hollowed out two parallel troughs which are

then filled with mercury. A wire bent as shown is then placed as a bridge with one end in each trough and floats on the surface of the mercury. The current entering at A crosses this bridge and leaves by B, the lines of force in this rectangular area all pointing up as shown by the arrows. As soon as the circuit is closed, the wire floats off towards C, thereby increasing the area ACB and consequently the number of lines of force embraced by the circuit.

The majority of the instruments, shortly to be described, operate in accordance with this law and it also explains the movement of all motors. It has already been shown (Par. 144) that, in a more general form, it accounts for the position assumed by magnetic needle.

372. Galvanometers.—A galvanoscope indicates by the movement of its needle both that a current is flowing and the direction of its flow. If this movement also affords a measure of the strength of the current, the instrument is called a *galvanometer*. There are many varieties of galvanometers but they may all be classed under one of two heads: first, those in which a needle moves in a field produced by a fixed coil and, second, those in which there is no needle but a suspended coil which swings in a fixed field. Of the latter class, the field may be produced by a permanent magnet or by a fixed coil. We shall now describe examples of each of the above.

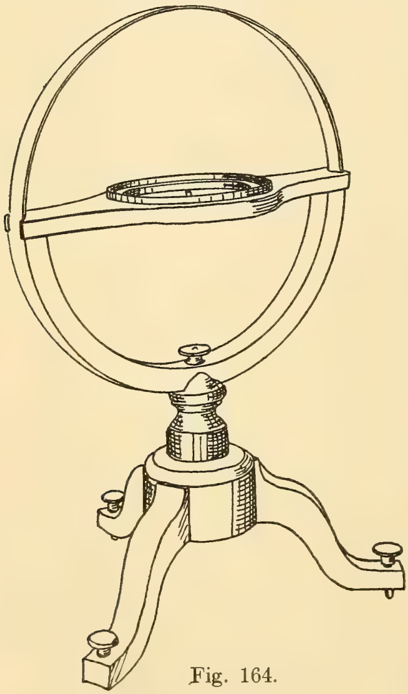


Fig. 164.

373. The Tangent Galvanometer.—This is an example of a galvanometer of the first class, that is, one with a needle moving in a field produced by a fixed coil. It consists (Fig. 164) of a vertical circular coil, more than one foot in diameter, mounted upon a base by which it may be accurately placed in the magnetic meridian. The coil is composed

of a single turn of heavy copper wire or copper ribbon. For measuring small currents it may consist of many turns of fine wire. Pivoted at the center of this coil is a short thick needle, generally less than an inch in length. Since it would be very difficult to read with any accuracy a scale engraved upon a circle whose diameter is only one inch, the needle is usually prolonged by light aluminum pointers. These have no magnetic effect but permit the use of a much larger graduated scale.

374. Measurement of Current by Tangent Galvanometer.—In order to measure a current by the tangent galvanometer, the latter is connected up in the circuit, its coil accurately placed in the magnetic meridian, the circuit closed and the angle of deflection of the needle read. If it be convenient to reverse the current, this is done, the new angle of deflection read and the mean of the two readings is taken as the correct one.

In Par. 146 we saw that "the magnetic field which, acting *at right angles to the meridian,* produces in a needle a deflection δ, is equal to the horizontal component of the earth's magnetism at that point multiplied by the tangent of the angle of deflection," or

$$f = H . \tan \delta$$

Again, in Par. 354 we saw that the field produced at the center of a circular coil of radius r by a current of I *absolute units* is

$$f = \frac{I . 2\pi}{r}$$

We therefore have

$$\frac{I . 2\pi}{r} = H . \tan \delta$$

whence

$$I = \frac{r}{2\pi} . H . \tan \delta$$

In this r is determined by measurement of the coil, H is obtained from observation (Par. 148), or from a table (Par. 175), δ is read from the galvanometer scale and $\tan \delta$ is obtained from a table.

If the galvanometer coil has n turns, the second expression for f becomes $f = \frac{I . 2\pi n}{r}$. The factor $\frac{2\pi n}{r}$, since it depends purely

upon the dimensions of the instrument, is called the *galvanometer constant*. Calling this G, we have

$$f = I . G$$

whence, if $I = 1, f = G$, or the galvanometer constant is equal to the strength of the field produced at the center of the coil by a current of one absolute unit.

In practice, it is more frequent to use the tangent galvanometer to compare currents rather than to determine them absolutely. Various currents are to each other as the tangents of the angles of deflection which they severally produce. If the deflection produced by a known current be ascertained, the determination of other currents is a simple matter.

375. Remarks on Principle of Tangent Galvanometer.—The deduction in the preceding paragraph is based upon two assumptions, neither of which is strictly accurate, although the error is

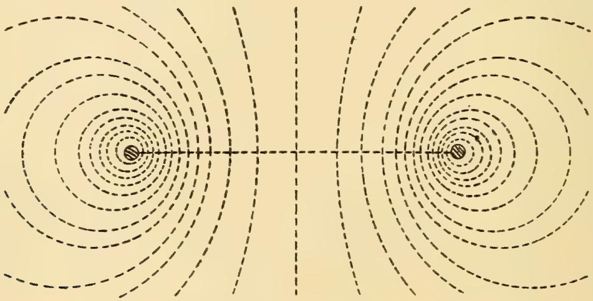


Fig. 165.

usually negligible. First, the deflecting force is supposed to be perpendicular to the meridian. Fig. 165 represents the field along the horizontal diameter of the coil of a tangent galvanometer, whence it is seen that the lines of force are curves (although slightly different from the circles shown in the figure), and therefore are perpendicular to this meridian only where they pierce the plane of the coil. They have, however, less curvature near the center of the field and this flatness increases with the diameter of the coil, for which reason the needle is made very short and the coil large. A still better remedy is to use two parallel coils and place the needle midway between them. The lines of force of the field in this case are sensibly parallel.

Second, the expression employed for the intensity of the field is determined for the center of the coil (Par. 354). The field, as in-

dicated in the figure, is much stronger near the coil and diminishes towards the center. The needle is therefore made short so that its poles do not extend into a field much stronger than that at the actual center.

376. The Sine Galvanometer.—The sine galvanometer, shown in its simplest form in Fig. 166, differs from the tangent galvanometer only in that the coil need not be so large and that the needle extends as nearly across the diameter of the coil as its

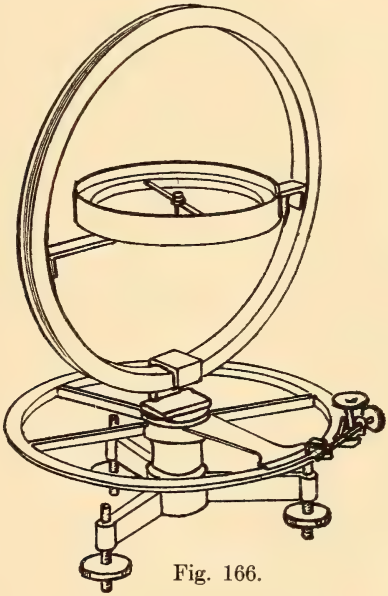


Fig. 166.

surrounding graduated circle will permit. The poles of the needle therefore lie in the strong field close to the coil and the instrument is more sensitive than the tangent galvanometer. The coil is free to rotate about a vertical axis and in more improved forms of the instrument there is a horizontal graduated limb from which may be read by a vernier the exact angle through which the coil has been turned. This limb, however, is not essential.

To use the instrument to measure a current, it is connected up in the circuit and accurately adjusted until the coil lies in the magnetic meridian. The horizontal graduated limb is then read and the circuit is closed, causing a deflection of the needle. The coil is then turned by hand in the direction of the deflection of the

needle until the needle is overtaken and lies once more in the plane of the coil. The deflecting force, or the field of the coil, is now perpendicular to the needle. The angle through which the coil has been turned is read from the scale on the horizontal limb. Should there be no horizontal limb, this angle can still be determined, for it is only necessary to take the reading of the needle, then break the circuit and take the reading of the needle when it has swung back into the meridian; the difference between these two readings is the required angle.

In Par. 147 it was shown that "magnetic fields acting *at a constant angle with the needle* are to each other as the sines of the respective angles of deflection." It follows that the current is proportional to the sine of the angle through which the coil has been turned; also, that different currents are to each other as the sines of these angles. The sine galvanometer can therefore be used to compare currents although it can not be used, like the tangent galvanometer, to measure currents absolutely.

Should the deflecting force be greater than the controlling force, the coil will never overtake the needle, and in such a case the instrument can not be used.

377. The Mirror Galvanometer.—The mirror galvanometer is an extremely sensitive form of instrument and is more frequently used as a galvanoscope than as a galvanometer, in fact, it was devised by Lord Kelvin to give indications of the exceedingly small currents transmitted by submarine cables. Its principle will be understood from Fig. 159. It consists of a vertical coil of many thousand turns of very fine insulated wire. The opening through the coil is barely half an inch in diameter and in the center of this there hangs, by a silk fibre, a very light glass mirror, about the size of a silver ten-cent piece. The mirror is slightly concave so as to focus in a long pencil any rays of light which fall upon it. To the back of this mirror there are glued three or four very light magnets made of short sections of watch spring. The controlling force of the earth's magnetism is neutralized by Haüy's method. The little mirror normally hangs with its plane parallel to the face of the coil, but when a current passes through the coil the magnets at the back of the mirror tend to turn in accordance with Maxwell's law until their lines of force coincide with those of the coil. A beam of light is caused to fall upon the mirror and is reflected back, producing a bright spot upon a blank wall or upon

a suitably-prepared scale. The slightest angular motion of the mirror is revealed at once by motion of the spot of light, the angular motion of the spot being twice that of the mirror and the radius being the distance from the mirror to the wall or scale. Thompson states that the most improved form of this instrument gives indications of a current as small as one fifty-four-thousand millionth of an ampere.

378. Suspended Coil Galvanometer.—In the galvanometers described in the preceding paragraphs, the coil carrying the current is fixed and the magnet rotates; in the form now to be described the magnet is fixed and the coil rotates. While not having the extreme delicacy of the mirror galvanometer, the suspended coil galvanometer is still of a high order of sensitiveness and is used by practical electricians where the most refined observations are required. There are many different forms and it is known by other names, such as the D'Arsonval galvanometer, the reflecting galvanometer, etc., but the principle of all is the same.

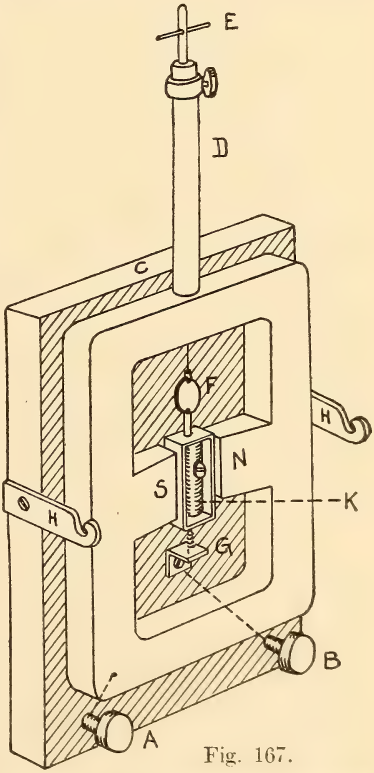


Fig. 167.

A usual form consists (Fig. 167) of a heavy rectangular frame of magnetized steel whose poles are *N* and *S*. This frame is mounted upon a wooden back *C* which may be fastened to a wall, mounted upon a tripod, or otherwise suitably supported. Through the center of the top of the frame is bored a hole into which is screwed a vertical brass tube *D*. In the upper end of this tube there fits a small brass spindle with a cross-bar handle *E*. This spindle may be turned about a vertical axis and may be raised or lowered and fastened in any desired position by the set-screw shown at the right. The movable coil is suspended from the spindle by means of a very delicate

phosphor-bronze filament. Silk or quartz fibres can not be used since the suspension must convey current to the coil. The coil, which swings in the space between the poles, consists of many turns of very fine wire wrapped upon a thin, light, elongated rectangular metal frame. Midway between the poles *N* and *S* there is fastened to the wooden back *C* a vertical soft-iron cylinder *K* which projects into the opening of the coil frame, almost entirely filling this space and leaving barely room for the coil to turn. This, as shown in Fig. 69, Par. 143, greatly concentrates the field in which the coil moves. Above the coil frame and supported by it is a small mirror *F*. Below the coil, a coiled phosphor-bronze filament connects to a small metal bracket *G* which in turn is connected from behind to the binding post *B*. The other binding post *A* is connected direct to the steel frame. A current entering at *A* travels up the steel frame to the brass tube, thence up this tube to the spindle, thence down the suspension to the coil, around the coil, thence through the lower filament to *G* and out by *B*. The coil hangs normally with its face to the front, the controlling force being the torsion of the phosphor-bronze suspension. If the coil does not hang properly, it can be made to do so by turning the spindle *E*. With the poles situated as represented in the figure, the lines of force of the field run from right to left. When a current flows through the coil, the lines of force of the coil are from front to rear, or the reverse; therefore, the coil, in accordance with Maxwell's law, turns either to the right or left. The coil, mirror and filaments are protected by a metal plate screwed to the frame and carrying a glass window through which the mirror may be observed.

In using the instrument, there is attached to the hooks *H H* an arm (Fig. 168) which carries at its farther end a telescope and a printed scale. The scale, which is usually divided into millimeters, is one-half meter from the mirror. By means of the telescope the reflection of the scale in the mirror is observed. Since the telescope inverts objects and the mirror reverses them right for left, the numbers on the scale must be engraved both upside down and reversed. Cross hairs in the telescope allow the scale to be read very accurately. When the coil, and consequently the mirror, is deflected by a current, it appears to the eye of the observer as if the scale moved across the field of the telescope. For moderate deflections of the coil the currents producing these deflections are

proportional to the number of scale divisions passed over by the vertical hair.

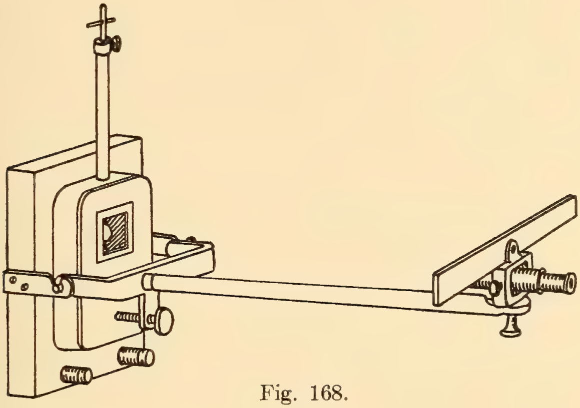


Fig. 168.

379. Damping.—In instruments in which readings are taken of the angular displacement of a needle, a coil, or a mirror, the moving part may oscillate for some time before coming to its final position of rest. This causes, in taking observations, a vexatious delay which it is very desirable to avoid. Any process by which, while not interfering with the freedom of movement of the part, it is made to come to rest quickly is called *"damping,"* and an instrument whose needle moves at once to the proper reading on the scale is said to be *"dead beat."* Damping may be brought about by (a) mechanical means or (b) electrical means. As an example of mechanical damping, a moving coil may have suspended below it a metal vane which is immersed in oil, the viscosity of the liquid slowing down the movement and preventing vacillation. Suspended coil galvanometers often have attached to the mirror a thin sheet of metal or mica which turns in a little closed box which it nearly fits. The confined air in this box acts something like the oil in the first case.

Electrical damping can not be thoroughly explained at present but depends upon the principle that a piece of metal moved in a magnetic field experiences forces which tend to stop the movement (Par. 430). This is the method employed in the suspended coil galvanometer just described. The metal frame upon which the coil is wrapped turns in the strong magnetic field between the poles and the soft-iron core and is thus brought quickly to rest.

380. Need of Galvanometer Shunts.—The currents which a reflecting galvanometer may measure are extremely small. Thus, if a pin be connected by a wire to one terminal of the galvanometer and a needle be connected to the other and the pin and needle be held tightly between the fingers, the contact of the two dissimilar metals with the slight moisture of the fingers will drive a sufficient current through the coil to cause the mirror to run entirely off the scale. In order therefore to measure even minute currents we must employ a shunt by which, as explained in Par. 301, only one-tenth, one-hundredth, or one-thousandth of the total current is permitted to flow through the instrument. Even in this case it is usual to insert in the circuit a resistance of 50,000 or 100,000 ohms by which the current is reduced to measurable intensity.

381. The Universal Shunt.—We saw in Par. 301 that the resistance of a galvanometer shunt must bear a fixed relation to the resistance of the galvanometer with which it is used and that shunts are not interchangeable and can be used only with the galvanometer for which they are constructed. The phosphor-bronze suspension of a suspended coil galvanometer is frequently broken and must be replaced by a new one, in doing which the resistance of the galvanometer is usually considerably changed and this change would render useless a shunt designed to accompany the original resistance. Reflection will show, however, that if we simply wish to compare currents *relatively* it is not necessary to know what fraction of the total current flows through the galvanometer, for if $1/x$th of a current I' flowing through a galvanometer produces a certain deflection, and if $1/x$th of a different current I'' produces a deflection twice as great, then the current I'' is twice as great as the current I'.

Carrying out the idea farther, Ayrton devised a *universal shunt* which may be used with any galvanometer and which can be so varied that, irrespective of the resistance of the galvanometer, the deflection produced is proportional to one-tenth, one-hundredth, or one-thousandth, etc., of the total current. This shunt is shown diagrammatically in Fig. 169. Five contacts (sometimes six) are arranged in the arc of a circle and marked, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$ and 0. Between these contacts are resistance coils A, B, C, D. If R be the total resistance, A is .9 of R, $A+B$ is .99 of R and $A+B+C$ is .999 of R. A common arrangement of these resistances is to have $A=9000$, $B=900$, $C=90$ and $D=10$ ohms, a total of 10,000 ohms.

The current enters by K and leaves by H. The arm attached to K can be placed on any desired contact. The galvanometer is connected in shunt with the total resistance as shown. Let the resistance of the galvanometer be x. With the arm on contact 1, let the total current be I, and the current through the galvanometer be I_g. The joint resistance from K to H is $\frac{Rx}{R+x}$ (Par. 293).

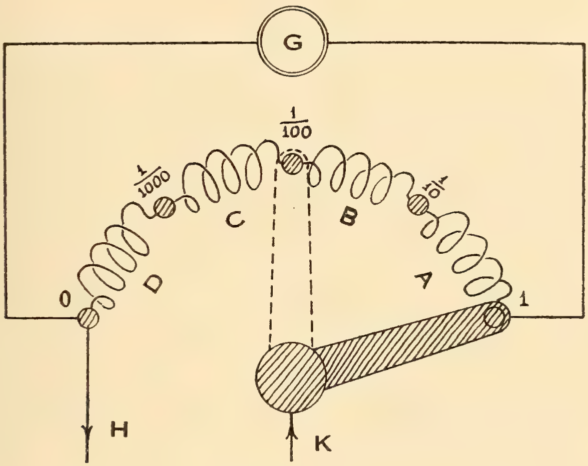


Fig. 169.

Hence

$$I_g : I = \frac{Rx}{R+x} : x$$

Whence

$$I = I_g \cdot \frac{R+x}{R} \qquad \text{(I)}$$

Suppose the arm to be placed on the $\frac{1}{100}$ contact. The joint resistance from K to H is now

$$\frac{(.99R+x)(.01R)}{.99R+x+.01R} = \frac{(.99R+x)(.01R)}{R+x}$$

If the total current be now I' and the current through the galvanometer be I'_g

$$I'_g : I' = \frac{(.99R+x)(.01R)}{R+x} : .99R+x$$

Hence

$$I' = I'_g . 100 . \frac{R + x}{R} \qquad \text{(II)}$$

From (I) and (II)

$$I' : I = 100 I'_g : I_g \qquad \text{(III)}$$

Or if D be the deflection produced by the first current and D' that produced by the second

$$I' : I = 100 . D' : D$$

or the ratio of the total current when the arm is on the $\frac{1}{100}$ contact, to the total current when the arm is on the 1 contact, is as one hundred times the deflection produced in the first case, is to the deflection produced in the second case.

It will be noted that x, the resistance of the galvanometer, does not appear in (III), hence the shunt may be used with any galvanometer.

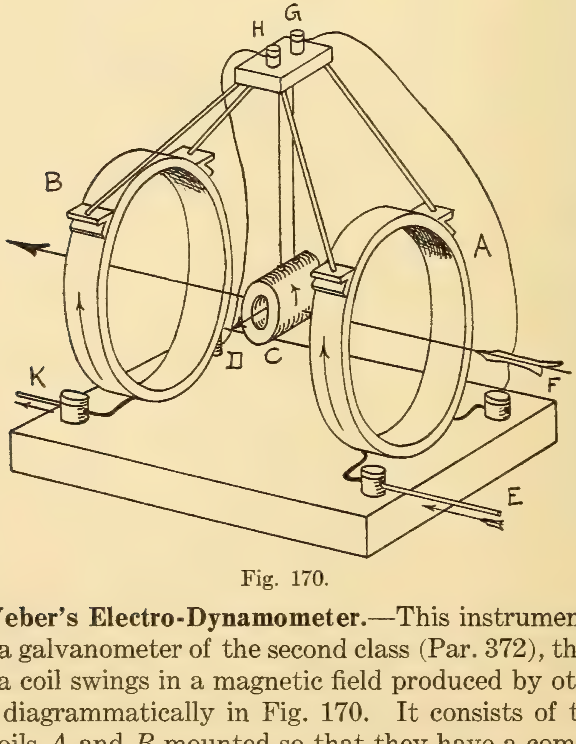


Fig. 170.

382. Weber's Electro-Dynamometer.—This instrument, an example of a galvanometer of the second class (Par. 372), that is, one in which a coil swings in a magnetic field produced by other coils, is shown diagrammatically in Fig. 170. It consists of two large parallel coils A and B mounted so that they have a common axis

and their planes are vertical. Midway between these there hangs by a bifilar suspension (Par. 127) a small coil C so arranged that its axis is in the same horizontal plane but at right angles to the common axis of A and B. As generally used the same current traverses all three coils. Entering at E it flows around the coil A and out to F, thence by the wire to G, thence down the slender wire suspension to C, around this coil, up the other suspension to H, down to D, around the coil B and finally out by K.

If the currents in the two coils flow as indicated by the small arrows, the field of AB will be from right to left; that of C from rear to front and therefore C, viewed from above, takes up a clockwise motion, or, in accordance with Maxwell's law, tends to turn so that its field coincides in direction with the field of AB. The angle of deflection is read, as in the mirror galvanometer, by means of a small mirror attached to the suspended coil. The controlling force is gravity which tends to pull the inner coil back to its primary position; the moment of this force being directly proportional to the sine of the angle of deflection, or

$$M_c = a \,.\, \sin \delta$$

The deflecting force is due to the interaction of the fields of the suspended and the fixed coils and since these fields are severally proportional to the currents flowing in the coils (Par. 354), the deflecting force is proportional to the *square* of the current. The moment of the deflecting force is proportional to the product of the square of the current and the cosine of the angle of deflection, or

$$M_d = b \,.\, I^2 \,.\, \cos \delta$$

When the coil comes to rest the two moments are equal and opposed, hence

$$b \,.\, I^2 \,.\, \cos \delta = a \,.\, \sin \delta$$

whence

$$I^2 = \frac{a}{b} \,.\, \tan \delta$$

or, the square of the current is proportional to the tangent of the angle of deflection. This fact might have been anticipated since reflection will show that the instrument is virtually a tangent galvanometer.

In making an actual observation a number of refinements must be observed in determining the constants a and b above, and it may also be necessary to allow for the effects of the earth's field.

Should the current through the instrument be reversed in direction, the fields in the coils will also be reversed but from the figure it will be seen that the tendency will still be for the movable coil to turn in a clockwise direction. Since this direction of deflection does not vary with reversal of the current, instruments of this class, that is, two-coil instruments, are employed in the measurement of *alternating currents,* or those currents which reverse many times per second.

383. Siemen's Electro-Dynamometer.—Siemen's electro-dynamometer, shown diagrammatically in Fig. 171, is in principle the

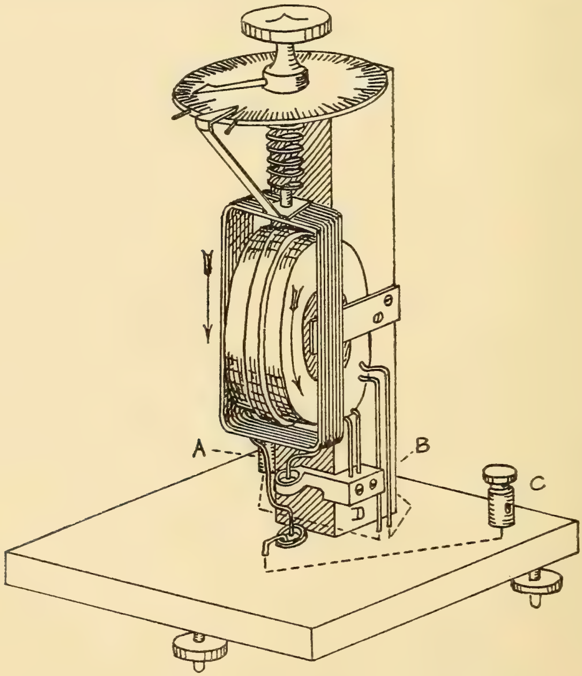


Fig. 171.

same as Weber's but differs in that the movable coil is external to the fixed, and that the controlling force is the torsion of a delicate coiled spring. The base and supporting upright are of wood. There are two fixed coils, one of a few turns of heavy wire for use with large currents, the other of many turns of a finer wire for use with smaller currents. The short coil is wrapped upon the long

coil. The terminal for one of these coils is the binding post A, that of the other coil the binding post B, and the remaining end of each coil is connected to the metal bracket D which at one end carries a little cup of mercury. One terminal of the movable coil dips into this; the other terminal dips into a similar cup just below the first, this last cup being connected by a wire to the binding post C. The movable coil is suspended either by a silk fibre or upon a pivot and is free to rotate about a vertical axis. It carries a needle or pointer which is bent over the edge of an upper circular scale. This scale may be graduated in degrees but more often in some arbitrary number of points, such as 400. If the current in the coils flow as indicated by the arrows, the field of the fixed coil is from left to right, that of the movable coil from rear to front and, viewed from above, the rotation of the movable coil is counter-clockwise. This movement is opposed by the torsion of the spiral spring attached to the upper part of the movable coil and by means of a projecting pin or stop is restricted to a few divisions of the graduated scale. At the center of this scale there is a milled head to whose end the upper end of the coiled spring is attached. Below the milled head there is a second pointer which, as the head is turned, sweeps around the graduated circle and indicates the angle through which the head has been turned.

When a current is flowing through the instrument, the movable coil is urged in a counter-clockwise direction. The milled head is turned in a clockwise direction and the torsion of the spiral spring, which varies directly as the angle through which the milled head is turned, tends to drag the coil back to its primary or zero position. When the coil has finally been brought back to this position, the pull exerted by the spring exactly balances the contrary moment exerted by the current.

Consider a vertical portion of the wire in the movable coil and an adjacent portion in the fixed coil. The force exerted between these portions is directly proportional to their length, to the products of the currents flowing in them, and inversely proportional to the simple distance between them (Par. 362). The length of the portions is constant, so also is the distance between them, since the coil is always brought back to its original position, therefore, the force between the portions, and consequently the force between the coils themselves, varies as the square of the current and also as the number of divisions of the scale over which the pointer

attached to the milled head has been turned. From this it follows that the current varies as the square root of the angle of torsion, or

$$I = K\sqrt{\delta}$$

δ being the number of divisions of the scale indicated by the pointer. The constant K is different for different instruments but is easily determined by passing through the instrument a known current I and noting the corresponding torsion δ.

In addition to its use in measuring currents, this instrument, as will be shown later (Chap. 36), may be used to measure electrical *power*.

384. Ballistic Galvanometer.—In the earlier attempts to measure the velocity of moving projectiles, use was made of a piece of apparatus called a *ballistic pendulum*. This consisted of a large pendulum with a very heavy and solid bob. The projectile was fired against and embedded itself in the bob, the blow causing the pendulum to swing through a certain angle which was recorded. Knowing this angle, the vertical height through which the weight had been lifted could be determined and, knowing the weight of the projectile, the velocity with which it struck the pendulum bob could be calculated.

When a charged body is discharged through a conductor, the charge in its passage is a veritable current but its duration is only momentary. If passed through a galvanometer, it gives to the moving parts a sudden impulse or blow comparable to the blow given to the pendulum by the bullet. If these moving parts be somewhat heavy and therefore rather slow in vibration, the current will have passed before any appreciable movement takes place. It can be shown that the sine of half of the angle of the first swing or "throw" of the needle is proportional to the charge which has passed through the coil. (See Gray, Absolute Measurements in Electricity, Vol. II, pp. 390-396.) Galvanometers used in this manner are called *ballistic galvanometers*. They are generally of the suspended coil type and must not be damped.

CHAPTER 31.

ELECTRIC MAGNETIZATION OF IRON AND STEEL.

385. Solenoid.—A cylindrical coil of wire whose length is great as compared to its diameter is called a *solenoid,* the Greek word *solen* meaning a tube. The successive turns of the coil are wrapped as closely together as the thickness of the insulating covering will permit but in diagrams it is usual, for the sake of clearness, to represent these turns as somewhat widely separated. To give an accurate shape to the coil it is generally wrapped upon a material core, such as a wooden rod or a tube of glass or paper, which afterwards may be withdrawn. In diagrams, to avoid confusion as to the direction in which the coil is wrapped, it is preferable to represent the core as in position.

The coil being a helix, the turns are inclined to the axis of the cylinder but each is electrically equivalent to a turn at right angles

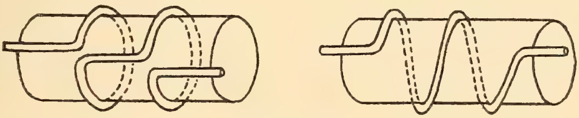


Fig. 172.

to the axis (Fig. 172) and a short portion parallel thereto and equal in length to the pitch of the coil. The effect of these longitudinal portions is neutralized if one end of the coil be brought back along the axis of the coil, or if the wire, the circular direction of its winding being unchanged, be wound back to the starting point, thus forming a second layer on top of the first.

386. A Solenoid Equivalent to a Bar Magnet.—If a current be passed through a solenoid, application of the right hand rule will reveal the fact, which indeed has already been shown (Par. 365), that the successive turns combine in the production of a field in the same direction. Thus (Fig. 173), all the lines of force inside of the solenoid run in the direction shown by the long arrow. A solenoid carrying a current is therefore magnetically equivalent

to a bar magnet. It has poles, it will attract magnetic substances, it will attract or repel the pole of a bar magnet and, if

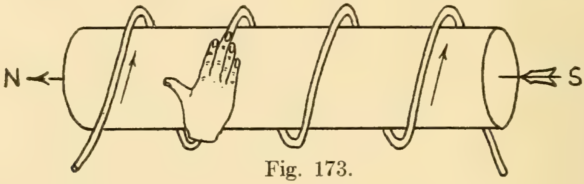


Fig. 173.

freely suspended, it will turn so as to place itself in the magnetic meridian.

387. Intensity of Field on the Axis of a Solenoid.—The intensity of the field at a point on the axis of a circular coil is (Par. 354)

$$H = \frac{2\pi I r^2}{x^3} \text{ dynes} \tag{I}$$

in which I is the current in absolute units, r is the radius of the coil, and x is the slant distance from the coil to the point on the axis.

Let P, Fig. 174, be a point on the axis of a solenoid through which a current I is flowing. If in each centimeter length of the

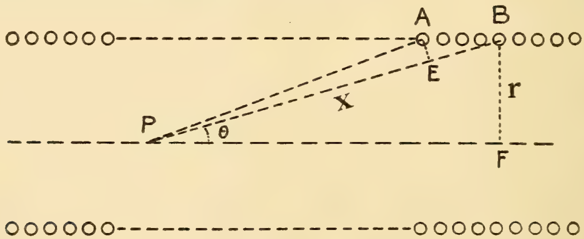


Fig. 174.

solenoid there be N turns, we may consider that around such unit of length there is flowing a *sheet* of current of strength NI. The current over a small portion AB is therefore proportional to the length of AB or $N.I.dl$.

Since $\frac{r}{x} = \sin\theta$, expression (I) can be written

$$H = \frac{2\pi I r}{x^2} \sin\theta \text{ dynes}$$

The field at P due to the current on AB is, therefore,

$$dH = \frac{2\pi N I . dl . r}{x^2} \sin\theta \tag{II}$$

From the figure, $\dfrac{AE}{AP} = d\theta$, or $AE = AP.d\theta$ (III)

From the similar triangles AEB and BPF

$$AE : AB = BF : BP$$

Hence $$AB = \frac{AE \times BP}{BF} \qquad \text{(IV)}$$

or, substituting from (III)

$$AB = AP.d\theta.\frac{x}{r}$$

and as AB decreases, AP approaches BP, hence

$$AB = dl = \frac{x^2}{r}d\theta$$

Substituting in (II)

$$dH = 2\pi NI.\sin\theta.d\theta \qquad \text{(V)}$$

Integrating

$$H = 2\pi NI(-\cos\theta) + \text{a constant}$$

The field due to the entire solenoid is obtained by taking this expression between the proper limits. If P be at the center of the coil and if the coil be so long that $\theta = 0°$ and $180°$, then

$$H = 4\pi NI \text{ dynes}$$

If P be at the mouth of the solenoid so that θ is $0°$ and $90°$,

$$H = 2\pi NI \text{ dynes}$$

or the field at the mouth of a long solenoid is one-half what it is at the center.

It is to be noted that since in these expressions for the field the radius of the coil does not occur, the intensity of the field would appear to be independent of the diameter of the solenoid. This, however, is not correct unless the further condition be expressed, a condition already introduced in the integration, that the various solenoids are geometrically similar. Should the radius of the solenoid be doubled or trebled, its length must be likewise doubled or trebled.

The length of wire required in similar solenoids varies as the square of their like dimensions and if the length be increased the diameter of the wire must be increased to overcome the increase in resistance, therefore, considerations of economy lead us to make the coil fit its core as closely as possible.

It has been shown that the field at the center of a long solenoid is very uniform. If the solenoid be wrapped upon a circular core, so as to return upon itself, the field at every cross-section is the same.

388. Ampere Turns.—In the discussion in the preceding paragraph the current I is in absolute units. If it be given in amperes it must be reduced to absolute units by dividing by ten. The expression for the field at the center of a long solenoid becomes in this case

$$H = \frac{4\pi}{10} \cdot N \cdot I \text{ dynes}$$

The field therefore varies with NI. This product remains a constant if N and I vary reciprocally, hence three amperes making five turns produce the same magnetic effect as five amperes making three turns, or as one ampere making fifteen turns, or as fifteen amperes making one turn. In any coil the product of the *total number of turns* times the current flowing in the coil is called the *ampere turns,* and this product appears as a factor in all expressions dealing with circular coils. We have already employed it in the discussion of the tangent galvanometer (Par. 374).

The constant $4\pi/10$ is equal to $1.2566+$; it is therefore sufficiently accurate in ordinary calculations to say that H, the field, or the number of lines of force per square centimeter at the center of a long solenoid, is one and a quarter times the ampere turns *per unit of length.*

389. Variation of Field of Solenoid with Current.—The fact that the field on the axis of a solenoid varies directly with the

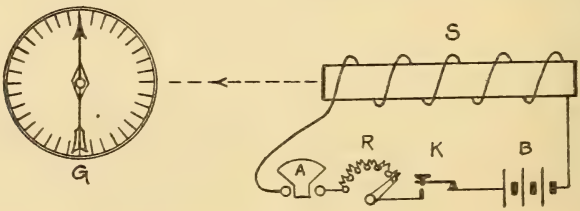


Fig. 175.

current may be shown experimentally as follows. In Fig. 175, S represents a solenoid, B a battery, K a key, R a rheostat (Par. 302), A an ammeter (a current-measuring instrument), and G a galvanometer with a short needle and long attached pointers poised over a graduated circle and placed so that the axis of the

solenoid prolonged passes through the pivot of the needle and is perpendicular to the magnetic meridian. By means of the rheostat, the current through the solenoid may be varied at will. The strength of the current is read direct from the ammeter. When the key K is closed, permitting a current to flow, the needle of the galvanometer is deflected. It will be seen that this is the case discussed in Par. 146 and that the deflecting force (which is due to the field of the solenoid) varies as the tangent of the angle of deflection. If, therefore, we lay off on a horizontal axis distances proportional to the current through the solenoid, the corresponding ordinates laid off proportional to the tangent of the angle of deflection will be proportional to the corresponding field. The points so determined will lie on a straight line passing through the origin (see OA, Fig. 176).

390. Effect of Material of Solenoid Core Upon the Field.—With the apparatus described in the preceding paragraph we may investigate the effect produced upon the field by varying the material of which the core of the solenoid is composed. Using cores of glass, rubber, wood, lead, copper, tubes of various gases or liquids or even vacuous space, no perceptible variation of the field is discovered, its strength remaining the same as when the solenoid enclosed only air. If, however, we insert a core of steel, the deflection of the galvanometer needle will indicate that the field has been increased several hundred times, that is, there are now several hundred times more lines of force traversing the solenoid than there were before the steel core was inserted. If the core be of soft iron, the increase is still greater; if it be of nickel, it is less than in the case of steel but much greater than in the case of air.

391. Permeability.—The great increase in the density of the magnetic flux (number of magnetic lines) when iron is inserted in the coil has been explained by saying that iron is more *permeable,* or has greater *permeability* than the other substances. When a beam of light falls upon a sheet of clear glass many more rays go through than when the beam falls upon a sheet of dark glass. We may consider that in each case there is a force tending to drive the rays through and that the dark glass offers greater resistance while the clear glass offers less, or is more permeable. So also there is a magnetizing force which tends to drive magnetic lines through the field of the solenoid. Air, wood, etc., offer a magnetic resist-

ance to this force and only a certain number of lines get through; iron and steel offer much less resistance, or are much more permeable, and permit many more lines to pass. To this magnetic resistance the name *reluctance* has been given. It follows that reluctance is the reciprocal of permeability or that the two are comparable to resistance and conductance, respectively.

392. Expression for Permeability.—We have seen that the field of a solenoid varies directly with the number of ampere turns per unit of length. It follows that the magnetizing force varies in the same manner, hence we may use H or 1.25 times these ampere turns (Par. 388), as a measure of the magnetizing force. If the magnetizing force which produced H lines per square centimeter in air produces B lines per square centimeter in iron, then the permeability of the iron is B/H. The accepted symbol for permeability is the Greek letter *mu*, μ, hence

$$\mu = \frac{B}{H}$$

Hopkinson found that a magnetizing force which produced 10 lines of force per square centimeter in air produced 12,400 per square centimeter in a specimen of wrought iron; the permeability of the iron was therefore 1,240.

The permeability of air, glass, and other non-magnetic substances is unity; that of bismuth, the most diamagnetic substance, differs from unity in the fourth place of decimals only.

393. Magnetic Saturation.—The conception of permeability as outlined in the preceding paragraphs loses some of its definiteness when it is found that for magnetic substances it is not a constant but is different for different magnetizing forces.

In Par. 389 it was stated that the field of a solenoid varies directly with the current. This is shown by the line OA in Fig. 176, in which the abscissae are laid off proportional to the magnetizing current and the ordinates proportional to the corresponding field. If we now insert in the solenoid a long soft-iron core, magnetically neutral, and gradually increase the current, we will notice three stages in the field produced: (a) for small values of the current it will increase slowly; (b) as the current is increased it will rise suddenly until a certain point is reached, after which (c) it will continue to increase but slowly. These stages are shown graphically in the curve OD. Since this curve represents the field pro-

duced by the solenoid and the core in conjunction, if we subtract from its ordinates the corresponding ordinates of OA, we will get the curve of magnetization of the core alone. The result is the curve OE. The upper portion of this being very nearly parallel to

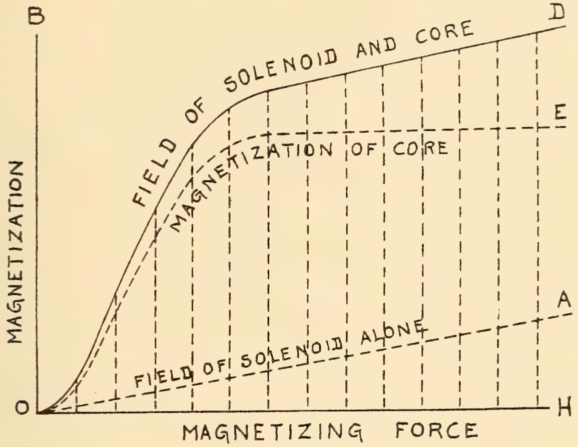


Fig. 176.

the horizontal axis indicates that the magnetization of the core would be but slightly increased by a further increase in the magnetizing current; in other words, the core is now *magnetically saturated.*

394. Curves of Magnetization.—As will shortly be shown, the designer of electrical machines and apparatus is frequently called upon to solve problems such as the following: Given an iron core of a certain size, shape and quality; required the number of ampere turns to produce in this core a flux of a certain strength. Among the data needed for the solution is not simply the permeability of the particular kind of iron of which the core is constructed but its permeability when the magnetic flux is of the strength called for in the problem. Such information is contained in tables but is more striking when presented graphically in the form of curves of magnetization. Fig. 177 represents these curves for five different qualities of iron and steel, whence it is seen that soft annealed iron may be both most easily and most highly magnetized and that hard steel is most difficult of magnetization. From the figure it is seen that for a magnetizing force of 5 the magnetization of soft

iron is 10,000, or the permeability is 2000, while for a force of 50 the magnetization is 16,000, or the permeability is only 320.

It will be noted that these curves all exhibit the three stages as described in Par. 393.

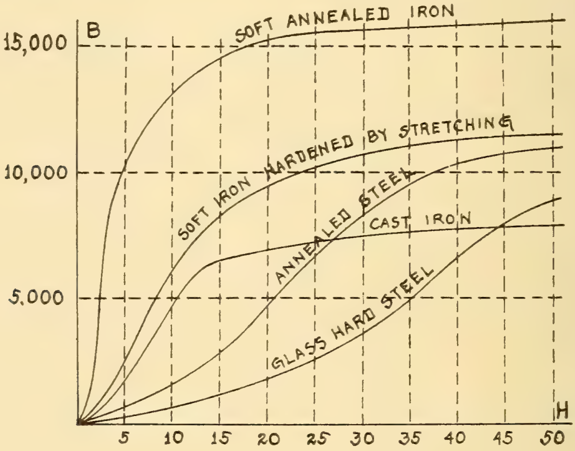
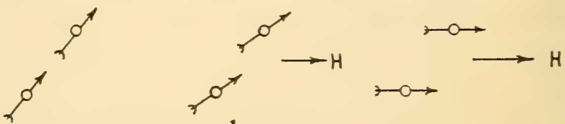


Fig. 177.

395. Ewing's Theory of Molecular Magnetism.—The accepted explanation of these phenomena is that advanced by Ewing and has already been given in part in Par. 153. The molecules of magnetic substances are inherently magnets but ordinarily exhibit no magnetic effects since they are grouped so as to mutually satisfy their individual polarities. Application of a magnetizing force disturbs this grouping, and exercises a directive effect upon the mo-



a b c

Fig. 178.

lecular magnets, causing them to take approximately a common direction so that they combine in the production of a magnetic flux. His theory, as the following will show, satisfactorily accounts for the three stages in the curves of magnetization. Let us take the simplest possible hypothetical case, that of two molecular magnets, and let the two small needles in *a*, Fig. 178, represent these molecules. If they be remote from other magnetic bodies

they will take up a position of equilibrium with their axes lying upon a common line. Let them now, as shown in *b*, be subjected to a magnetizing force *H*. If *H* be feeble the needles will move slightly but will not swing entirely to the right because they are pulled back by their mutual attraction. However, as *H* increases, this attraction will finally be overcome and the needles will then whirl suddenly to the right as shown in *c*. This corresponds to the stage of saturation. The needles, because of their action upon each other, are not strictly parallel nor can they ever become so. Further increase of *H* can only pull them a little more nearly parallel. If the magnetizing force be discontinued, the needles will not fly back at once to their original position but will linger and may require a slight jar to cause them to turn back.

Ewing's theory has been corroborated experimentally. A great many small magnetic needles were distributed side by side upon a long board which was then inserted in a coil and the needles allowed to come to a position of equilibrium. The arrangement was then subjected to a gradually increasing magnetizing force and the resulting fields were determined and plotted as described above. The result was a curve showing the three stages of the usual magnetization curves. Furthermore, when subjected to a demagnetizing force the curve went through the cyclic changes described in the following paragraphs.

396. Hysteresis.—Suppose that beginning with a magnetically neutral specimen of soft iron and applying a gradually increasing magnetizing force we should determine and plot the corresponding curve of magnetization. Suppose that having reached a point where a magnetizing force *OD* (Fig. 179) produces a magnetization *DA*, we should reduce the magnetizing force to zero. It will be found that the magnetization is by no means reduced to zero but persists or lingers after the withdrawal of the force and has some such value as *OC*. That portion of the curve representing the change from *A* to *C* is concave to the horizontal axis. If now the magnetizing force be reapplied, the curve of magnetization will not retrace the path *ANC* but there

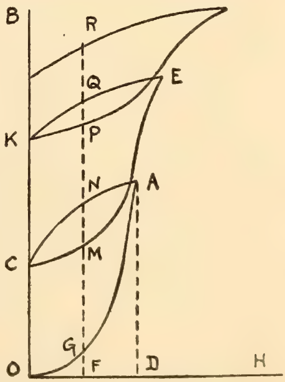


Fig. 179.

will be a tendency for the magnetization to linger at the value *OC* and it will increase at first at a slower rate than it decreased, the corresponding portion *CMA* of the curve of magnetization being convex to the horizontal axis. If at some other point *E* the magnetizing force be again reduced to zero and then reapplied, a similar loop *EQKPE* will be traced, and so on, the magnetization always hold ng back or conforming reluctantly to the changes in the magnetizing force. To this phenomenon the term *hysteresis*, a lag or lagging, is applied.

397. Further Data on Permeability.—The magnetizing force *OF*, Fig. 179, produces the various degrees of magnetization corresponding to *FG, FM, FN, FP, FQ, FR*, etc. Which of these is to be taken in determining the permeability of the specimen? It is seen that the notion of permeability is even more indefinite than was pointed out in Par. 393, and that in order that it may be of any practical use we must know the previous magnetic history of the specimen with which we are dealing. It can easily be shown that even though a specimen be magnetically neutral, its permeability, if it has recently been demagnetized by a single reversal of the current, is very different from what it is if it has never been magnetized at all. The usual understanding, therefore, is that when the permeability of iron or steel of a certain quality is given, it refers to a specimen which has not previously been magnetized and, furthermore, the permeability has been determined by the application of a continually increasing magnetizing force without reversals.

398. Cycle of Magnetization.—If a specimen of soft iron be magnetized, then demagnetized, then magnetized to an equal degree in the opposite direction, then demagnetized, and finally again subjected to the original magnetizing force, it will pass through a cycle of magnetization represented by the curve in Fig. 180. When the magnetizing force has first been reduced to zero the magnetization of the specimen is still proportional to *OC*. In order to remove this *residual magnetism* an opposite or negative magnetizing force *OF* must be applied. Since after the withdrawal of the magnetizing force the iron still retains a portion of the magnetism, we may say that the iron clings to this magnetism with a force equal to the force *OF* which must be employed to cause its relinquishment. The force which must

be applied to remove the residual magnetism is called the *coercive force.*

The broken curve in Fig. 180 represents a cycle of magnetization of a specimen of hard steel, whence it is seen that the coercive

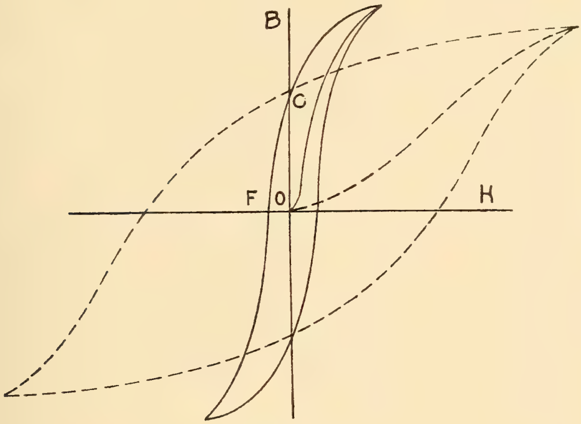


Fig. 180.

force is very much greater than in the case of iron. This has already been shown in Par. 155.

399. Energy Loss Due to Hysteresis.—In raising a weight a certain amount of work must be performed. If after the weight is raised it be released, it will in its fall restore the same amount of energy. In magnetizing a bar of iron or steel work is likewise performed but when the magnetizing force is withdrawn the entire amount of energy is not given back, in other words, there is a loss.

In Par. 358 we saw that the work expended in changing the field within a coil carrying a current is IN ergs, in which I is the current in absolute units and N the increase or decrease in the number of lines embraced. If there be n turns in the coil, the expression becomes nIN ergs, but n being a constant the work is always proportional to the product of the current by the change in the number of lines embraced.

In Fig. 181, AL is the average magnetizing force as the number of lines embraced by the coil increased from OE to OF. But we have seen that the magnetizing force is proportional to the current, therefore AL is proportional to the current and the

area of the rectangle $AL \times EF$ is proportional to IN or to the energy expended while the magnetization increased from OE to OF. In a like manner the area of the rectangle FM is proportional to the energy expended while the magnetism increased from OF to OK. The sum of these elementary rectangles, or the area $SDGO$, represents the total energy expended in magnetizing the iron to the stage OG. It follows that the area FDG represents the energy restored as the magnetization falls to OF, and the difference between these two areas represents lost energy. The energy lost during a complete cycle is proportional to the entire shaded area enclosed by the loop. It will be seen from this that the lost energy is much less in the case of soft iron than it is in the case of steel. This lost energy reveals itself in the form of heat, the temperature of the core rising. It represents waste which in the case of certain alternating-current machines may assume serious proportions. It is largely on this account that the best and softest iron is used in the cores of transformers (Par. 431). Ewing has shown that the energy consumed in subjecting one ton of soft iron to 100 cycles of strong magnetization per second is about sixteen horse-power and the energy loss for a very hard tungsten steel is twenty times greater.

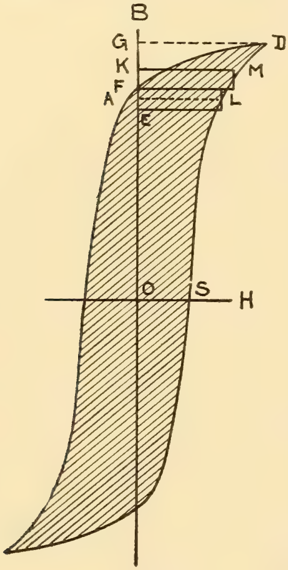


Fig. 181.

400. Law of Magnetic Circuit.—In Par. 387 it was shown that the intensity of the field at the center of an indefinitely long solenoid is

$$H = 4\pi NI$$

in which N is the number of turns per centimeter and I is the current in absolute units. Actually it is impracticable to employ very long magnetizing coils but by substituting the proper values in the integral in the paragraph referred to, it can be shown that in applying the above formula to coils whose length is not less than six times their diameter, the error committed does not exceed one per cent. If the

length of the magnetizing coil be l and if the total number of turns be n, the above expression can be written

$$H = \frac{4\pi nI}{l} \qquad \text{(I)}$$

Suppose this coil to be wrapped uniformly around an iron ring whose length is l and whose permeability is μ. The flux or *induction* per square centimeter (the word "induction" being used in the sense of "crop of lines of force produced") is

$$B = H\mu = \frac{4\pi nI \,.\, \mu}{l}$$

If the cross-section of the core be A, the total induction is

$$\phi = \frac{4\pi nI \,.\, A \,.\, \mu}{l}$$

This may be put in the following form

$$\phi = \frac{4\pi nI}{\frac{l}{A} \times \frac{1}{\mu}}$$

In Par. 391 it was shown that reluctance is the reciprocal of permeability, therefore representing $1/\mu$ by $\mathcal{R}$, the above becomes

$$\phi = \frac{4\pi nI}{\frac{l}{A} \times \mathcal{R}} \qquad \text{(II)}$$

Ohm's law may be written (Par. 285)

$$I = \frac{E}{\frac{l}{A} \times \rho}$$

The similarity of these two expressions is striking. In the case of electricity, the current varies directly as the electro-motive force and inversely as the resistance; in the case of the magnetic field, the flux varies directly as the *magneto-motive force* and inversely as the *reluctance.*

From (I) $4\pi nI$, the magneto-motive force, is equal to $H.l$ in which H is force in dynes and l is length in centimeters, therefore this magneto-motive force is measured in *work,* or ergs. It will be recalled that the electro-motive force between two points is also measured (Par. 72) by the *work* expended in moving a unit charge from one point to the other.

From (II) it is seen that, like resistance, the reluctance varies directly as the length and inversely as the cross-section of the magnetized body, and also as the factor $\mathcal{R}$, which may be called the *specific reluctance* or the *reluctivity* of the body. It can be shown by the method used in Par. 288 that specific reluctance is measured by the reluctance of a centimeter cube of the substance.

The foregoing analogy is not complete. The resistance of a conductor kept at a constant temperature does not vary with the current; on the other hand, the permeability, and hence the reluctance, does vary with the flux.

401. Calculation of Flux.—It is seldom that the magnetic circuit is a complete iron path as assumed in the preceding paragraph. It most frequently is intersected by air gaps and is composed of portions which differ in permeability. In such a case the total reluctance is the sum of the separate reluctances in series. As an illustration, suppose we are required to calculate the flux through a magnetic circuit as shown in Fig. 182 consisting of an iron horseshoe-shaped portion M whose length is l_1, cross-section A_1 and permeability μ_1, and a cylindrical iron armature B, whose average length is l_2, cross-section A_2, and permeability μ_2, the armature being separated on either side from the horseshoe frame by an air gap of length l_3, cross-section A_3 and permeability unity. The flux, if I be in amperes, is

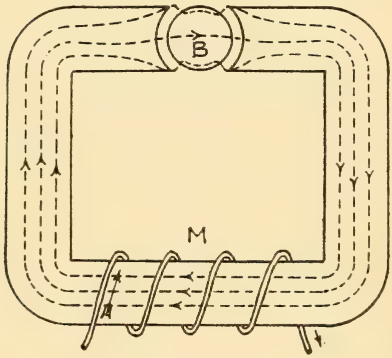


Fig. 182.

$$\phi = \frac{4\pi n I}{10\left(\frac{l_1}{A_1\mu_1} + \frac{l_2}{A_2\mu_2} + \frac{2l_3}{A_3}\right)}$$

As an alternative problem we may be required to calculate the ampere turns to produce a required flux in a given circuit. This involves the solution of the above equation for nI but is complicated by the decrease in permeability with increase in flux. The permeability under the conditions of the problem is best obtained from tables or from the corresponding curves of mag-

netization (Par. 394). It may also be necessary to make allowance for a certain amount of leakage of flux which occurs at the air gaps.

The foregoing calculations are not exact but they enable the designer of electrical machinery to approximate very closely to the solution of his problems.

402. Diamagnetism.—In Par. 122 reference was made to diamagnetism, or the property possessed by certain bodies, notably bismuth, which causes them to be feebly repelled from the poles of a magnet. Various attempts have been made to account for this phenomenon, the explanation now accepted being based upon the theory that the permeability of these diamagnetic substances is less than that of the surrounding medium. Fig. 183 represents a block of bismuth placed in a magnetic field. The bismuth being less permeable than the surrounding air, it crowds off to the right and left a portion of the field. The tension along the lines of force causes the bismuth to move from the stronger into the weaker field, or away from the magnet.

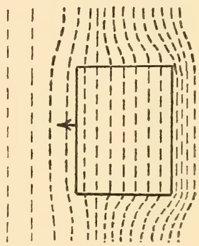


Fig. 183.

This hypothesis is corroborated by the fact that a glass tube filled with a solution of an iron salt is paramagnetic when suspended in air between the poles of an electro-magnet, but becomes diamagnetic when surrounded by a denser or more concentrated (and hence more permeable) solution of the same salt.

CHAPTER 32.

ELECTRO-MAGNETS.

403. Electro-Magnets.—The combination of a coil with a core of a magnetic substance, usually soft iron, which is made a magnet by the passage of a current through the coil is called an *electro-magnet.* The first electro-magnets were made in 1824 by the English scientist Sturgeon. At that time insulated wire had not been invented and his magnets were made by insulating the core by a thick coating of varnish and wrapping the wire on top of this, the successive turns being so spaced that they did not touch each other. In 1826, Joseph Henry of Albany discovered how to insulate wire by a silk covering. This enabled him to wrap the wire more closely and to put on several layers and he soon produced electro-magnets remarkable for their power. In 1831 he constructed one whose iron core weighed less than sixty pounds yet could support over a ton.

404. Rules for Polarity of Electro-Magnets.—After the facts brought out in the preceding chapters it is perhaps unnecessary to give a rule for determining the polarity of an electro-magnet. Should such be needed, the simplest is the right hand rule, which is merely a variation of the rule given in Par. 345. *Place the palm of the right hand upon the coil, the fingers pointing in the direction of the flow of the current* (Fig. 173); *the extended thumb will point to the north pole of the magnet.* Another rule frequently used is the following. Face the pole of the manget; if the magnetizing current flows around it in a clockwise direction it is a south pole; if in a counter-clockwise direction it is a north pole.

405. Value of Electro-Magnets.—Electro-magnets are used extensively and for very varied purposes, their value depending upon the three following characteristics.

(1) Their great power. They can be made very much more powerful than the strongest permanent magnets and they can also be made of much greater size.

(2) Control of magnetism. The magnetism is perfectly under the control of the operator and, like an electric light, may be turned off or on at pleasure.

(3) Control from a distance. The control can be exerted even at distances of several hundred miles.

406. Tractive Power of Magnets.—In Par. 66 it was shown that a unit charge placed near a plane charged to a uniform surface density δ is acted upon by a force of $2\pi\delta$ dynes. A frequently employed conception of magnetism is that the intensity of magnetization is due to the number of unit poles spread over the polar or terminal surface of the magnet (Par. 133). If N (Fig. 184) be the pole of a bar magnet and if S be a bar of soft iron or other magnetic substance placed so near N that all the lines of force which emerge from N enter S, then there will be as many

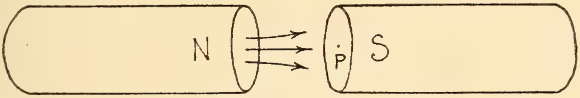


Fig. 184.

unit poles upon S as there are upon N. The force between N and S will be one of attraction. If we consider that the magnetism upon N is uniformly distributed and equivalent to δ unit poles per square centimeter, then the same course of deduction as followed in Par. 66 will show that a unit pole at P is attracted with a force of $2\pi\delta$ dynes. If the sectional area of N be A, there are upon N, $A\delta$ unit poles. There are an equal number upon S, each of which is acted upon by a force of $2\pi\delta$ dynes, therefore, the total attraction between N and S is

$$F=2\pi\delta \times A\delta = 2\pi . A . \delta^2 \text{ dynes} \qquad \text{(I)}$$

Since from each unit pole there radiate 4π lines of force (Par. 145), the total number per square centimeter between N and S is $H=4\pi\delta$, whence $\delta = H/4\pi$.

Substituting in (I) above we have

$$F=A . \frac{H^2}{8\pi}$$

or the tractive force exerted by a magnet is proportional to the square of the number of lines of force per square centimeter of pole surface.

Ewing states that by using very high magnetizing force a magnetic pull of over 225 pounds per square inch has been obtained.

A curious consequence follows from the above. By decreasing the pole area we increase the tractive power of the magnet. This is because as we decrease the area we increase the number of lines of force per square centimeter and the tractive power varies as the square of this number. This is the explanation of the fact referred to in Par. 124, namely, that if one end of a bar magnet be square and the other end be rounded, the rounded end will exert the greater pull. The powerful electro-magnets used in hospitals to extract particles of iron from the eye have long conical poles.

The above expression for the tractive power seems to indicate that this power is independent of the distance between the pole and the body, but actually the force does fall off very rapidly as we recede from the pole. The explanation is that as we increase the air gap we increase very greatly the reluctance of the magnetic circuit and this in turn decreases the flux or H. (Par. 401.)

407. Shape of Electro-Magnets.—Since the pull of a magnet varies as the square of the flux per square centimeter and since this flux varies inversely as the reluctance of the magnetic circuit, electro-magnets, as a rule, are designed so that the air gaps in the circuit are as small as possible. The majority therefore are either of the horseshoe pattern or bent to three sides of a rectangle. The magnetizing coil may be wrapped over the whole length of the horseshoe, or only on the central part or *yoke,* but most frequently two coils are used, one being wrapped on each leg of the core. In small instruments these coils are called *spools* or *bobbins.* The dimensions and relative proportions of the parts of these magnets are varied according to the use to which they are to be put.

408. Use of Electro-Magnets.—The uses to which electro-magnets are put may be classed under two general heads; (a) for creating the magnetic fields required for the operation of certain electrical machines and (b) for exerting a tractive effort or pull. The use for creating fields will be described when the subject of electrical machinery is reached. The second heading embraces a most varied class of uses among which are (1) lifting weights,

(2) operating annunciators, call and alarm bells, etc., (3) telegraphy, (4) operating automatic switches, (5) regulating the feed of the carbons of an arc light, regulating clocks from a master clock, etc. Only a few of these can be described.

409. Lifting Weights by Electro-Magnets.—Electro-magnets are largely used in handling scrap iron, steel billets, boiler plates, etc. The magnet employed is shown in section in Fig. 185. The core is a short and heavy one-piece casting consisting of an inner cylindrical core surrounded by an annular space in which the magnetizing coil is wound, the whole being called an *iron-clad electro-magnet.* When the current is turned on, the inner core becomes one pole and the outer ring the other. Owing to the large cross-section and little length of the iron and to the shortness of the air gap when a piece of iron is in contact with the poles, the pull is very powerful. This magnet, suspended from a derrick, is lowered upon the pile of scrap iron that is to be moved, the current is turned on, the magnet with the clinging mass of iron raised, swung over to where it may be desired, the current turned off and the iron dropped. In handling such objects as boiler plate, it avoids the necessity of using and adjusting hooks, chains or ropes. The coil is thoroughly protected from accidental injury, a sheet of brass usually being inserted in the annular space.

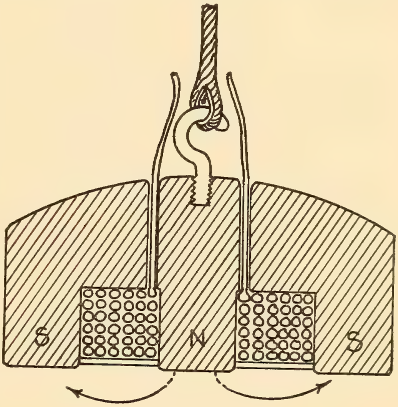


Fig. 185.

410. Electric Bells.—A common form of electric bell is shown diagrammatically in Fig. 186. It consists of the bell or gong G, the hammer H, the electro-magnet M, the battery C (usually one or two dry cells), and the push button D. The hammer is a metal knob on a slender arm pivoted at P and bearing at its middle the soft iron armature A. A delicate spiral spring S is attached to the arm and exerts upon it a pull from the magnet. At the back of the armature there is a slender brass strip which makes contact at B with an adjustable screw. When the button

D is pressed, closing the circuit, a current flows from *C* to *D*, thence to *B*, thence to *P*, thence through the coils of *M* and back to *C*. The cores of *M* are magnetized by this current, attract the armature *A*, causing the hammer to strike the bell, but at the same time break the circuit at *B*. The circuit being broken, *M* is no longer magnetized, the spring *S* pulls the armature back to its

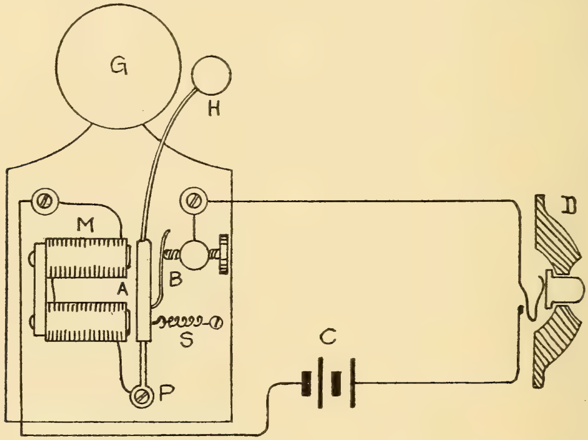


Fig. 186.

original position, and the contact at *B* is restored. This causes the hammer to strike the bell again and so on, a rapid succession of blows being given so long as the button is pressed. Arrangements of this kind for rapidly making and breaking a circuit are called *interrupters*.

411. The Electric Telegraph.—The word "telegraph" meant originally to convey messages by exchanging signals at a distance. During the wars of Napoleon there was developed a system of semaphore signals by which messages could be transmitted rapidly from point to point. We read in the contemporary accounts of the campaign in the Spanish peninsula that Napoleon *telegraphed* his instructions from Paris to his corps commanders in the field.

The sending of signals by means of electricity was tried by many. An insulated wire between two points was given a static charge which caused a pith ball at the far end of the wire to stand out. If a charged body be moved near the other end of the wire corresponding movements could be produced in the pith ball.

Sparks from a Leyden jar were transmitted over a wire in accordance with a prearranged code. Use was made of the electrolytic effect of a current. Twenty-six separate wires, each marked to correspond to a certain letter of the alphabet, were stretched between two points and at the receiving station the ends of these wires dipped into an acidulated solution. A single wire led from the solution to the ground. At the sending end a voltaic pile was used, one pole of which was "grounded." When the other pole was touched to one of the twenty-five wires, the circuit was complete and bubbles of gas appeared at the corresponding end at the receiving station. These various methods failed mainly because of the lack of a steady source of electricity. This difficulty was overcome by the invention in 1836 of the Daniell cell. In the following year Congress was induced to make an appropriation of $30,000 for the erection between Baltimore and Washington of a line to test the system invented by Morse. This proved successful and with minor variations is in operation to-day over the greater part of the globe. It is estimated that there are now over five million miles of land telegraph lines in use.

412. The Morse Telegraph.—The principle of the Morse telegraph will be readily understood from the following. In the diagram (Fig. 187) *K* is the sending and *M* the receiving station.

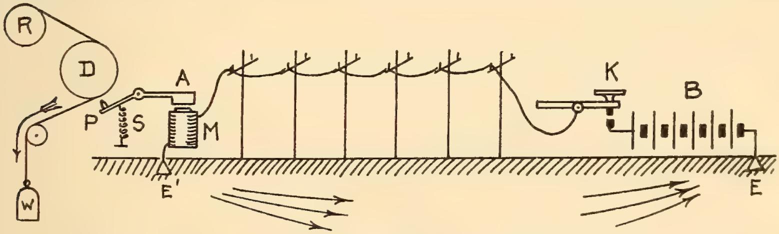


Fig. 187.

R is a roll of paper ribbon which is slowly unwound by clockwork in the direction shown by the arrow. *B* is a battery of Daniell cells, one pole of which is grounded at *E*. When the key *K* is closed the current travels from the battery over the line to the electro-magnet *M*, thence to the ground at *E′*, thence back through the earth to *E*. When *M* becomes magnetized, the iron armature *A* is pulled down. This causes the end *P* of the lever to rise and to press a pencil against the moving ribbon at *D*. When the key *K* is opened, the circuit is broken and a little

spring S pulls the pencil away from the ribbon. The length of the pencil mark on the ribbon varies therefore with the length of time that K is kept closed and the Morse alphabet is accordingly made up of a system of dots, dashes and intervals or spaces. If there be in the face of the drum D a groove, and if P instead of being a pencil is a hard and smooth stylus which presses above this groove, there will be produced in the ribbon long and short indentations.

While the foregoing gives the principle of the Morse telegraph, in actual practice certain conditions arise which cause a considerable modification in the simple arrangement described above. These are the following:

(1) Each station must be able both to send and to receive.

(2) The line must be so arranged that intermediate stations may be operated.

(3) If the key K, Fig. 187, be left open, the circuit is broken and it would be impossible for an operator at M to send a signal to K. Accordingly, in the American system the key K is kept *closed* when not in use, in other words, there is a current constantly flowing over the line. This would appear to be a wasteful method and is avoided in the European system, but actually the current (and the consequent waste) is very small, and since the European system requires a greater number of batteries, the cost is about the same.

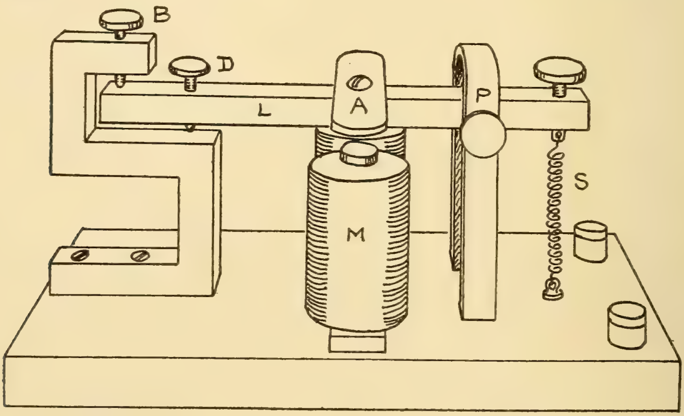


Fig. 188.

(4) It was soon discovered that the signals could be read by ear and therefore the recording apparatus is now generally

omitted and in its place is substituted a *sounder*, an instrument shown in simplest form in Fig. 188. A horizontal brass lever L, pivoted at P, is pulled down at one end by the spring S until the other end is pressed up against the adjustable contact B. The lever carries on its upper side the crosswise soft iron armature A and below this armature is the electro-magnet M. When a current flows through M the core is magnetized, A is attracted and the lever is pulled down until the contact D strikes the brass frame just below, making a loud click. When the current is broken the spring S causes the lever to fly up and strike B, making a second click. The interval between these successive clicks determines whether the sound be a dot or a dash.

(5) The currents employed are only a few thousandths of an ampere (not entirely through choice but because of the resistance

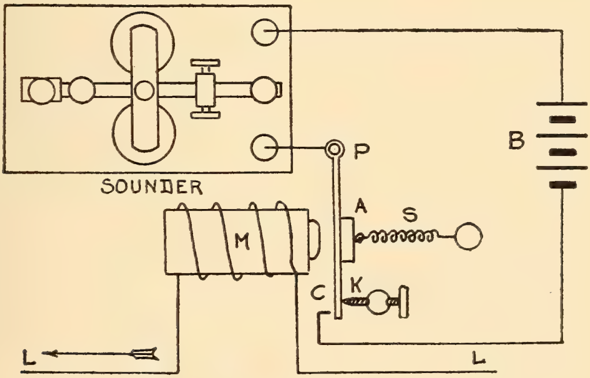


Fig. 189.

of the line), and are usually not strong enough to actuate directly either the recording device or the sounder. Morse overcame this difficulty by means of a *relay*, an electro-magnet so placed in the main circuit that when a current flowed the magnet attracted an armature which in its movement closed an auxiliary circuit, thereby throwing in a local battery which supplied the necessary current to operate the recording apparatus. This arrangement is shown diagrammatically in Fig. 189 in which M is the electro-magnet in the main line LL, A is the armature, hinged at P and drawn up against the adjustable stop K by the feeble tension of the spring S. When a current passes through M, the armature A

is attracted and makes contact at C, thus throwing in on the sounder the auxiliary battery B. The armature is therefore really a switch or key for the local circuit.

413. The American System.—The operation of the American system will be understood from Fig. 190. The operator in Boston, preparatory to signalling, opens the switch S of his *sender*. This

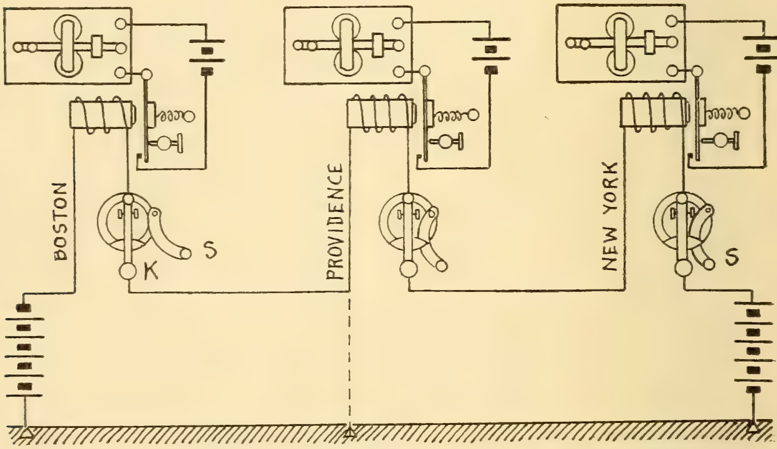


Fig. 190.

breaks the circuit and stops the current in the line. When he closes his key K, the circuit is restored, a current flows, each of the electro-magnets pulls down its relay armature thus causing every sounder to click. A signal made at one station is therefore repeated at every station on the line. Should the New York operator wish to interrupt, he opens his switch S, thus breaking the circuit. The Boston operator is aware of this at once because his own sounder ceases to click, and he at once closes his switch and awaits instructions from New York. Whenever a message is completed, the operator must at once close his switch.

Should a break occur in a line, it is still possible to use the remainder. Thus, should a break occur between Providence and Boston, the Providence operator by grounding his line, as shown by the dotted line, restores communication with New York. Should the break be between Providence and New York, he must ground his line to the right of his key.

414. Overload Switch.—Should a short circuit occur on an electric-lighting or on a power circuit, serious injury may result. Various automatic devices are employed to afford protection in such cases. We saw in Par. 306 the use of *fuses* for this purpose. There have been devised many kinds of switches which automatically break the circuit when the current exceeds a certain maximum for which they are set. These are called *circuit-breakers* or *overload switches,* the word "load" in electric parlance meaning current. They are therefore analogous to safety valves.

One of these is shown diagrammatically in Fig. 191. The switch *A* when closed makes contact through a curved arm with two points *B* and *C*. A stout spring, *S*, tends to throw the switch in the direction shown by the arrow but is prevented from doing so by a hook *H* which engages in a corresponding hook on the trigger *T*. The current enters at *E*, passes thence to *B*, thence through the switch to *C*, thence around the coil *G* and out by *F*. Within the coil *G* there is a soft iron core. As the current increases in strength, the coil exerts a greater and greater pull upon this core until finally it is lifted bodily. As it moves upward it strikes the trigger *T*, releasing the switch which is then thrown forcibly up, thus breaking the circuit. The farther the core is inserted in the coil, the more easily it is lifted, therefore, by means of the screw *K*, the switch may be set to trip at any desired limit.

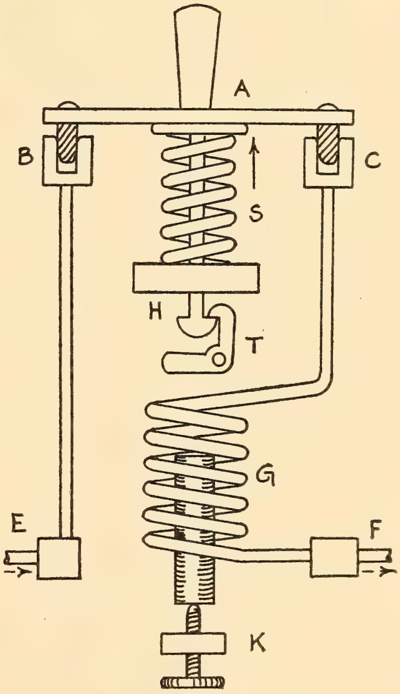


Fig. 191.

415. Underload Switch.—Automatic switches are also in use which trip when the current falls *below* a certain minimum. One form is shown diagrammatically in Fig. 192. An arm, pivoted at *P*, carries at one end a weight *W* and at the other end an arc of

wire whose extremities dip into mercury cups. The current, flowing as shown, passes around M, thence to the first mercury cup, thence across the arc to the second cup and out. The armature A is attracted and held by the electro-magnet M. When the current decreases below a certain point, M can no longer hold A, the weight W falls and lifts the ends of the arc out of the mercury cups, thus breaking the circuit. Instead of these "mercury break" switches, preference is now given to forms similar to the overload switch, described in the preceding paragraph, the switch being thrown open by a compressed spring when the current falls below a certain minimum.

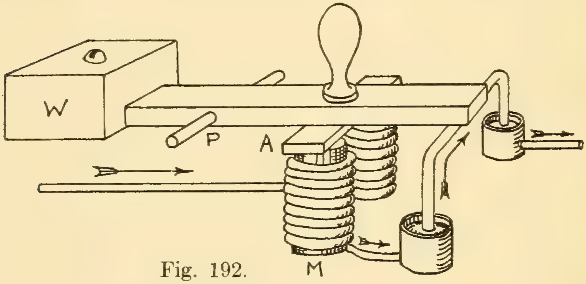


Fig. 192.

At first sight it is not clear why an underload switch is needed. The following is an example of its use. Fig. 193 represents a storage battery B being charged by current from a generator G through an underload switch S. It was shown in Par. 245 that in order to drive a current through the battery, the E. M. F. of the generator should be about ten per cent greater than that of the battery. Suppose that by some accident during the charging,

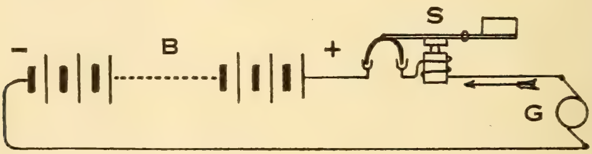


Fig. 193.

such as the belt slipping, the generator should slow down or should stop. The moment the E. M. F. of the generator falls below that of the battery, the battery would at once begin to discharge back, and the resistance of the generator being very small, the discharge would amount to a short circuit. However, before a current can be reversed it must die down and pass through zero, therefore, before the battery could discharge, the underload switch would trip and thus protect it.

CHAPTER 33.

INDUCTION.

416. Faraday's Discovery of Induction.—In Fig. 194, *C* is a hollow cylindrical coil of wire connected in circuit with a galvanometer *G*, and *M* is a magnet held above the coil. If the magnet

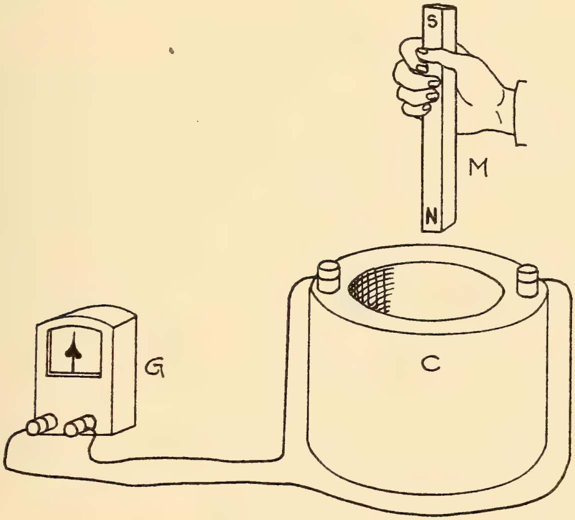


Fig. 194.

be quickly thrust into the coil, the galvanometer needle will be deflected indicating a current in *C*, but the deflection is only momentary and if the magnet after insertion be held motionless, the needle will at once return to its zero position. If, after the needle has come to rest, the magnet be quickly withdrawn from the coil, the galvanometer will again indicate a momentary current but in this case in a direction opposite to that produced by the insertion of the magnet. The more rapidly the magnet is inserted or withdrawn, the greater the momentary current as indicated by the greater deflection of the galvanometer needle. If the magnet be reversed end for end, the currents will likewise be reversed. Finally, if the magnet be held motionless and the

coil be moved, the same results are obtained, that is, the motion of the magnet and coil need only be relative.

These facts were discovered by Faraday in 1831. Their importance can hardly be overestimated since they are the basis of nine-tenths of the present commercial production of electricity. The currents produced in the coil by these movements are said to be *induced* and the phenomenon is called *induction.*

If there be a break in the circuit of the coil there will be an induced E. M. F. but no current, and, to avoid repetition, it is to be borne in mind hereafter that whenever reference is made to induced E. M. F. there will also be an induced current in the same direction, provided the circuit be complete.

We have already seen (Par. 403) how a magnet may be produced by the electric current; the above shows the reverse process, the production of an electric current by means of a magnet. It must, however, be noted that in the production of a magnet by means of a current there is an expenditure of electrical energy, while in the production of a current by means of a magnet there is no loss of magnetism and the magnet suffers no diminution in strength. More physical energy is required to move the magnet or the coil relative to each other than is required if a soft iron bar of equal weight be substituted for the magnet, and this extra energy is the source of the electrical energy.

417. Faraday's Second Discovery.—Since inserting into the coil an unmagnetized bar of iron or steel, otherwise exactly similar

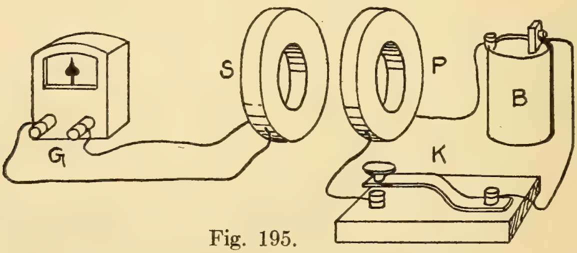


Fig. 195.

to the magnet, produces no effect, it follows that the current must have been produced, not by the movement of the magnet alone but by the movement of the field surrounding the magnet. Since this field consists of space traversed by lines of force, we may state that if lines of force are thrust into or withdrawn from a circuit, an E. M. F. is induced in the circuit. It is not necessary that the

magnet be actually inserted in the coil provided it be so moved as to alter the number of lines of force traversing the coil. It follows logically from the foregoing that induced currents may be produced by using lines of force produced otherwise than by magnets, that is, by currents.

In Fig. 195 *B* represents a battery, *P* a coil of wire and *S* a second coil near to the first and connected to the galvanometer *G*. There is no electrical connection between *P* and *S*. With *K* closed and a current flowing in *P*, the galvanometer will indicate a momentary induced current in *S* if *P* be moved nearer to *S*, and a momentary current in the opposite direction if *P* be moved farther from *S*. This production of an induced current by varying the position of a current with reference to a circuit was the second of Faraday's discoveries in induction. To the coil *P* he applied the name *primary*, and to the coil *S*, the one in which the current is induced, the name *secondary*.

Without varying the position of *P* and *S*, a momentary current is induced in *S* whenever *K* is closed, and one in the opposite direction when *K* is opened. These are but extreme cases of the general case above, for to close the key is equivalent to bringing up a current to *P* from an infinite distance, and to open the key is equivalent to removing the current in *P* to an infinite distance.

In the case of the magnet, induction took place only while the magnet was moving; so in this case induction takes place only while the current in the primary is changing, or while the primary with current flowing is being shifted in position relative to the secondary.

418. Inertia of Electro-Magnetic Fields.—A physical explanation of induction may be given if the following preliminary conceptions be grasped.

(a) The space embraced by an electric circuit is at any given time pervaded by n lines of force. If the convention be adopted that lines in one direction are positive, then those in the opposite direction must be considered negative and therefore n may have any value, positive, or negative, or zero.

(b) Positive and negative lines of force neutralize each other, in other words, a sufficient number of lines of force of one kind may be introduced into a field of the opposite kind to weaken the field, or to reduce it to zero, or to reverse it.

(c) Electro-magnetic fields possess a property which has been termed *electro-magnetic inertia* and which is analogous to the inertia of matter. Inertia is a property of matter by which the matter resists any change of its state with respect to rest or motion. Thus, a body at rest resists being put in motion and a body in motion resists being accelerated, retarded, turned aside, or stopped. This resistance manifests itself only so long as the change in the state of the body is being made and disappears the instant the change is accomplished. Electro-magnetic inertia may be said to be the property by which electro-magnetic fields resist any change in the number or direction of their lines of force. This resistance manifests itself as E. M. F. and corresponding current in the circuit, which current tends to produce lines of force of such number and kind as to keep the original number constant. Like the inertia of mass, it reveals itself only while the change in the number of lines in the field is taking place and vanishes as soon as the change has taken place.

419. Explanation Applied to Magnet and Coil.—To illustrate, consider the case of the magnet and the hollow coil (Fig. 196).

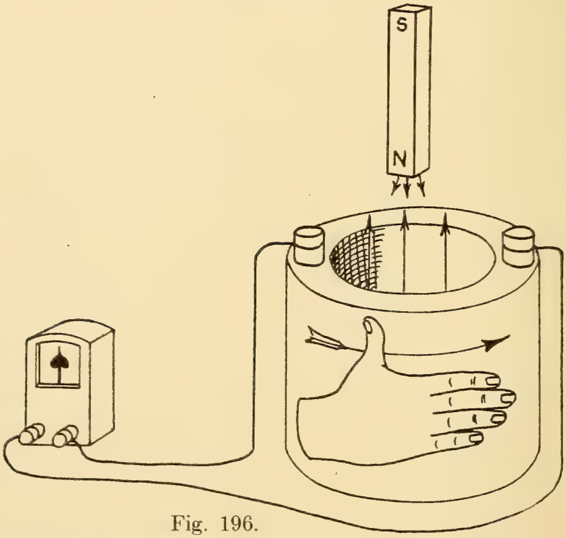


Fig. 196.

At the outset, the number of lines in the field of the coil may be considered zero. If we thrust in the magnet in the direction shown in the figure, we push in lines of force from above downward.

The current induced in the coil is in such direction as to produce lines of force upward, that is, tending to neutralize those which are being inserted and thus keeping the original number in the field unvaried. Applying the right hand rule (Par. 404), we see that, looking down into the coil from above, the induced current will be counter-clockwise.

Had the magnet been reversed and the south pole been inserted in the coil, the lines of force would have been thrust in in a negative direction, or pointing upwards, which must be considered as a *decrease* in the number in the original field. The induced current would therefore have been in such direction as to send lines of force downward, that is, viewed from above, it must have been clockwise.

Upon withdrawing the magnet in the first case, we decrease the number of lines embraced by the coil. The induced current is in such direction as to compensate for this withdrawal by producing lines running downward, hence, looking at the coil from above, the current is clockwise.

Similarly, withdrawing the magnet which had been inserted south end foremost produces a counter-clockwise induced current.

420. Explanation Applied to Two Coils.—Consider the case of the two coils as described in Par. 417. Upon closing the key (Fig. 197) the current flows around P as indicated. This produces in

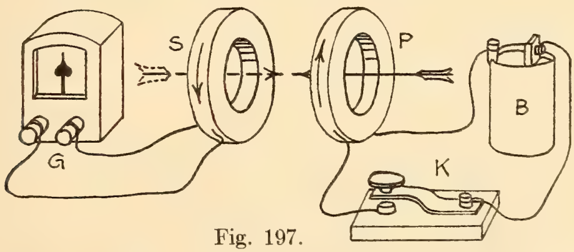


Fig. 197.

the coil P lines of force in the direction shown by the large arrow, and as the two coils are now placed, some of these lines pass through S thus changing the number of lines in the latter's field. The current induced in S is in such direction as to produce lines of force opposed to those coming from P. This current, viewed from P, is therefore counter-clockwise.

Similarly, when K is opened the effect is to withdraw these lines of force from S and the current induced in S is in direction to

produce others to replace those being withdrawn, hence, seen from P, the current is clockwise.

With the current flowing in P, changes in the position of P with respect to S vary the number of lines through S and induce currents in S in accordance with the principles just given.

421. Rule for Direction of Induced E. M. F.—A simple rule for remembering the direction of the induced E. M. F. (and current) in a coil is the following. *Look through the coil in the positive direction of the lines of force; a decrease in the number enclosed induces a clockwise E. M. F.; an increase induces a counter-clockwise E. M. F.*

422. Right Hand Rule for Direction of Induced E. M. F.—There are certain cases where the beginner may be perplexed as to the application of the foregoing rule. Thus, the conductor under

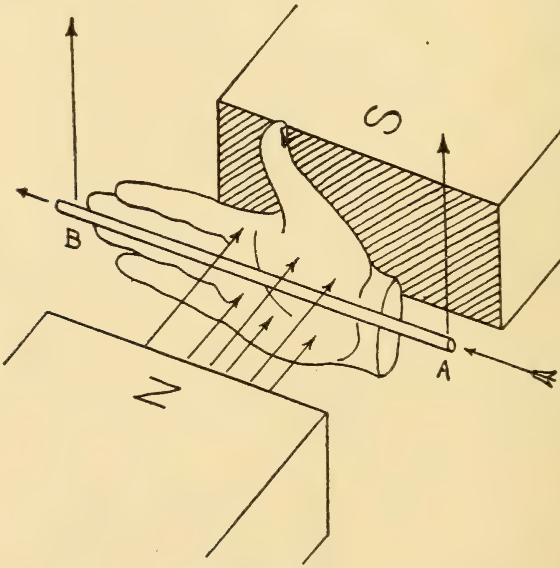


Fig. 198.

consideration may not form a coil but may be a straight piece of wire, or there may be a coil but its position may be in doubt, only a portion of it being visible. For example, the coils on the armature of a dynamo are often interwoven in an intricate manner and further concealed by a covering of insulating material, yet it may be necessary to determine the direction of the induced E. M. F.

In such cases the following right hand rule seems to be the simplest. *Place the right hand upon the conductor, the thumb pointing in the direction of its motion, the palm turned to receive the lines of force of the field; the extended fingers will indicate the direction of the induced E. M. F.*

In Fig. 198 the conductor *AB* is moving upward and the direction of the induced E. M. F. is from *A* to *B*.

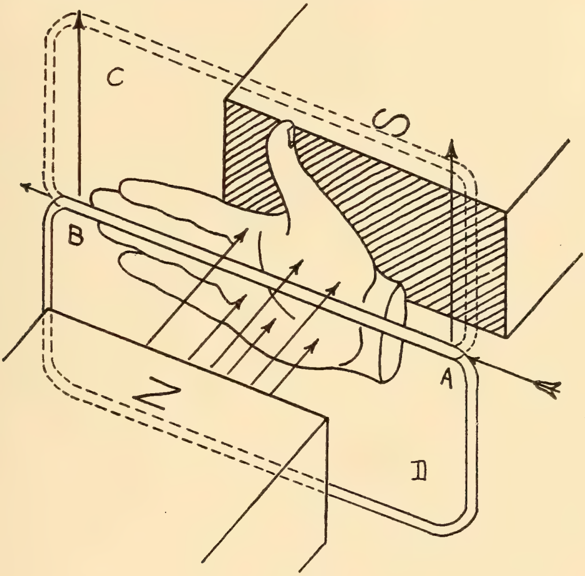


Fig. 199.

These two rules are of course perfectly compatible. For example, suppose *AB* (Fig. 199) to be a part of either the coil *ABC* or of the coil *ABD*. If it be *ABC*, the upward movement will carry it out of the field, there will be a *decrease* in the number of lines embraced and the induced E. M. F. will be clockwise, or from *A* to *B*. If it be *ABD*, the upward movement will carry it into the field, there will be an *increase* in the number of lines embraced and the induced E. M. F. will be counter-clockwise, or again from *A* to *B*.

If the plane of the coil be moved parallel to the lines of force, or if the coil be moved parallel to itself in a uniform field, there is no increase or decrease in the number of lines embraced and consequently no induced E. M. F. This same conclusion may be

derived from Par. 358. To induce E. M. F. there must be an expenditure of energy, but since the number of lines embraced by the coil is unaltered, there is no such expenditure. From another point of view it may be considered that in each half of the coil there is induced an equal E. M. F. but these being in opposite directions, the resultant E. M. F. is zero.

423. Mechanical Production of Electric Current.—Since the insertion of a magnet into a coil induces a momentary current and the withdrawal of the magnet induces a momentary current in the opposite direction, it is possible to construct a machine by which a reciprocating motion is given to a magnet which alternately enters and recedes from a coil and thus induces an alternating current in the coil and in its circuit. Such a machine would be of low efficiency. But we have also seen (Par. 417) that it is not necessary to actually insert the magnet into the coil provided it be so moved as to vary the number of lines of force through the coil. For example, it could be swept across the mouth of the coil. This is the basis of the construction of modern machines for generating electric current. A number of coils are fixed radially upon the outer circumference of a circle which rotates within a larger circle upon whose inner circumference are attached magnets, or they may interchange places and the magnets may rotate and the coils remain fixed. As the coils and the magnets sweep by each other at high speed, alternating currents are induced in the coils and are drawn off and utilized. Such machines are called *generators* and are explained in detail later on.

424. Cutting Lines of Force.—Electro-motive force is induced by varying the number of lines of force embraced by a circuit. A line of force is a closed curve. A circuit is also a closed figure. Therefore, like two links of a chain, in order that a line of force may be inserted into or withdrawn from a circuit, one or the other must be cut and it is usually the line of force. Hence, on account of the conciseness of the expression, it is convenient and customary to speak of the E. M. F. generated by "cutting lines of force." It must, however, be remembered that, as was shown in Par. 422, in speaking thus we mean by the number cut the number by which the original field embraced by the circuit has been increased or decreased, for when a circuit is moved parallel to itself across a uniform field, there are certainly lines cut, but since the original number embraced is unvaried, there is no E. M. F. induced.

425. Relation Between Rate of Cutting of Lines of Force and the Resulting E. M. F.—In Par. 416 it was shown that the more rapidly the field embraced by the coil is varied, the greater is the induced E. M. F. The relation between the induced E. M. F. and the rate of cutting of lines of force may be deduced as follows.

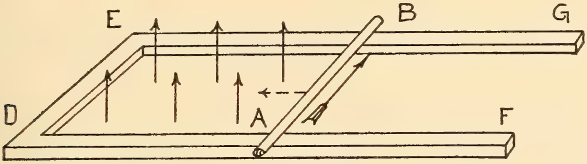


Fig. 200.

Let EG and DF, Fig. 200, represent two parallel metal rails connected across DE and embracing between them a uniform field whose positive direction is upward. Let AB be a wire resting across these rails. If this wire be slid along towards DE, there will be induced a current I which will flow around the enclosed rectangle in the direction $ABED$. If the movement of the wire and the resulting flow of current continue for a time dt, the total quantity of electricity which is moved around the circuit is $Q = I.dt$, whence $I = Q/dt$. If during this time the number of lines of force embraced by the rectangle be decreased by dN, the work done (which has resulted in moving these Q units around the circuit) is (Par. 358) $W = I.dN$.

Substituting in this the expression for I above, we have

$$W = Q \cdot \frac{dN}{dt}$$

The E. M. F. induced in the circuit being E, if the circuit be cut at any point there will be a difference of potential E between the opposite sides of the resulting gap. In Par. 72 it was shown that the difference of potential between two points is measured by the work expended in moving a *unit quantity* of electricity from one point to the other. Since, from the above, it required an expenditure of $Q.dN/dt$ ergs to move Q units through this difference of potential, to move one unit requires dN/dt ergs, hence

$$E = \frac{dN}{dt}$$

or *the induced E. M. F. varies directly with the rate of cutting of the lines of force.*

Had the coil consisted of n turns, the work done would have been $W=Q.n.dN/dt$ (Par. 358) and hence

$$E = n.\frac{dN}{dt}$$

or the induced E. M. F. also varies directly with the number of turns in the coil.

It is a simple matter to confirm experimentally the foregoing conclusions.

426. Absolute Electro-Magnetic Unit of E. M. F.—If a coil embraces N' lines of force and after an interval t embraces N'', the average E. M. F. generated is

$$E = \frac{N' - N''}{t}$$

If $N' - N''$ be positive, there has been a decrease in the number of lines embraced and the induced E. M. F. is positive or clockwise. If it be negative, the induced E. M. F. is negative or counter-clockwise.

If in the above expression $N' - N''$ be unity and t be one second, E becomes unity, whence *the absolute electro-magnetic unit of E. M. F. is defined as that E. M. F. induced by cutting one line of force per second.*

427. The Practical Unit of E. M. F., the Volt.—The absolute unit of E. M. F. is entirely too small for practical purposes, and even a unit corresponding to the E. M. F. produced by the cutting of one million lines per second is extremely small. In deciding upon a practical unit, the Paris Congress of Electricians in 1881 might have taken the E. M. F. produced by cutting one million, or ten million, or one hundred million, or even one billion lines of force per second, but in this selection they were probably guided by the following considerations. Before the adoption of a unit of E. M. F., the need for such a unit had been felt and it was quite the custom to take as an every-day standard of comparison the E. M. F. of a Daniell cell, the most constant cell then in general use. In the older books we frequently find E. M. F. specified in terms of that of so many Daniell cells. To disturb these conceptions as little as possible, the practical unit was selected as that one which most nearly approximated to the E. M. F. of a Daniell cell. The practical unit of electro-motive force, *the volt, is there-*

fore defined as the E. M. F. produced by cutting one hundred million (10^8) *lines of force per second.* The volt is therefore equal to 10^8 absolute units of E. M. F. The average E. M. F. of a Daniell cell is 1.07 volt (Par. 206).

If in Ohm's law, $I = E/R$, we substitute for I its value in absolute units $I \times 10^{-1}$, and for E its value $E \times 10^8$, we see that for R we must put $R \times 10^9$, therefore, the ohm is 10^9 absolute units of resistance.

428. Eddy Currents.—In the preceding paragraphs we have considered currents induced in coils when the flux embraced by the coils is varied. The phenomenon of induction is still more general and whatever the shape of a conductor, that is, whether it be a sphere, or a plate, or an irregular lump, currents are induced in it whenever there is an increase or decrease in the number of lines of force penetrating the body.

In 1824 Gambey observed that a compass needle set to oscillating above a sheet of copper came to rest much more quickly than when placed above a wooden board. This observation was investigated by Arago who made the additional discovery that a disc of copper rotated either above or below a needle produces a deflection of the needle in the direction of the rotation, and if rotated rapidly enough would cause the needle also to take up a motion of rotation. This experiment is noteworthy since the endeavor to account for the movement of the needle led Faraday to the discovery of induction as outlined in paragraphs 416 and 417 above.

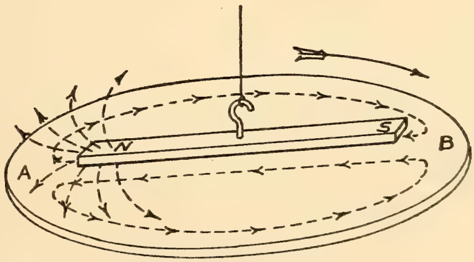


Fig. 201.

The movement of the needle may be explained as follows: NS, Fig. 201, represents a needle suspended above a copper disc which latter is caused to rotate in a clockwise direction. Consider at any one instant a strip AB along the diameter of the disc and

parallel to the needle above. The lines of force from the north end of the needle radiating in all directions, some of them penetrate the disc. The strip AB is therefore a conductor moving across a magnetic field and application to each half of AB of the right hand rule for direction of induced currents (Par. 422) shows that a current flows from B to A, returning by the right and left as shown by the broken lines. But, such a current will, in accordance with Oerstedt's rule (Par. 345), cause the north pole of the needle to move off in a clockwise direction.

Such induced currents flowing around through the mass of the conductor and returning upon themselves, are, from analogy with the circular whirls produced in running streams, called *eddy currents*.

Reflection will show that if the copper plate in the above experiment be suspended by a thread and the needle be rotated just below it, the plate will take up a motion of rotation in the same direction. On account of the feebleness of the needle, it is customary, in showing this fact experimentally, to employ an electromagnet. The principle involved in these experiments is applied in the induction motor, a machine to be described later.

429. Foucault's Experiments.—Foucault arranged a copper disc to rotate like a circular saw between the poles of an electro-magnet. When the current was off, the only energy required to rotate the disc was that to overcome the friction of its bearings, but as soon as the cores were magnetized, resistance to the turning was experienced. If, in spite of the resistance, the disc was forced to rotate, it rapidly grew hot. Foucault showed that this heating was due to the circulation of the eddy currents in the copper. If narrow radial slits were sawed in the disc, thus interrupting the paths of these circular currents, the resistance to turning and the accompanying heating effect disappeared. On account of these experiments, eddy currents are often spoken of as *Foucault's currents*, but the two names are synonymous.

In order to produce an electric current there must be an expenditure of energy. This heating effect therefore represents waste energy and is of much importance in any electrical apparatus in which the flux is frequently varied, such as electro-magnets, transformers and electric generators and motors, especially those employing alternating currents. To avoid this loss of energy, and also to avoid excessive heating, the cores of electro-magnets

are sometimes made of bundles of soft-iron wires, and the cores of transformers and of the field magnets and armatures of electric machines are laminated, or built up of many thin sheets of soft iron, the principle being that since the eddy currents flow in closed curves whose planes are perpendicular to the lines of force of the core, they may be checked if the cores be split up by planes parallel to the lines of force.

430. Lenz's Law.—If a copper cylinder be suspended by a thread so as to hang between the poles of an electro-magnet, and if this thread be twisted and then released, the cylinder by its weight will cause the thread to untwist and, if the current be turned off, will rotate rapidly. If now the current be turned on, the rotation will be instantly checked as if an invisible brake had been applied.

The principle involved in this phenomenon was given by Lenz in the form of a general law to the effect that *the currents induced by moving a conductor in a magnetic field are of such direction that their reaction tends to stop the movement which produces them.*

The following illustration will make this clear. Fig. 202 represents the same arrangement of two rails and a sliding wire as ex-

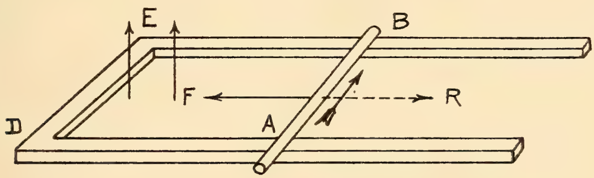


Fig. 202.

plained in Par. 425. If AB be pushed in the direction F, a current will be induced flowing from A to B (Par. 422). AB is therefore a conductor carrying a current and placed in a magnetic field. By Par. 352 it is acted upon by a force in the direction R, that is, diametrically opposed to F.

The foregoing affords the correct explanation for the electrical damping referred to in Par. 379.

431. Transformers.—It was shown in Par. 425 that the E. M. F. induced in a coil varies with the number of lines of force introduced or withdrawn in a given time. The flux produced within a coil varies with the permeability of the core. If a coil be wrapped

upon a soft iron core, a current flowing through this coil will produce many more lines of force within the coil than would be produced if the inner core were absent. The inductive effect is, therefore, very greatly increased by inserting in the coil an iron core.

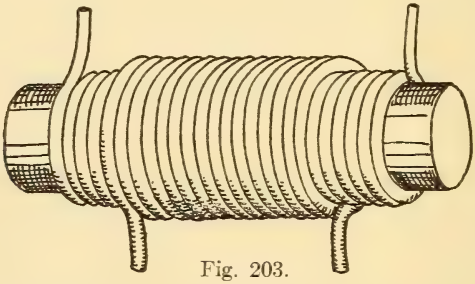


Fig. 203.

Fig. 203 represents an iron rod upon which is wrapped the primary coil and on top of this the secondary. It will be seen that any lines of force produced in the primary must of necessity be embraced by the secondary. The following consideration will show that this arrangement may be still further improved. The lines of force which emerge from one end of the iron core must pass through the air to enter the other end. This long air-gap in the magnetic circuit very materially reduces the total flux (Par. 401). It is therefore better to bend the iron rod into a ring, or similar closed figure, so that the entire paths of the lines of force will lie in iron.

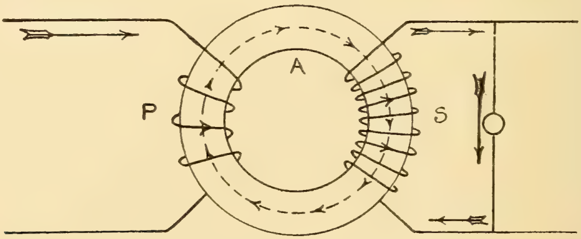


Fig. 204.

Faraday devised the arrangement shown in Fig. 204, a soft iron ring A, on one side of which is wrapped the primary P, and on the other side the secondary S. When a current I is sent through P as indicated, clockwise lines of force are produced in the iron core A. When these lines penetrate S, a current I' is induced, its

direction being as shown. If the current I produces N lines of force and if there are n turns in P, the work done in P is InN ergs (Par. 358). If there are n' turns in S, the work done as these N lines penetrate S is $I'n'N$ ergs. The work in the two coils being equal,

$$InN = I'n'N$$

and since in each coil this work is done in the time t, we may write

$$In\frac{N}{t} = I'n'\frac{N}{t}$$

But (Par. 425) nN/t is the E. M. F. in the primary and $n'N/t$ is that in the secondary. Representing these by E and E' respectively

$$IE = I'E'$$

or

$$I : I' = E' : E$$

that is, the currents are to each other inversely as the number of turns in the respective coils; the voltages are to each other directly as the number of these turns. In the secondary coil, the current and voltage vary reciprocally, that is, as one increases, the other decreases so that their product is constant. Should there be ten times more turns in the secondary than in the primary, the induced current in the secondary will be only one-tenth of that in the primary, but its voltage will be ten times greater.

Since either coil may be used as the primary, the other one being the secondary, it is possible with this arrangement to transform at will a changing current (i. e., one which is increasing or decreasing) into another whose voltage is either higher or lower than that of the original current. For this reason it is called a *transformer*, this particular one being known as *Faraday's ring transformer*. Those which increase the voltage are called *step up transformers;* those which lower it are called *step down transformers.*

The assumption above that the work in the secondary is equal to that in the primary is not strictly correct. There is always some magnetic leakage and some of the lines produced in the primary do not penetrate the secondary. Again, a part of the energy of the primary is wasted in producing eddy currents in the core and another portion in overcoming hysteresis (Par. 399). This waste, however, is reduced to a minimum by constructing

the core of thin punchings of soft iron of the shape shown in Fig. 205. This lamination of the core avoids eddy current losses (Par. 429); and the two coils being wrapped one above the other around the central portion and the magnetic circuit being complete to the right and left, the leakage is very small. In the best modern transformers, the total loss is less than three per cent.

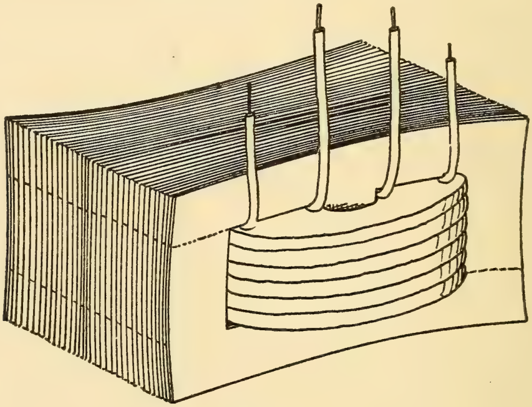


Fig. 205.

Since induction is an effect of changing currents only, transformers have no application to steady currents but find their most useful employment in connection with alternating currents. They will therefore be discussed further when that subject is reached.

432. Self-Induction.—The induction considered in the preceding pages and revealed by E. M. F. induced in one circuit by varying the field of another and neighboring current, is called *mutual induction*. Induction is, however, still more general and inductive effects are produced in a circuit by varying the field produced by the current flowing in the circuit itself. This phenomenon is called *self-induction*.

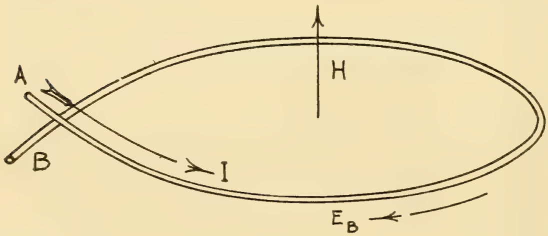


Fig. 206.

For example, if a current I be sent around the circular coil AB (Fig. 206), a field will be produced within this coil in the direction

H. But, we have seen (Par. 421) that if lines of force be thrust into this coil in the direction H, there will be induced an E. M. F. in the direction E_B, that is, opposed or counter to the original E. M. F. Therefore, the effect of self-induction is to oppose any increase in the current, and this explains why when a circuit is closed the current is retarded and does not instantly rise to its full value. It is also seen that if a current flowing in this circuit be decreased, the self-induction of the circuit delays this decrease and causes the current to linger, so that, in general, we may say that self-induction tends to prevent any change in the field embraced by a circuit and, consequently, in the current flowing in the circuit.

If a piece of soft iron be inserted in the coil AB, the strength of the field H is greatly increased (Par. 390). Hence, the induction of a circuit embracing a magnetic substance is very much greater than the induction of the circuit alone.

433. Measure of Self-Induction.—Since induction is common to all circuits and since, especially in dealing with alternating currents, it must frequently be taken into account, it is necessary that we should have some definite measure of this property and some concrete unit by which we may give concise expression to its value.

If we had to deal with circular coils, each of a single turn, we could use the term "induction" in its primitive significance of "crop of lines of force produced" (Par. 400), and could measure induction by the change in the number of lines embraced by the coil when the current was increased or decreased one unit. But this simple conception is complicated by the fact that the inductive effect varies with the geometric form of the circuit. For example, suppose that in a given circular coil of wire an increase of one unit in the current should increase by two the number of lines embraced. If the wire be now coiled into two smaller circles, but otherwise not changed, an increase of one unit in the current would again add two lines, but these two lines would penetrate each turn of the coil and the counter E. M. F. produced would be twice that produced in the original circuit. Finally, if the wire be folded at its middle point and then made into a coil (Par. 315) the unit current would again produce the two lines but they would be in opposite directions and hence (b, Par. 418) the resultant field would be zero and there would be no counter E. M. F. pro-

duced. It is agreed, therefore, to use the term "induction" in the sense of "cutting of lines of force." Thus, in the illustration above, if two lines be cut twice, the cutting is four, and in the last case the cutting is zero. From this point of view, therefore, the absolute unit of self-induction is the induction of that circuit in which a change of one absolute unit of current produces a cutting of one line of force. This unit has received no name. The practical unit of self-induction, however, is called the *henry* and is defined as *the induction of that circuit in which a change of one ampere in the current produces a cutting of one hundred million* (10^8) *lines of force.* The henry is therefore 10^9 absolute units of self-induction.

In the above definition, the question of time is not involved, that is, it is immaterial whether the change takes place rapidly or slowly.

434. Inductance.—The total cutting of lines of force caused by a change of one ampere in a circuit is called the *inductance* of the circuit and is represented by the symbol L. It follows that if the current change I amperes, the total cutting of lines of force will be $N = LI$. If this change takes place in t seconds, the average rate of cutting will be N/t or LI/t, which, as we have seen (Par. 425) is the counter or back E. M. F. produced in the circuit. This may be expressed thus

$$E_B = -LI/t$$

the negative sign indicating that the induced E. M. F. is opposed to the impressed E. M. F. In order that E_B should be expressed in volts, the above must be put in the form

$$E_B = -L \cdot \frac{I}{10^8 \times t}$$

If, however, as is usually the case, L be expressed in henrys (cutting of 10^8 lines), this reverts to the form

$$E_B = -LI/t$$

If in this last expression I be one ampere, t be one second and E_B be one (negative) volt, L becomes unity, whence we may say that *a circuit has an inductance of one henry if, when the current is varied at the rate of one ampere per second, an opposing E. M. F. of one volt is set up in the circuit.*

If the current does not vary at a uniform rate, the instantaneous value of the counter E. M. F. is

$$E_B = -L \cdot \frac{dI}{dt}$$

This is true for simple coils, since the field of a coil varies directly with the current, but it is not strictly true of coils with magnetic cores, because as these cores approach saturation the field ceases to vary directly with the current.

435. Expression for Inductance of a Coil.—An expression for the inductance of a coil may be deduced as follows: If a change of I amperes in the current flowing in the coil varies the field of the coil by ϕ lines of force, and if the coil consists of N turns, the total cutting is ϕN. If this takes place in t seconds, then

$$E_B = -\frac{\phi N}{10^8 \times t} \text{ volts}$$

But in the preceding paragraph we saw that

$$E_B = -LI/t \text{ volts}$$

L being the inductance of the circuit *in henrys.* Equating these expressions and striking out common factors

$$LI = \frac{\phi N}{10^8} \qquad \text{(I)}$$

In Par. 400 it was shown that the flux produced by a current of I amperes in a coil of N turns, l centimeters long and of r centimeters radius, wrapped upon an iron core of permeability μ is

$$\phi = \frac{4\pi N I \pi r^2 \mu}{10 . l}$$

This was deduced under the supposition that the core was ring-shaped, but it may without great error be applied to coils with straight cores. Substituting in (I) above and striking out common factors, we have

$$L = \frac{4\pi^2 N^2 r^2 \mu}{10^9 l}$$

L being the inductance of the coil in henrys.

Had the core been of air or other non-magnetic substance, μ in the above expression becomes unity.

436. Helmholtz's Equation.—If an E. M. F. E be impressed upon a circuit of resistance R, the current produced will, by Ohm's law, be E/R amperes. If, however, there be inductance in the circuit, a counter E. M. F. of $L.dI/dt$ volts will be produced (Par. 434). This, if acting alone in the circuit, would produce a current of

$$-\frac{L}{R}\cdot\frac{dI}{dt} \text{ amperes}$$

The current actually produced is therefore

$$I=\frac{E}{R}-\frac{L}{R}\cdot\frac{dI}{dt} \text{ amperes} \tag{I}$$

If E and L are constant, the variables in this expression are I and t. By transposing and dividing, (I) may be put in the form

$$\frac{R}{L}\cdot dt=\frac{dI}{\frac{E}{R}-I} \tag{II}$$

The integral of the first member is $\frac{Rt}{L}$. The integral of the second member is $-\log\left(\frac{E}{R}-I\right)+$ a constant, whence

$$\frac{R.t}{L}=-\log\left(\frac{E}{R}-I\right)+\text{a constant} \tag{III}$$

To find the value of the constant, place $t=0$. The first member becomes zero, and I disappears from the second member, for at the instant $t=0$, no current is flowing. The constant therefore $=\log\frac{E}{R}$

Substituting in (III) and changing signs throughout,

$$-\frac{R.t}{L}=\log\left(\frac{E}{R}-I\right)-\log\frac{E}{R}$$

$$=\log\left\{\frac{\frac{E}{R}-I}{\frac{E}{R}}\right\}$$

Whence
$$\frac{\frac{E}{R} - I}{\frac{E}{R}} = \epsilon^{-\frac{Rt}{L}}$$

ϵ being 2.7183, the base of the natural system of logarithms. Solving for I,

$$I = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$$

or

$$I = \frac{E}{R}\left(1 - \frac{1}{\epsilon^{\frac{Rt}{L}}}\right) \qquad \text{(IV)}$$

From this equation, first deduced by Helmholtz, we may determine the instantaneous value I of a current in a circuit of resistance R and inductance L at any time t after the circuit is closed. If the inductance of the circuit be very small, that is, if L be very small as compared to R, the second term in the parenthesis in (IV) disappears and the current rises almost instantly to its maximum value. If, however, the inductance be

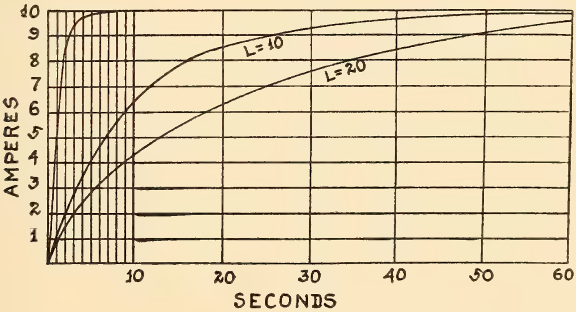


Fig. 207.

great, as in the case of the coils around a large electro-magnet, the rise of the current may be gradual. This is shown graphically in Fig. 207 in which the curves represent the growth of the current urged by an E. M. F. of ten volts through circuits of a resistance of one ohm and inductances of one, ten and twenty henrys, respectively. If the inductance be one-tenth of a henry, the current at the end of one second will have reached a value of 9.9996 amperes, while with an inductance of 20 henrys, this value is not reached in three *minutes*.

437. Induced E. M. F. at Make and at Break.—The E. M. F. induced when a circuit carrying a current is broken, is, on account of the great rapidity with which the lines are removed, much greater than that induced when the circuit is closed, or made. Interesting experiments have been devised to show this but the following considerations will show that they are hardly needed.

First, when the wires attached to the terminals of an ordinary dry cell are touched together, an E. M. F. is induced counter to the E. M. F. of the cell. Reflection will show that it must be less than the E. M. F. of the cell (that is, less than about 1.4 volts), for if it were greater, a reverse current would be sent through the cell, and if it were equal, no current would flow, both of which suppositions are absurd.

Second, when the wires are separated, the E. M. F. induced is many times greater than that of the cell, for it throws a spark across the gap which the E. M. F. of the cell itself could not do.

438. The Induction Coil.—In gasoline engines, the mixture of vapor and air, in the proper proportions to produce the most powerful explosion, is introduced in the cylinder and must be ignited just as the piston is at the proper point in its stroke. The ignition of this explosive mixture is generally brought about by an electric spark. We have seen (Par. 93) that to produce a spark across a gap of even one-hundredth of an inch requires at least 300 volts, and this is considerably increased by the pressure of the vapor in the cylinder. It would be impracticable to transport in an automobile a battery large enough to supply this voltage direct, but, by utilizing the principle of the transformer as applied in an *induction coil,* the necessary voltage may be obtained from two or three cells.

The induction coil, shown diagrammatically in Fig. 208, consists of a cylindrical core A (made of a bundle of soft iron wire so as to avoid eddy currents), upon which is wrapped the primary coil, a few turns of heavy wire, and on top of this, the secondary coil, usually many thousand turns of fine wire. In the large induction coil of the Military Academy, the primary consists of 208 feet of one-sixth inch copper wire and the secondary of 49.3 *miles* of wire, 1/133 of an inch in diameter. In the circuit of the primary there is a battery B of two or three cells, a key K, and an interrupter I, similar to the one described in Par. 410. The

ends of the secondary terminate in the adjustable spark gap *S*. If used for ignition purposes, the spark gap is located in a *spark plug* which is screwed into the cylinder of the engine.

The operation of the coil is as follows: When the key *K* is closed, a current flows through the primary circuit and establishes a field from right to left through the coil. The core *A* becomes magnetized and attracts the armature of the interrupter *I*, thereby breaking the circuit. The effect of breaking the circuit is to withdraw suddenly the flux through the core and this induces in the

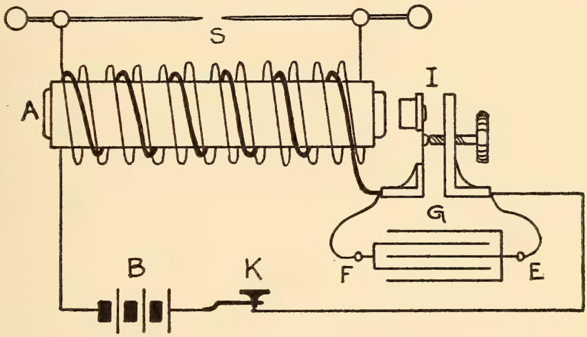


Fig. 208.

secondary a direct E. M. F. which (Par. 431) is as much greater than the E. M. F. of the primary as the number of turns in the secondary is greater than the number in the primary. In other words, the coil acts as a step up transformer. The voltage in the secondary is high enough to cause a rush of sparks across the gap *S*. When the circuit is restored at the interrupter, the current again flows through the primary and re-establishes the field in the coil, but the induced E. M. F. at *make* is much less than that at *break* (Par. 437), and sparks are not generally produced.

To cause the production of sparks when the piston is at the proper point in its stroke, the key *K* is closed by a revolving cam, a part of the engine.

It should be remarked that the invention of the induction coil antedates by many years the invention of internal combustion engines, and that these coils have other important uses besides that of ignition.

439. Use of Condenser.—The action of an induction coil is much improved by shunting across the break of the interrupter

a condenser, shown diagrammatically at G in Fig. 208. A correct explanation of its operation involves a discussion of *capacity*, as will be shown when the subject of alternate currents is reached. For the time being, however, the following explanation will suffice. As preliminary thereto, we assume that (a) the charge which may be given to a condenser varies with its capacity and with the difference of potential between its terminals (Par. 93), and (b) the induced E. M. F. at *break* is a hundred or more times greater than that at *make* (Par. 437).

At make, when the current is flowing across I, the amount of charge in G depends, from the above, upon the difference of potential between E and F. This is the IR drop from E through I to F, and is very small, consequently, the charge in G is small. At break, the self-induced current in the primary continues to flow in the same direction, therefore, the field in the coil is maintained for a brief interval. Moreover, the induced E. M. F. being great, and the circuit being broken at I, E is at a much higher potential than F, and a large charge flows into the condenser. At the next instant, the induced E. M. F. dies out, there is now no difference of potential to maintain the charge in G and the condenser discharges backward through E, K, B, to F. This discharge passes through the primary with great energy and opposite in direction to the original current. It therefore not only pushes out the flux which ran from right to left through the coil, but establishes a flux in the coil in the opposite direction, the cutting of lines of force, and hence the inductive effect, being much greater than that produced by simply breaking the circuit in the primary. At the next succeeding make, the current through the primary must rise slowly for, before it can establish a field in the core, it must push out the negative field already there. Therefore, the condenser suppresses any sparks at make and increases the intensity of the sparks at break.

440. The Bell Telephone.—A very important application of the principle of induction is the telephone. The original form, as invented by Graham Bell in 1876, is shown in section in Fig. 209. It consists of a cylindrical, hard-rubber case expanded at one end and containing a long bar-magnet M. Just in front of the pole of the magnet, but not in contact with it, is a diaphragm D of thin sheet iron, similar to that used for tintypes. Around the same pole of the magnet is wrapped a coil C whose free ends are

attached to the terminals *T*. Wires extend from these terminals to the other end of the line and are there attached to a second instrument, a duplicate of the first.

When sound waves strike upon the diaphragm, they set it in vibration and it alternately approaches and recedes from the magnet. As it approaches the magnet, the air gap between the two is reduced and, the diaphragm being of iron, additional lines of force extend from the magnet to it. As it recedes, the number of lines decreases. Since these lines pass through the coil *C*,

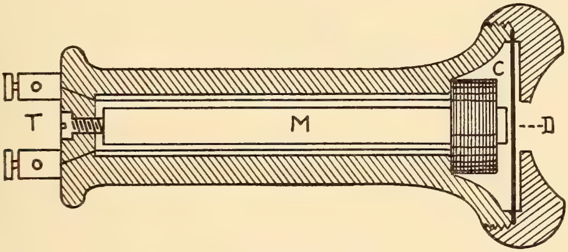


Fig. 209.

variations in their number set up induced currents in the coil, and hence in the circuit of which it forms a part. As these currents flow in one direction through the coil at the far end of the line, they increase the strength of the enclosed magnet and the diaphragm is drawn in. As they flow in the opposite direction, they weaken the magnet and the diaphragm springs back. The vibrations at the near end of the line are therefore reproduced at the far end, and this causes the sounds to be repeated. It is thus seen that the Bell telephone was originally intended to be used both as a transmitter and as a receiver. As a transmitter, it was used as a mouth-piece; as a receiver, it was held to the ear. In more recent receivers, instead of a simple bar-magnet as described above, a slender horseshoe magnet with soft iron pole pieces is used, but the principle is the same.

441. The Transmitter.—The E. M. F. induced by the vibration of the diaphragm of the Bell telephone is necessarily very small. The current which it can drive over a long line of considerable resistance is therefore very feeble, so feeble in fact as to restrict its use to short distances. This difficulty was first overcome by the Blake transmitter. More recent transmitters embody the same principle but are improved in details.

typical form is shown diagrammatically in section in Fig. 210. It consists externally of a metal case with a suitably shaped hard-rubber mouth piece. Within, there is a diaphragm, insulated from the case, and a cylindrical metal box. In the back of this box there is a carbon disc and in the front a second, the space between the two being packed with carbon granules. The front carbon disc is bolted to the diaphragm. The sides of the box are lined with insulating material. A wire connected to the diaphragm runs to a battery of several cells, whence the circuit is completed through the primary of a small induction coil (a step up transformer), thence through the metal frame supporting the transmitter back to the enclosed metal box, through the back carbon

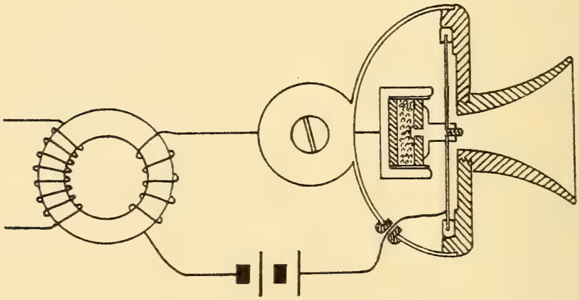


Fig. 210.

disc, through the carbon granules to the front carbon disc and thence to the diaphragm. There is an arrangement, shown in Fig. 211, by which this circuit is broken when the telephone is not in use. When the telephone is in operation, a current flows through the circuit but the resistance of the carbon granules is large and the actual amount of the current is small. When the diaphragm is set in vibration by the sound waves, it compresses the granulated carbon which, as we have seen (Par. 285), reduces the resistance of the carbon and allows a greater current to flow from the battery through the primary. The current through the circuit therefore varies with the sound waves and the voltage in the primary is stepped up by the transformer so that the resistance of the line, the secondary, may be overcome. The transmitter is seen to be somewhat analogous to the relay used in telegraphy (Par. 412).

442. Operation of Telephone.—From the foregoing, each telephone consists of a receiver, a transmitter, a transformer and a battery. It must include some device, usually a bell, by which calls may be received, and also some arrangement by which other stations may be called. Finally, when the telephone is not in use the circuit of the battery must be broken, otherwise the battery would soon run down.

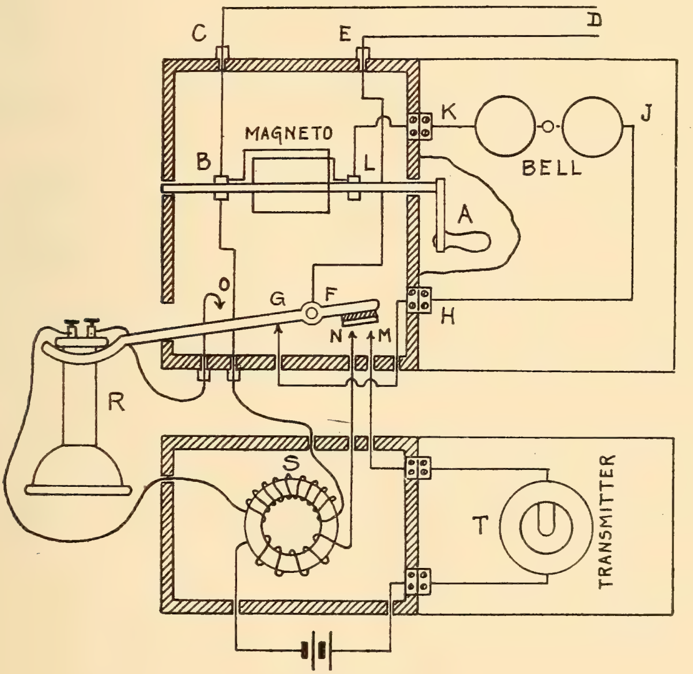


Fig. 211.

There are many telephone systems in use. Fig. 211 represents a common form, the hinged doors of the boxes being shown as swung to one side. Its operation is as follows:

(a) To call a station. With the receiver on the hook switch, as represented, the crank handle *A* of the magneto is turned. (The magneto is a small generator whose operation will be explained in Part V.) A current traverses the following path: *B-C-D-E-F-G-H-J-K-L*. At the second station *D*, the circuit is precisely the same and the bell rings at both stations.

(b) To receive a message. The receiver is removed from the hook and held to the ear. The hook, freed from the weight of the receiver, rises and breaks the circuit at G but closes it at M, N and O. The current coming in from D follows the route E-F-O-R-S-B-C.

(c) To send a message. The hook being up, the transmitter-battery-primary circuit is closed at NM. Currents through this circuit are stepped up in the secondary S and follow the route given above.

CHAPTER 34.

AMMETERS AND VOLTMETERS.

443. Electrical Quantities to be Measured.—The modern development of the science of electricity has been accompanied and greatly aided by the production of ever improving instruments of precision for the rapid and accurate measurement of certain electrical quantities. The principal of these quantities are:

1 Resistance,
2 Strength or intensity of current,
3 Electro-motive force,
4 Electrical power.

The measurement of resistance was explained in Chapter 26 and in the present chapter we are concerned with the measurement of current and of electro-motive force.

444. Electrical Effects Used in Measurements.—Electricity not being matter, and hence being imponderable and without physical dimensions, must be measured indirectly by its effects. These are usually classed under four heads, viz.:

1. *Thermal.*—A current flowing through a conductor heats it.

2. *Electro-magnetic.*—A current flowing through a conductor produces about it a magnetic field. (a) If flowing near a poised magnetic needle, the needle will be deflected, or, (b) if flowing around a soft iron core the latter will be magnetized.

3. *Electro-chemical.*—(a) A current flowing through acidulated water will decompose the same, releasing its component gases hydrogen and oxygen, or (b) flowing through a solution of a metallic salt will decompose the salt, depositing the metal upon the cathode or plate by which the current leaves.

4. *Physiological.*—A current flowing through a living or recently living body will produce certain effects such as muscular twitchings and contractions, and in a living being cause more or less painful sensations.

Of the above, the first three may be and are used in electrical measurements.

445. Effect Best Adapted for Measurement.—The effect best adapted for measurement may be arrived at by a consideration of the following experiments after Professor Ayrton. In Fig. 212

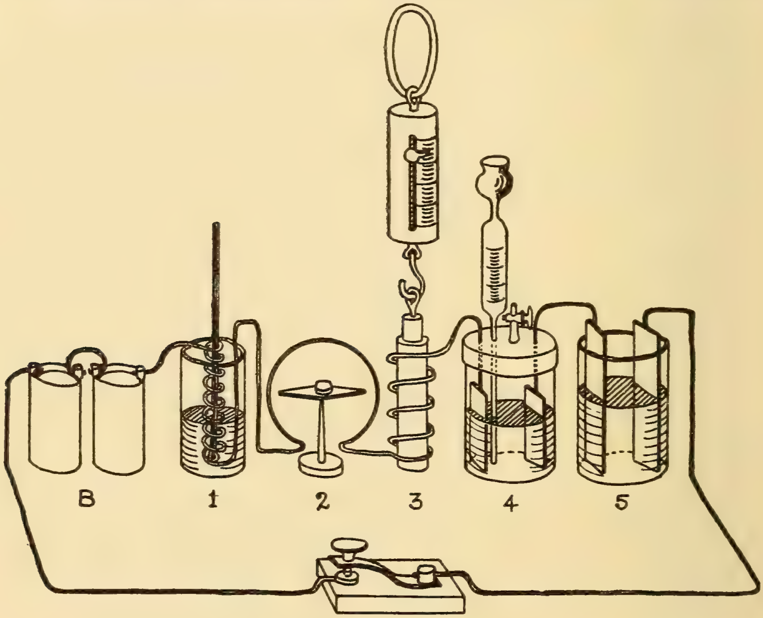


Fig. 212.

B represents a battery with which are connected in series the various pieces of apparatus 1, 2, 3, 4, and 5, through which therefore the same current flows.

1 is a thermometer around whose bulb the conducting wire is wrapped and which dips into some oil, a non-conductor of electricity.

2 is a magnetic needle in whose vertical plane and around whose pivot as a center the wire is bent in a circle.

3 is a soft iron core around which the wire is wrapped. On top of this core is a piece of soft iron fastened to the hook of a spring balance.

4 is a glass jar upon which is screwed an air-tight cover. Through this run the two wires, each terminating in a platinum plate

dipping into the acidulated water with which the jar is partly filled, and also a glass tube extending nearly to the bottom of the jar, its upper portion expanded and graduated as shown.

5 is a glass jar partly filled with a solution of copper sulphate into which dip two copper plates to which the wires are attached.

If now the key be closed and the current be allowed to flow for a short time, t, the following effects will be noted:

1. The thermometer will indicate a rise in temperature.

2. The needle will be deflected through a certain angle and will remain constantly at that angle as long as the current flows.

3. The soft iron core will become magnetized and will attract the iron block so that a force of x ounces must be exerted upon the spring balance to tear the block free.

4. Gas will be released at the surface of the two platinum plates in 4 and its pressure will force a certain number of cubic centimeters of the liquid up into the graduated tube.

5. The cathode copper plate in 5 will be found to have increased in weight due to the deposition of fresh copper upon its surface.

446. Second Experiment.—If, beginning under the original conditions of the preceding experiment, the key be closed an interval, t', say twice as long as the original t, the following will be observed:

1. The thermometer will indicate a temperature in general greater than that produced by the first experiment but bearing no definite relation to the same.

2. The needle will be deflected through the same angle as before.

3. The same pull will be required to release the soft iron block from the electro-magnet.

4. Twice the volume of gas will be released in 4.

5. Twice the weight of copper will be deposited on the cathode in 5.

Assuming that the current has been the same in these two experiments, we may conclude—

(a) That the temperature indicated by the thermometer in 1 varies in some indeterminate manner with the time and that consequently the heating effect is not suitable for measurement.

(b) That the electro-chemical effects vary directly with the time and hence if reduced to a common unit of time will give a definite measure.

(c) That the electro-magnetic effects are independent of time and give a direct measure without reduction.

447. Third Experiment.—A third experiment will throw further light upon this subject.

In Fig. 213 B represents, as before, a battery.

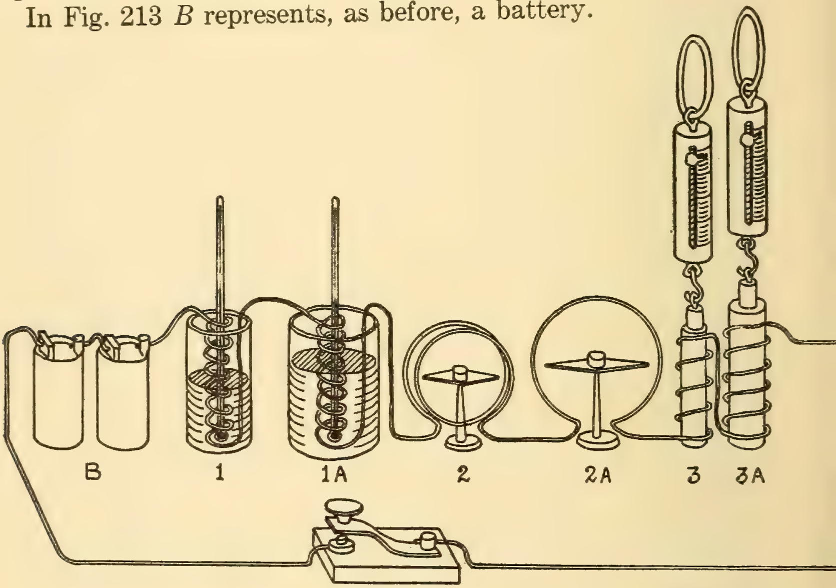


Fig. 213.

1 and 1A thermometers, as before, but 1A has more turns of the wire around its bulb than has 1 and they may be in different sized jars which contain different amounts of oil and perhaps different kinds of oil.

2 and 2A magnetic needles with circular coils in their vertical plane, the coil around 2 being of less diameter and of a greater number of turns than that around 2A.

3 and 3A electro-magnets differing in size and in the number of turns of the wire.

4, 4A and 4B gas voltameters, 4 being two in parallel, 4A a large one with plates far apart, 4B a small one with plates closer together.

5, $5A$ and $5B$ copper voltameters arranged similarly to the gas voltameters.

The key now being closed for an interval t, during which the same current flows through the entire system, the following will be observed:

1. The two thermometers will indicate a rise of temperature but the indications will not be the same and will bear no apparent relation to each other.

2. The magnetic needles will be deflected and will remain constantly deflected as long as the current flows but the angles will differ in the two cases and will bear no apparent relation to each other, except that the deflection is greater in the instrument with the greater number of turns.

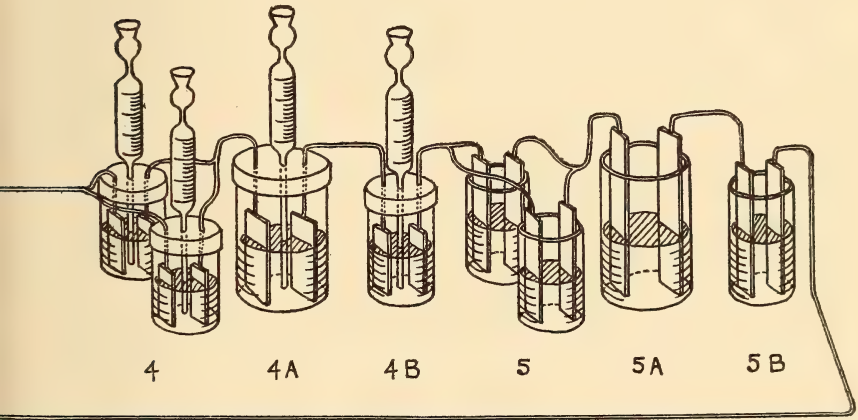


Fig. 213.

3. The electro-magnets will require pulls of x and y ounces respectively to separate the iron blocks but these pulls will bear no apparent relation to each other.

4. The amount of gas released in each of the two gas voltameters in series and the sum of the amounts released in the two in parallel will be exactly equal.

5. The amount of copper deposited in each of the two copper voltameters in series and the sum of the amounts deposited in the two in parallel will be exactly equal.

We conclude from the above:

(a) That the heating effect is unsuitable for measurement.

(b) That the electro-magnetic effect, while constant for the same current for any one instrument, is yet a function of the mechanical arrangement of the instrument and would be different for every different instrument.

(c) That the electro-chemical effect is, within wide limits, independent of the size, shape, and arrangement of the instruments.

448. Electro-Chemical Effect Selected as Standard.—As a logical consequence of the above, the electro-chemical effect has been selected as a standard for the measurement of electrical currents and the Act of Congress of July 12, 1894, legalized the resolution of the International Congress of Electricians of the preceding year and defined the *practical* unit of current, the ampere, as that unvarying current which flowing through an aqueous solution of nitrate of silver deposits silver at the rate of .001118 gram per second.

449. Why Silver Selected.—The current is defined in terms of silver deposited, partly because silver is one of the precious metals and when deposited from solution can be dried and weighed without appreciable error due to increase of weight by oxidation or other chemical change, but mainly because it combines high atomic weight (107.9) with monovalency while the next most suitable metal, copper, whose atomic weight is 63.6, is bivalent, so that a given current flowing for a given time will deposit nearly three and a half times as great a weight of silver as of copper. Silver is therefore used in delicate measurements of small currents but it is expensive and for large currents copper is employed.

450. Reason for Weight Selected.—It may naturally be asked why this particular weight of silver was selected instead of some even number, such as .001 gram for instance. The reply to this is that the absolute C. G. S. unit of current had already been defined, the definition being based upon electro-magnetic effects (Par. 355), and from many elaborate and accurate experiments the amount of silver deposited by the unit current, and hence the amount deposited by an ampere, had been determined.

451. Unsuitableness of Electro-Chemical Effect for Industrial Needs.—While, as shown above, the electro-chemical effect is selected as a standard, in its practical application to most in-

dustrial needs it has certain insuperable objections. The principal of these are (a) time consumed in a determination and hence inability to take instantaneous observations and (b) lack of sensitiveness and hence inability to measure small effects.

(a) For example, just as the steam engineer must without intermediate calculations be able to read his steam gauge at any moment, so the electrician should be able to read at any instant his voltage and current. The determination of a current by a voltameter observation is a laborious matter of hours, while what is needed is an instrument which can be read just like a steam gauge instantly and with a minimum expenditure of labor. To make another comparison, to use a voltameter is as if a person desiring to find out the time was compelled to take a set of astronomical observations and by tedious calculations arrive at his result. Naturally it is simpler, and in most cases preferable, to read from a clock even though it should be several minutes fast or slow.

(b) Again, the sensitiveness of a voltameter is not great and can hardly be increased. Many currents with which electricians have to deal are so small that they would have to flow for days before they would produce enough chemical effect to be susceptible of accurate measurement and even this supposes what is very doubtful, that is, that a current could be kept constant for that length of time.

452. Electro-Magnetic Effect Best for Practical Measurements.—As we saw in the account of the preliminary experiments in Pars. 445, 446, and 447, the magnetic needle in each case instantly took up a certain position and retained it as long as the current remained constant. This then is the basis of the majority of instruments in practical use.

453. Why not Selected as Standard.—The question now arises why then was not the electro-magnetic effect selected at the outset as the standard. The reply is that it is well-nigh impossible to construct two galvanometers which shall be duplicates, and it would be even more difficult to construct a duplicate following the specifications which such a definition would have involved. On the other hand, as we have seen above, the electro-chemical effect is, within wide limits, independent of instrumental size and shape and accurate measurements can be made with such appara-

tus as is found in any laboratory. An instrument maker could therefore accurately calibrate a galvanometer by the somewhat tedious voltameter method, as explained in the next paragraph, and thereafter use this calibrated galvanometer as a standard for the rapid calibration of others.

454. Calibration of Galvanometer.—The galvanometer to measure current is calibrated by connecting it in series with a voltameter, noting the point at which the needle stands, determining the current by means of the voltameter and marking the galvanometer scale to correspond, then repeating this, varying the current, and so on.

For small currents it is not possible to calibrate the galvanometer directly by this method, but since galvanometers follow the fixed law that the deflecting force is directly proportional to the number of turns in the coil, it may be calibrated as follows. It is first calibrated for large currents as explained above, with say only one turn in the coil. The soil is then re-wrapped with finer wire and say 100 turns are put on. A small current is now sent through the coil and produces a deflection which corresponds to i amperes in the original calibration. We know that the effect of the actual current has been multiplied 100 times by the number of turns, consequently the current is actually only $i/100$ amperes and the scale can be so marked, and so on.

The sensitiveness of a galvanometer can be increased to a very high degree. Ayrton states that it is possible to measure accurately with one a current so small that it would have to flow for a million years through a voltameter before it produced as much chemical action as a current of one ampere could produce in one hour.

455. Difference between Ammeters and Voltmeters.—The galvanometers used to measure current are called *Ammeters;* those to measure voltage are called *Voltmeters.* The moving parts of an ammeter and of a voltmeter, of the kind shortly to be described, are indistinguishable. They both move under the effect of the current which flows through them. Ohm's law can be written $E=RI$. As applied to a voltmeter or to an ammeter, R is the instrumental resistance and is constant, whence it is seen that the voltage is always some constant times the current through the instrument and it might be thought that one and the same

instrument could be used either as a voltmeter or as an ammeter. If its scale were graduated in amperes, the readings need only be multiplied by the constant R to convert them to volts, or there might perhaps be two parallel scales under the same needle, one reading amperes and the other volts. If, as will be shown later (see Par. 474), an additional piece of apparatus be employed, the foregoing conclusion is correct, but alone, ammeters and voltmeters are not interchangeable. The following explanation of their use will make it clear why they are not.

456. Essential of Measuring Instruments.—The first requirement of every measuring instrument is that when used it should not alter the quantity which it is to measure. Consequently, neither the ammeter nor the voltmeter when properly connected should change the resistance in the original circuit. Should this resistance be changed, the current will change in accordance with Ohm's law and this will also involve change in voltage. It is interesting to see how these two instruments fulfill this requirement by apparently diametrically opposite methods.

457. Ammeters.—An ammeter measures the current flowing in the circuit at the point at which it is connected. It is inserted in series in this circuit and should it have any appreciable resistance it would reduce the current, that is, change the quantity it is to measure. The resistance of an ammeter must therefore be so small that its effect on the current is negligible.

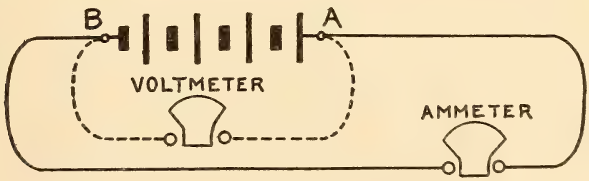


Fig. 214.

458. Voltmeters.—A voltmeter measures the difference of potential between the *two* points to which it is connected. These two points are never adjacent but in general are far apart electrically. For example, they may be the terminals of a battery (Fig. 214) or the brushes of a dynamo or the leads of an electric light circuit. Two cases may arise: (a) there may be a broken circuit between the two points, or (b) there may be between them

a closed circuit over which a current is flowing. In either case, in order that the original status of the circuit as regards current should be changed as little as possible, the resistance of the voltmeter must be great.

(a) If the circuit between the two points be broken, the resistance between them may be considered as infinite, and no current flows. When the voltmeter is inserted, therefore, its resistance must be so great that the current which flows through it is so small as to be negligible.

(b) If a current is flowing between the two points, in order that it may be inserted between them and yet not disturb the original circuit, the voltmeter must be connected in shunt. The voltmeter and the original circuit are therefore in parallel and constitute a divided circuit whose resistance is less than that of the original circuit (Par. 293). In order to alter the original resistance as little as possible the resistance of the voltmeter must be as great as possible. This statement hardly requires proof but may be shown mathematically as follows: let R be the resistance of the original circuit between the two points and x be the resistance of the voltmeter. The joint resistance is (Par. 293)

$$\frac{Rx}{R+x}$$

This may be written

$$R - \frac{R^2}{R+x}$$

whence it is seen that the joint resistance is less than the original resistance by the fraction $\frac{R^2}{R+x}$ and approaches the original resistance as this fraction approaches zero, which it does as x increases.

Practically, the resistance should not be made excessive for enough current must be let through the voltmeter to actuate the moving parts. The average resistance of a voltmeter reading up to 100 volts is about 15,000 ohms.

459. Summary.—To sum up—

(a) The moving parts of an ammeter and of a voltmeter are the same.

(b) An ammeter is always connected in series and its resistance should be as near zero as possible.

(c) A voltmeter between two points in a circuit carrying a current must always be connected in shunt and its resistance should be great, so great that the current through it is negligible.

460. Numerical Example, Voltmeter Between Two Points of a Circuit.—The following numerical example will bring out the effect of altering the resistance of a voltmeter.

Suppose we wish to measure with a voltmeter the difference of potential between AB, the terminals of the battery represented in Fig. 214. Suppose the E. M. F. of the battery to be 10 volts, the internal resistance to be 1 ohm, the external resistance 9 ohms. The current is $\frac{E}{R+r} = \frac{10}{9+1} = 1$ ampere. The internal drop is $Ir = 1 \times 1 = 1$ volt, hence the difference of potential between A and B is 9 volts. To measure this we connect up as shown. Suppose the resistance of the voltmeter to be 9 ohms. The joint resistance between A and B is now $9/2 = 4.5$ ohms, the current is $\frac{10}{4.5+1} = 1.8+$ amperes and the difference of potential between A and B is $4.5 \times 1.8 = 8.1$ volts or 0.9 less than it was before the voltmeter was connected up.

Suppose the resistance of the voltmeter to be 91 ohms. The external resistance becomes $\frac{9 \times 91}{9+91} = 8.19$ ohms and the current $1.08+$ amperes. The difference of potential between A and B is now $1.08 \times 8.19 = 8.85$ volts, or only .15 volt less than the original voltage.

Again, increase the resistance of the voltmeter to 991 ohms. The external resistance becomes 8.919, the current, 1.008 and the difference of potential between A and B, $1.008 \times 8.919 = 8.99$ volts, or only .01 less than the original voltage.

The scales of voltmeters, even of small range, are hardly ever graduated closer than to the nearest tenth and by estimate the position of the needle can be read to the nearest hundredth, therefore the above reading is within the limits of accuracy of the instrument. A further increase in the resistance would still further increase the theoretical accuracy. The resistance of the

usual voltmeter is considerably greater than the 991 ohms assumed above.

461. E. M. F. of a Cell or Battery.—Let Fig. 215 represent a cell or battery whose E. M. F. is E and whose internal resistance

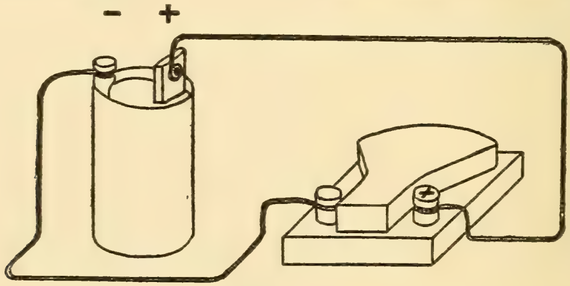


Fig. 215.

is r, and suppose it to be connected up with a voltmeter whose resistance is R. The current through the circuit is by Ohm's law

$$I = \frac{E}{R + r}$$

which obviously decreases as R increases. The above may be written

$$IR + Ir = E$$

whence
$$IR = E - Ir$$

But IR, the external drop, is what the voltmeter reads and this is always less by the quantity Ir, the internal drop, than E, the total E. M. F. However, this internal drop decreases as I gets smaller, and we have shown above that I gets smaller as R, the resistance of the voltmeter increases, therefore, the greater the resistance of the voltmeter, the more nearly the latter will read the E. M. F. of the cell or battery.

462. Classification of Ammeters and Voltmeters.—Ammeters and voltmeters may be classified in a number of ways.

1st, according to the kind of current for which they are intended as those for

(a) Direct Current,

(b) Alternating Current.

Some alternating current instruments may, by taking certain precautions, be used with direct currents but direct current instruments can not be used with alternating currents.

2d, according to the principle upon which they work, as
(a) Hot Wire Instruments,
(b) Moving Iron Instruments,
(c) Moving Coil Instruments,
(d) Induction Instruments.

3d, according to the controlling force, as those with
(a) Gravity Control,
(b) Spring Control,
(c) Magnetic Control.

Bifilar control, control by torsion and control by the earth's magnetism can not be used in these instruments and gravity control is unsuitable for the portable class.

4th, according to the manner of their use, as
(a) Portable,
(b) Switchboard.

5th, according to the arrangement of their scales, as
(a) Dial Instruments, those whose pointer moves over an arc of a circle like the face of a clock.
(b) Edgewise Instruments, the scale being on the surface of a cylinder which may be either horizontal or vertical, the pointer moving parallel to the elements of the cylinder. They occupy less space on the switchboard than the dial instruments.

Dial scales and horizontal edgewise scales usually have the zero on the left, but for some purposes it is of advantage to have the zero at the center although this shortens the available scale length by one-half. For instance, a zero center ammeter may be used to measure the current used in charging a storage battery and also the current given out in the opposite direction by the battery on discharge.

The number of kinds is so great that the mere enumeration of them would be voluminous, therefore description will be limited to certain typical forms in general use in this country.

463. Hot Wire Instruments.—In these the current flows through a long and thin platinum wire one end of which is fastened rigidly, the other directly or through a system of multiplying levers to a movable needle. The wire is drawn taut by a spring fastened to the needle. When a current flows through the wire it is heated

and expands. The slack is taken up by the spring and this causes the needle to move over the scale. Since the heating effect varies as the square of the current, the divisions on the scales of these instruments can not be evenly spaced. They may be used with both direct and alternating currents. They are not largely used.

464. Moving Iron Instruments.—These are also called "soft iron" and "gravity control" instruments, and are largely used abroad and to a less extent in this country. They may be used for both direct and alternating currents. There are many kinds, but the following will illustrate their principle. Fig. 216 represents an end view. *C* is a hollow cylindrical coil around which the current flows. *A* is the end of a bar of soft iron attached rigidly to the coil or to its frame, its length parallel to the axis of the cylinder. *B* is a second bar of soft iron parallel to the first and attached to the axis *D*, which is free to rotate. *P* is the pointer and *W* is an adjustable weight of non-magnetic metal, both attached to the axis *D*. The instrument can be used in but one position and when the weight *W* is properly adjusted the pointer *P* is on the zero of the scale. Suppose a current to flow around the coil; the bars *A* and *B* inside of the solenoid will both be magnetized with their north poles in the same direction. They will therefore repel each other, *B* will move off to the right, the pointer will sweep across the scale and the weight *W* will be lifted and oppose an increasing torque to the movement.

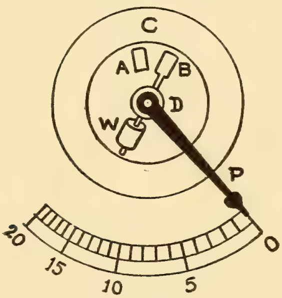


Fig. 216.

In a second class of these instruments the moving iron piece is drawn into a solenoid around which the current flows.

Like the preceding, the scales of these instruments can not be evenly spaced; moreover, they are liable to error due to residual magnetism in the soft iron bars and may give different readings for the same current depending upon whether the current has previously been increasing or decreasing. These disadvantages may more than compensate for the advantage of unvarying control.

465. Need of Ammeter Shunts.—We saw in Par. 457 that an ammeter is inserted in series in the circuit and should oppose no

resistance to the current. Some ammeters must measure very large currents, so large that the conductor must have a cross-section of a number of square inches. It is impracticable to construct an instrument whose coils should even approach such size, therefore the current is divided at the instrument and some very small but constant fraction is sent through the coil. This division is made by means of a shunt (Par. 301). For small portable instruments the shunt is within the case and such are said to be *self-contained.* For larger switchboard instruments the shunt is generally a separate piece of apparatus.

466. Switchboard Shunts.—These are also called "station shunts." They consist of two heavy copper terminals *A* and *B*, Fig. 217, which are connected by one or more strips or sheets *C* of

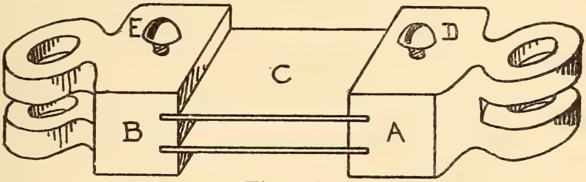


Fig. 217.

a special alloy of very small temperature coefficient. The strips are used, instead of one piece of the same cross-section, so as to offer more surface for cooling. On each terminal there is a binding screw *D* and *E* to which the leads to the instrument, flexible insulated wire cords six or eight feet long, are attached. Fig. 218 shows an ammeter and its shunt in position.

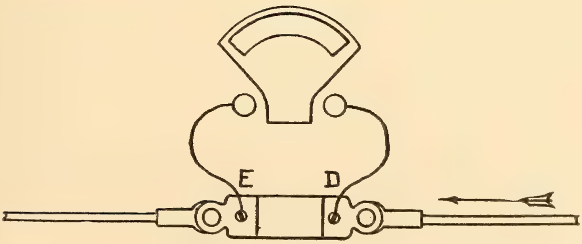


Fig. 218.

Suppose the resistance of a station shunt for an ammeter reading as high as 5000 amperes to be .00001 ohm; therefore, with full current the drop from *D* to *E* is .05 volt, and as the resistance of the instrument and its leads is .5 ohm, the maximum current

through it is 0.1 ampere. The resistance of the leads is taken into consideration in calibrating the instrument and they should on no account be altered by lengthening or shortening. They and the shunt are numbered to correspond to the instrument with which they are to be used and can not be used with any other. These leads confer a two-fold advantage; 1st, they permit of the position of the ammeter being shifted about at pleasure and without the expense caused by additional lengths of heavy copper mains or the trouble caused by the mechanical labor in bending and arranging these mains; 2d, the ammeter can be placed at such a distance from the mains that it is unaffected by the field produced around them by even very powerful currents.

467. The Weston D. C. Ammeter.—The Weston instruments are both in construction and accuracy among the best. In

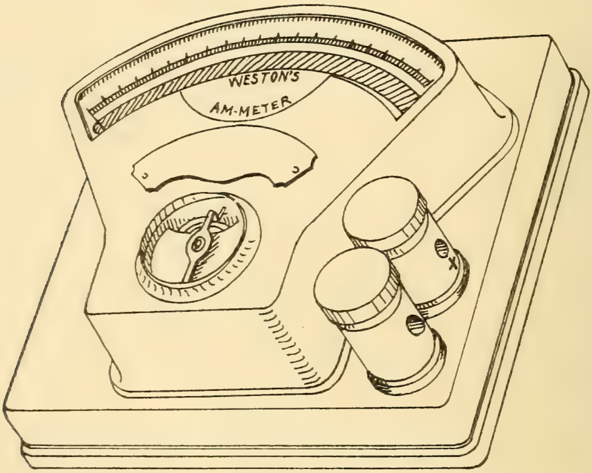


Fig. 219.

principle they are d'Arsonval galvanometers (Par. 378) with certain changes by which, while overcoming the structural weakness of the d'Arsonval instrument and making it fit for portable use, the requisite sensitiveness is retained. These changes are (a) substituting for the phosphor-bronze suspension filament suspension of the coil by pivots in watch jewels; (b) control by coiled hair springs instead of by torsion of the suspending filament; (c) use of a pointer of aluminum instead of reflection from a mirror; (d) accurate balancing of the coil, enabling the instru-

ment to be used in any position; (e) improved damping, making the instrument absolutely dead beat (Par. 379).

They are of many types. One of the usual forms of portable ammeter is represented in Fig. 219. Its case is of pressed brass or copper mounted upon a wooden base. In the larger switch-board instruments the case is of cast iron which has the advantage of shielding the instrumental field from perturbations due to external fields.

Within the case and nearly filling it is a permanent horseshoe magnet *M* (Fig. 220). To this are attached the soft iron pole pieces *N* and *S* which include between them a cylindrical opening. Were these pole pieces as represented in *a* in the following figure the greater part of the lines of force would cross the field at the

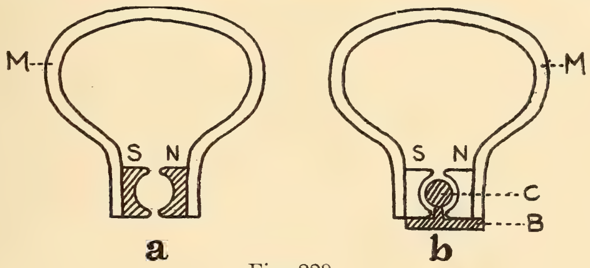


Fig. 220.

points where the horns of *N* and *S* approach each other most closely. The field would therefore be crowded at these points and thin at the intermediate points. However, as shown in *b*, a soft iron cylinder *C* bolted to a brass cross bar *B*, which is in turn bolted to the pole pieces, is fastened concentrically in the space between the pole pieces. The air gap between the cylinder and the pole pieces being very small and the permeability of the cylinder being large, the lines of force are evenly distributed and the field is very uniform (Par. 143). Pivoted in watch jewels so as to turn in this air gap is the rectangular coil. It is of very fine wire wrapped upon a light aluminum frame. Upon the axis of the coil are mounted from top to bottom the upper spiral spring, the aluminum needle, and below the coil the lower spiral spring coiled in opposite direction to the first. The needle, to combine lightness and stiffness, may be in cross-section either tubular or like an inverted V. The end which travels over the scale is, in portable instruments, compressed sidewise like a knife-blade and

in switchboard instruments terminates in an arrow-head. The rear end of the needle extends beyond the axis and carries an adjustable counterweight. There are also similar weights at right angles to the needle and by these the moving parts are so balanced that the instrument may be used in any position.

The binding posts by which the current enters and leaves may be placed, as shown in Fig. 219, both on one side, or may be both at the top or may be on opposite sides. For those instruments whose zero is at one end of the scale the post by which the current must enter is marked conspicuously + as shown in Figs. 219 and 222.

In portable instruments with self-contained shunt, the latter is a strip of alloy arranged similarly to the switchboard shunt described in Par. 466 above. The fraction of the current which flows through the coil flows first to the upper coiled spring, around this spring to its insulated hub, thence to the coil, around the coil and out by the lower coiled spring. Fig. 221 illustrates the actuating forces. The lines of force of the field run from N to S, the current flows up the right hand side of the coil and down the left, the lines of force of the coil run as shown by the short arrows. According to Maxwell's law the coil will therefore turn in a clockwise direction. Reflection will show that this could not be used with an alternating current.

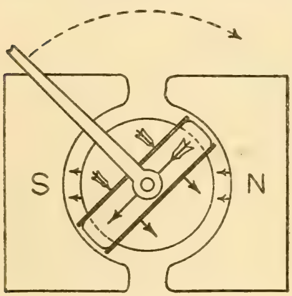


Fig. 221.

The field being very uniform and the resistance to torsion which the coiled spring offers increasing directly with the angle through which the coil turns, the scale is regularly spaced. Parallel to the scale and just beneath it is fastened an arc of a mirror. By covering the reflection of the needle in the mirror by the needle itself, the observer makes sure that the eye is always at the same angle with reference to the needle and to the plane of the scale and errors due to parallax are avoided.

The aluminum coil frame rotating in the strong magnetic field in the narrow air gap makes the instrument very dead beat. The damping effect varies as the square of the magnetic strength.

468. Weston Portable D. C. Voltmeter.—This instrument closely resembles the preceding. The one represented in Fig. 222

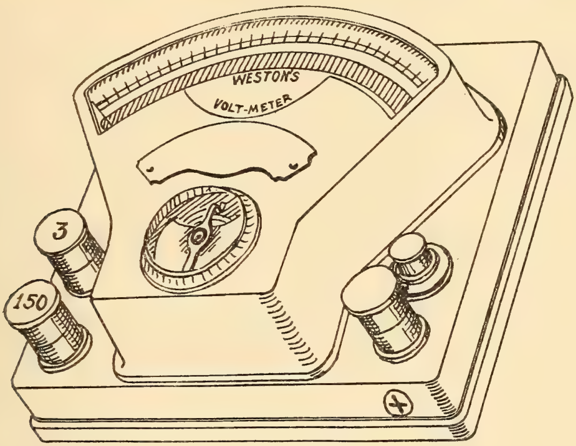


Fig. 222.

differs externally in having above the + binding post on the right a push-button switch by which the current through the instrument may be closed or broken at will, and on the left two binding posts by either of which the current may leave. The object of these is explained below. Internally it differs in having no shunt but a single circuit in which is a resistance coil. Suppose connection to be made with upper left hand binding post and circuit com-

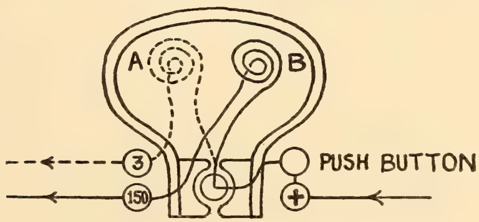


Fig. 223.

pleted. The current enters on the right (Fig. 223), through the button switch, thence through the rotating coil, thence through the resistance coil A and out. The resistance of the coil A, in the particular instrument represented in the figure, is so adjusted that a difference of potential between the terminals of the instrument of 3 volts will drive enough current through to carry the

needle entirely across the scale. The maximum reading is therefore 3 volts, the scale is graduated and numbered on the lower side accordingly, and the corresponding binding post is plainly marked 3.

If connection be made at the lower left hand binding post the current after leaving the moving coil passes through the resistance coil *B* and out. The resistance of *B* is so adjusted that the instrumental resistance is now 50 times greater than before, therefore a voltage 50 times greater, or 150 volts, would be required to carry the needle entirely across the scale. This binding post is therefore marked 150 and the upper side of the scale is numbered to correspond.

The two scales are usually selected so that the larger is ten or some multiple of ten times the smaller, therefore the graduation of the two scales is the same and it is only necessary to use two sets of numbers.

The resistance through the smaller coil of a 15-150 voltmeter of this class was found to be 1772 ohms, that through the larger coil 17,720 ohms.

These instruments are calibrated by comparison, usually through a potentiometer, with standard cells. The importance of accuracy in calibration will be realized when the statement is made that in electric lighting an increase of 3 per cent above the normal voltage shortens the useful life of a lamp one-half while a decrease of 4 per cent below normal reduces the candle power of the lamp one-fifth.

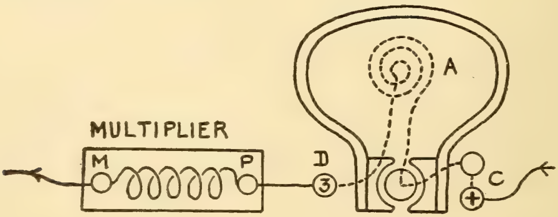


Fig. 224.

469. Multipliers.—The foregoing will enable us to understand an auxiliary piece of apparatus used with voltmeters and called a multiplier. If there be connected in series with a voltmeter, as shown in Fig. 224, a resistance *MP* which is so adjusted that the resistance between *C* and *M* is ten times what it is between

C and *D*, to produce a given deflection of the needle will require a difference of potential between *C* and *M* ten times greater than that between *C* and *D*. Hence to get the correct difference of potential between *C* and *M* the readings of the scale must be multiplied by ten. Therefore, a multiplier is a resistance which, when connected in series with a voltmeter, has the effect of multiplying the value of the scale divisions by a certain factor. This factor is usually marked upon the case of the multiplier.

Multipliers are not interchangeable but must be used with the particular voltmeter for which they are constructed. The second coil *B* in Fig. 223 is in effect a self-contained multiplier.

470. The Weston D. C. A. C. Voltmeter.—Consider Fig. 221 and suppose the current to be alternating. The direction of the

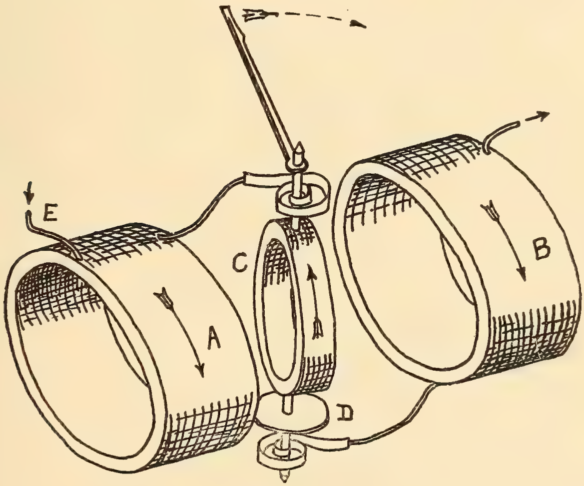


Fig. 225.

field due to the permanent magnet remains constant while that through the coil changes with change of direction of the current. Hence at one instant the needle would tend to turn in a clockwise direction and at the next instant in a counter-clockwise direction and if it moved at all would only quiver. Therefore, such instruments cannot be used with alternating currents.

The Weston D. C. A. C. voltmeter, to overcome this objection, employs the principle of the dynamometer (Par. 382). There is no permanent magnet but within the case and perpendicular to the middle of the scale arc there is a thin tubular brass frame

around which are wrapped many turns of fine wire. This cylinder is separated into two parts by a narrow gap in its middle (shown diagrammatically and much exaggerated in Fig. 225) and in this gap there turns a vertical axis which carries the needle, controlling spiral springs, movable coil, etc., as in the instruments already described. The movable coil *C* is circular instead of rectangular and normally its plane makes an angle of 45° with the axis of the cylinder. The current enters at *E*, flows around the coil *A*, thence to the upper spiral spring, then around the movable coil but opposite to its direction around *A*, thence to lower spiral spring, thence around coil *B* in same direction as around *A*, thence through a resistance coil and out.

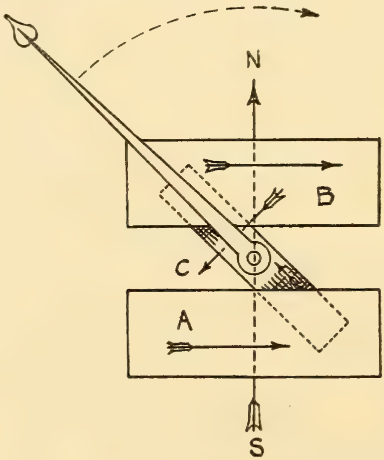


Fig. 226.

The current flowing as shown by the arrows in Fig. 226, the field of the fixed coils will be in the direction *SN*, that of the movable coil will be in the direction *C* and, in accordance with Maxwell's law, the needle will move in a clockwise direction. When the current reverses its direction both fields are also reversed and the tendency is still for the needle to turn in a clockwise direction, hence this instrument can be used for both alternating and direct currents.

When the movable coil has turned until it is at right angles to the outer coils the deflecting force is of maximum effect. The graduations of the center of the scale are therefore more widely spaced than those towards the extremities.

The movable coil turning in a weaker field than in the D. C. instruments, the damping effect is much less. To check the oscillations of the needle and bring it more quickly to rest, there is near the bottom of the coil shaft a circular plate D (Fig. 225) against which a light spring brake can be made to press.

471. The Thomson Inclined Coil Instruments.—These are primarily intended for alternating currents and in principle do not differ greatly from the one just described, that is there is a movable inner coil which rotates in the field of the fixed outer coil.

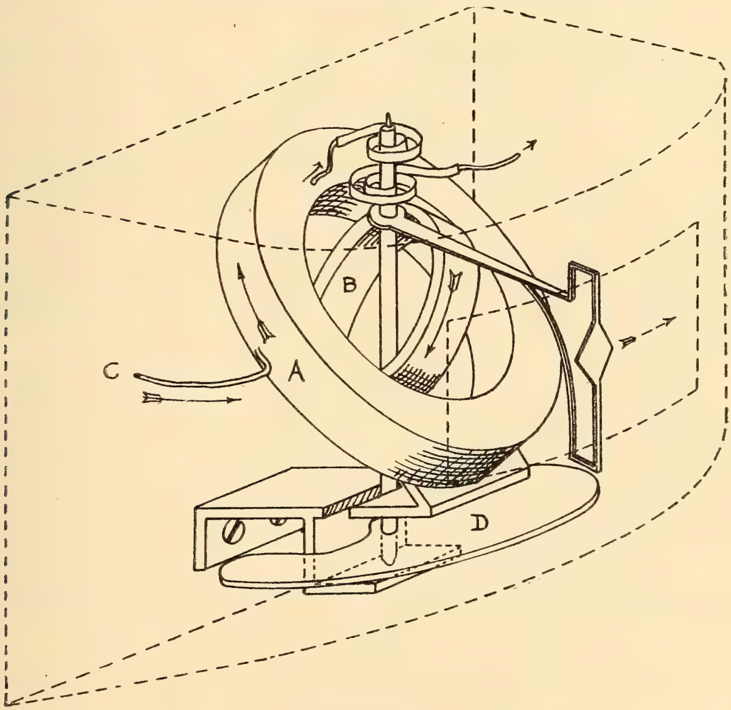


Fig. 227.

Figure 227 represents diagrammatically one of these instruments, a voltmeter with edgewise scale. The fixed coil A makes an angle of 45° with the horizontal base of the instrument. Rotating vertically through the center of this coil is the shaft which carries the two non-magnetic (phosphor bronze) spiral control springs, the needle, the movable coil B and a crescent-shaped aluminum disc D. The plane of the rotating coil makes an angle

of 45° with the base of the instrument and is also placed askew to the plane of the fixed coil. The current enters at C, flows around A in the direction shown by the arrow, thence to the upper spiral spring, thence around the coil B in the direction shown, thence to the second spiral spring and out through a resistance coil. According to Maxwell's law, the rotating coil tends to turn until its axis is parallel to that of the fixed coil and the needle travels across the scale to the right.

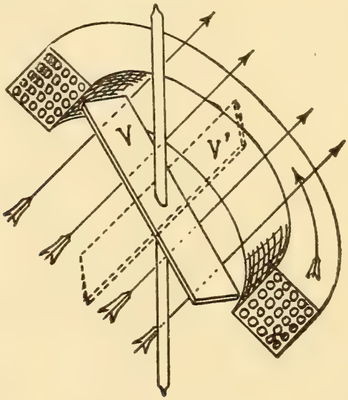


Fig. 228.

The inclined coil ammeters differ from the voltmeters just described in having no rotating coil but in its place a vane or flat sheet of soft iron V (Fig. 228) mounted upon the axis at the same angle as that made by the axis of the rotating coil in the voltmeter. When a current flows through the fixed coil, the vane tends to turn to the position V' parallel to the lines of force through the fixed coil (Par. 143).

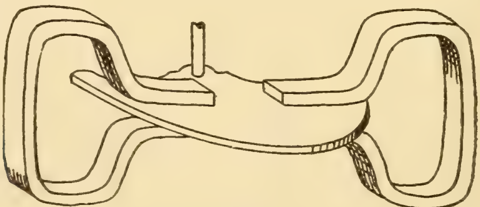


Fig. 229.

In the switchboard instruments of this type, damping is effected by the aluminum crescent D in Fig. 227 turning between the jaws

of two jew's-harp shaped permanent horseshoe magnets as shown in Fig. 229. In the portable instruments a friction brake or air vane is used.

472. Use of Transformers with A. C. Instruments.—Alternating current ammeters, due to the effects of self-induction in the coils,

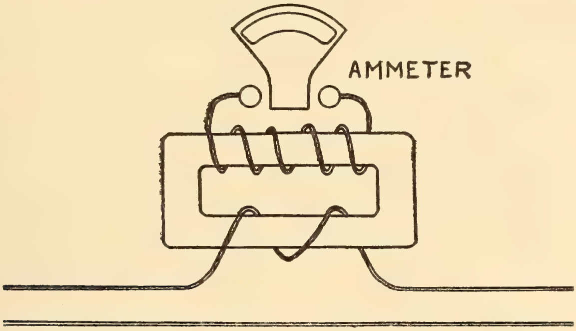


Fig. 230.

do not work satisfactorily with shunts and if the current to be measured is of such size that in a D. C. instrument a shunt would be used, the current through the ammeter is stepped down by means of a series transformer as shown in Fig. 230.

On the other hand, if the pressure in an alternating current circuit exceeds about 1000 volts, it is not considered safe to bring this

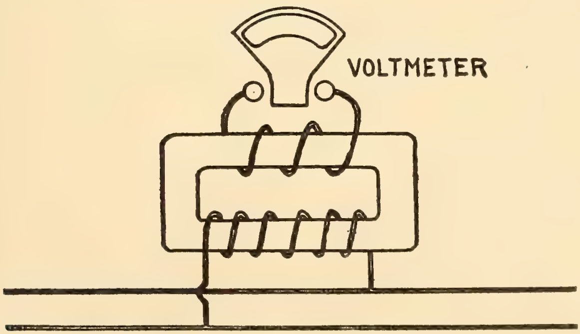


Fig. 231.

voltage direct to a voltmeter and it is stepped down by a potential transformer as shown in Fig. 231. These instruments are, of course, graduated to read the current or the voltage in the primary circuit.

473. Millivoltmeters.—If there be constructed an instrument like the voltmeter described in Par. 468 but of very much less internal resistance (10 instead of 1700 ohms) a slight difference of potential between its terminals will drive enough current through the coil to move the needle over an extended portion of the scale. The scale can therefore be graduated to show much smaller fractions of a volt than is possible in an ordinary voltmeter. Such an instrument reading to thousandths of a volt is called a millivoltmeter.

474. Millivoltmeters as Ammeters.—Suppose a millivoltmeter to be connected to the extremities of a shunt AB as shown in Fig. 232. Suppose it has a scale reading to 300 millivolts and that its resistance, including that of the leads which accompany it, is

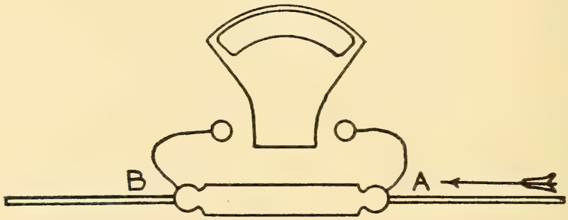


Fig. 232.

10 ohms. A difference of potential between AB of three-tenths of a volt will throw the needle entirely across the scale. In this case the current through the instrument is from Ohm's law $\frac{E}{R} = \frac{.3}{10} = .03$ ampere. Suppose a current of 300 amperes to be flowing in the main circuit. At A it divides inversely proportional to the resistances of the shunt AB and of the instrument and its leads. If the resistance of AB be made $\frac{10}{9999}$ ohm, then 299.97 amperes will flow through AB and .03 ampere through the instrument and the needle will move entirely across the scale. The divisions on the scale will therefore correspond to the amperes in the main circuit.

If the resistance of AB be made $\frac{10}{999}$ ohm, then 30 amperes in the main circuit will cause the needle to move entirely across the scale and the scale divisions will each correspond to one-tenth of an ampere.

Finally, if the resistance of AB be made $\frac{1}{99}$ ohm, the scale divisions will correspond to one-hundredth of an ampere in the main circuit.

It is therefore possible, by employing a shunt, to use a millivoltmeter as an ammeter.

475. Millivoltmeter Shunt.—Instead of separate shunts as described above, several are usually assembled in one case as represented in Fig. 233. The current to be measured is always brought in at the upper right hand post and leaves by one of the others in the upper row. The millivoltmeter is connected with the corresponding posts in the lower row. The circuits are as shown in the figure which represents connections made to read a current of a maximum of 1.5 amperes. The current in the case represented enters at A and leaves at B. AC is a heavy copper bar. D, D', D'' represent diagrammatically strips of resistance alloy. The numbers on the binding posts indicate the number of amperes to produce a total scale deflection of the needle when connection is made at the corresponding post. These shunts and their leads must be used with the particular instrument for which they are constructed.

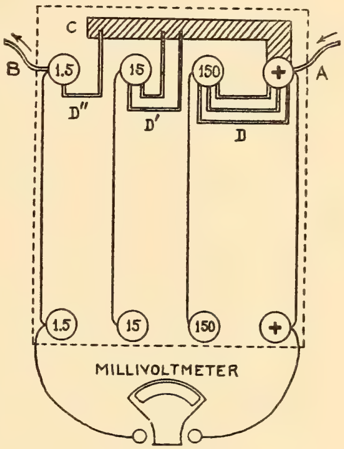


Fig. 233.

CHAPTER 35.

HEATING EFFECT OF ELECTRIC CURRENT.

476. Work Done by Electric Current.—To produce an electric current, an expenditure of energy or a performance of work is required. According to the fundamental principle of mechanics, this energy is not lost but only transmuted and must be given back in one form or another by the current. In a cell, for instance, there is an expenditure of chemical energy which results in moving Q units of electricity through a difference of potential V. The work done is therefore $W = QV$ (Par. 72). Since there is no current unless there be a complete circuit, each of these Q units of electricity must return to its starting point and in doing so passes back through the same difference of potential through which it was moved, or gives back the energy expended upon it in the first place. A current flowing in a circuit must, therefore, perform work of some kind. (a) In Par. 215 we saw that a current always heats the conductor through which it flows. (b) It may, in addition, perform electrolytic (chemical) work, or (c) it may, through the medium of machinery, do mechanical work, or finally, (d) it may do magnetic work. Energy is also expended by the current in establishing a magnetic field about the conductor, but this energy need not be considered for it is restored when the circuit is broken. If the current performs neither chemical, nor mechanical, nor magnetic work, then its entire energy is spent in heating the circuit.

We shall now examine into this heating effect of the current.

477. Determination of Laws of Heating Effect.—An experimental determination of the laws governing the heating effect of a current was made by Joule with an apparatus similar to that shown in Fig. 234. Through the cork of a wide-mouthed glass jar containing turpentine, or some similar non-conducting liquid, were run two heavy wires and a thermometer, T, all of which dipped below the surface of the liquid. Between the ends A and B of the large wires, there was connected a slender bare wire of

known resistance, preferably of manganin (Par. 289). The jar was then connected in series with a battery, a key and an ammeter. Upon closing the key, the current flowed through the circuit and heated the small wire, which, in turn, heated the turpentine. The strength of the current was read from the ammeter. The increase in temperature of the turpentine was determined by the thermometer, whence, knowing its weight and its specific heat, the number of heat units gained could be determined. The length of time that the current flowed was also measured. As a result of this

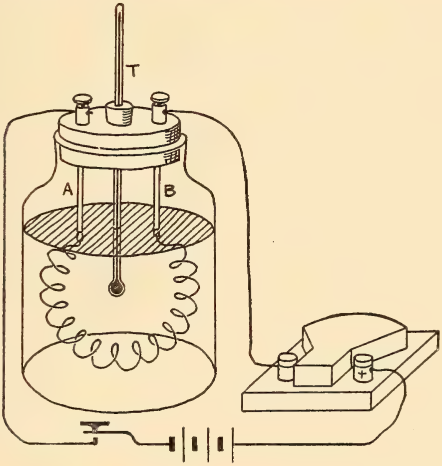


Fig. 234.

experiment, Joule found that the amount of heat produced varied (a) as the square of the current, (b) as the resistance of the conductor and (c) as the length of time during which the current flowed.

478. The Joule.—Representing by H the quantity of heat produced, Joule's results may be given mathematical expression as follows:

$$H = I^2Rt$$

If in this expression I be one ampere, R be one ohm, and t be one second, we have $H = 1$. This electric unit of heat, the quantity of heat produced by a current of one ampere flowing for one second through a resistance of one ohm, has been named *the joule*. It is, however, a redundant unit since we already have in the C. G. S. system the *small calorie*, the amount of heat required to

raise one gram of water through one degree Centigrade (Par. 11). The joule is a shade less than one-quarter of a calorie. One joule is .24 of a calorie and hence one calorie is 4.2 joules. If, therefore, H represents the number of calories produced, Joule's law becomes

$$H = I^2Rt \times 0.24$$

479. Theoretical Deduction of Joule's Law.—Joule's law, as given in the preceding paragraph, may also be deduced from theoretical considerations. Thus, suppose a current of strength I is flowing through a simple conductor whose resistance is R. The difference of potential between the ends of this conductor is IR (Par. 298), and is measured by the work done in moving a unit quantity of electricity from one point to the other (Par. 72). If the current flows for t seconds, the total quantity conveyed between the two points is $Q = It$, therefore, the total work done is $IR \times It$, or I^2Rt, this energy being spent solely in heating the conductor. To reduce this to ergs, I and R must be expressed in absolute units. Since one ampere $= 10^{-1}$ absolute units of current and one ohm $= 10^9$ absolute units of resistance (Par. 427), we have the total energy expended $= I^2Rt \times 10^7$ ergs. In Par. 11 it was shown that the small calorie is equivalent to 4.2×10^7 ergs. To reduce the above expression to calories, we must, therefore, divide it by this number, hence

$$H = (I^2Rt \times 10^7) \div (4.2 \times 10^7) = I^2Rt/4.2 = I^2Rt \times 0.24$$

which is the same as the expression in the preceding paragraph.

480. Electric Heating of Wires.—When a current flows through a wire, the wire is heated. The heat generated in the wire is conveyed away, mainly by radiation and convection. The rate at which this heat is dissipated increases as the temperature of the wire exceeds that of the surrounding medium. The temperature of the wire continues to rise until the loss of heat by radiation, etc., exactly balances the amount generated by the current. Reflection will show that since the heated air in a room ascends, a wire upon the ceiling will radiate its heat more slowly than if lower down, also, since the insulation upon a wire hinders the escape of the heat, the temperature of an insulated wire carrying a current will exceed that of the same sized bare wire. If the escape of heat be still further impeded by enclosing the wire in a wooden moulding, as is sometimes done, its temperature may reach a point where

the insulation becomes charred or even where the woodwork is set on fire. For this reason, most insurance companies forbid the use of these wooden ceiling strips and specify that wiring must either be exposed or enclosed in non-combustible conduits, and must be so proportioned that its temperature shall never exceed a certain allowable maximum.

481. Calculation of Temperature.—The dissipation of heat by a wire varies with the material of which the wire is composed and with the nature of its surface, with the extent of this surface, with the excess of its temperature over that of the surrounding medium and with the nature of this medium. If, when its temperature is 1° C above that of the surrounding medium it emits e calories per second per square centimeter of surface, it will emit Te calories per square centimeter per second when its temperature is T° C above. If its length be l centimeters and its diameter be d centimeters, its surface is πdl square centimeters and its emission per second is $Te\pi dl$ calories. During this time, the calories generated per second by the current are $I^2R \times 0.24$, hence when the temperature becomes constant,

$$Te\pi dl = I^2R \times 0.24$$

Substituting for R its value (Par. 285)

$$\frac{l.\rho}{\frac{1}{4}\pi d^2}$$

and solving for I, we have

$$I = d^{\frac{3}{2}}\left(\frac{T.e.\pi^2}{.96 \times \rho}\right)^{\frac{1}{2}}$$

Applying this to wires of the same material, ρ is constant, and if the wires attain the same temperature, T is constant, hence, the current to raise these wires to the same temperature varies as the square root of the cube of the diameter of the wires. This formula enables us to calculate the size of the fuse wires (Par. 306) which will melt when the current reaches a certain maximum. If the fuse wire be of tin, its specific resistance ρ is 13×10^{-6} ohms, and e is about .00025. Its melting point being 230° C, T is 230 minus the temperature (Centigrade) of the surrounding air.

482. Localizing the Heating Effect of a Current.—If a current passes through two portions of a circuit, each of the same resistance, the amount of heat developed in each will be the same. If

one of these portions be a large wire, several hundred yards long, and the other be a small wire, only a few inches in length, the heat will still be the same in amount but in the case of the large wire it will be distributed over its entire length and, on account of the great radiating surface, there will be no perceptible rise in temperature. On the other hand, the heat is concentrated in the small wire, which can not dispose of it by radiation, and the temperature of the small wire therefore rises. Such is the principle upon which the employment of electricity for heating and for lighting is based. The current is brought to the required spot through wires of but little resistance and is then passed through a short length of high resistance, the development of heat being thereby localized. If the portion of the circuit is to be heated to incandescence, as for example the filament in an incandescent lamp, its length must be short and its resistance high. If it is merely to be warmed, its length must be greater and its resistance less. The following examples will make this clear.

483. Electric Fuzes.—Electric fuzes are of many kinds. Fig. 235 represents in section an ordinary blasting fuze, which is also variously designated as a primer, a cap, or a detonator. It consists of a copper case *A*, which contains the explosive, usually

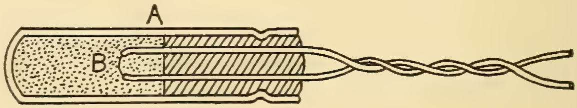


Fig. 235.

mercuric fulminate, and which is closed by a plug of wood, or wax, or sulphur or some similar cementing material. Through this plug pass the lead wires which come of various lengths to suit the depth of the drill holes in which the blast is to be fired. The inner ends of the lead wires are connected by a fine platinum "bridge" *B*, about .001 inch in diameter and one quarter of an inch long. About this bridge there is usually wrapped a wisp of gun-cotton. The passage of the current heats the platinum bridge and ignites the gun-cotton; this, in turn, ignites the fulminate and causes the main charge to explode. These fuzes afford the simplest and safest method of firing high explosives, and the only certain method of blasting under water and of causing a number of charges to explode simultaneously. They are fired from a safe distance,

the current usually being supplied from a hand magneto, although it may be furnished by a battery or taken from any other convenient source. In the military service, they are used to explode submarine mines and to fire heavy artillery. For this latter use they are charged with black powder instead of with the fulminate.

484. Electric Welding.—If two metal bars connected to the terminals of a generator be touched together, the current flowing through the resistance along the surface of contact will cause the local production of great heat. Such is the principle of the electric welding process devised by Elihu Thomson. The bars to be welded are brought together, the necessary current is turned on and in a very short time the metal softens. If now the bars be pressed together, a weld results. In this manner steel axles two inches square are joined in a little over a minute and a half.

Alternating current is used almost exclusively. It was shown above (Par. 477) that the heating effect varies as the *square* of the current. By a simple step down transformer (Par. 431) an alternating current may be transformed into another whose voltage is low but whose amperage is great. Thus, in welding the rails of trolley lines, the current is taken from the line itself but is transformed down, currents as great as 25,000 amperes being employed, and the rails in the mean time being squeezed together with a pressure of thirty-five tons.

485. The Electric Arc.—If the wires attached to the terminals of a battery or of a generator be touched together, completing the circuit, there will be a rush of current which, on account of the localized resistance, will, as we have just seen (Par. 484), develop great heat at the point of contact. If the wires be now separated about an eighth of an inch and if the E. M. F. between them be not less than about forty volts, the current will continue to flow, being conveyed by the vapor of the metal volatilized by the intense heat, and brilliant light will be emitted by the glowing ends of the wires and by the incandescent vapor between. This will continue for only a few seconds for the ends of the wires will rapidly melt off. If the terminal wires be attached to carbon rods which are then touched together and separated, the same brilliant light will be produced but in this case it will last much longer since the carbon is infusible. The flame, or rather the stream of incandescent vapor between the carbons, is really a

flexible conductor composed of volatilized carbon and has the properties of any other conductor carrying a current. For instance, it is surrounded by a magnetic field of its own and if placed in another magnetic field will tend to move off to one side (Par. 356). Because of the interaction of its field with that of the earth, it is generally somewhat curved, and on this account it was named the *electric arc*. If the field be strong enough, the arc may be pushed so far to one side as to be extinguished. A form of apparatus utilizing this principle to prevent accidental arcs when switches are opened is called a "magnetic blow-out."

As long as the arc is maintained, the carbons consume away slowly but at different rates, the positive carbon wasting much more rapidly than the negative. The tip of the positive carbon becomes hollowed out into a little pit, called the *crater;* on the other hand, the tip of the negative carbon seems to receive the particles torn away from the positive carbon and assumes a rather pointed outline.

The chief source of the light of the arc is the crater of the positive carbon, the arc itself emitting but little. The maximum temperature, however, exists in the arc. This temperature, the highest yet attained, is said to be about 3500° C, or twice that required to fuse platinum. In this arc the most infusible substances are promptly melted and even vaporized.

486. The Electric Furnace.—Although the light and the intense heat produced by the electric arc have been known for over one hundred years, it was not until the development within the last thirty years of machines for supplying continuously the required current that the use of the arc for illuminating purposes became commercially practicable, and its utilization on a large scale as a source of heat dates from the still more recent development of such great sources of power as Niagara Falls.

Electric furnaces may be divided into two general classes according as the body to be heated is or is not a conductor. If it be not a conductor, it must either be placed beneath the arc, the heat of which is both radiated and reflected down upon it, or it must be intimately mixed with powdered carbon or other conducting substance, the passage of the current through which produces enough heat to raise the temperature of the body to the required point. If it be a conductor, it may be made one electrode of an immense arc and be heated both by the heat radiated from the

remaining electrode and by that produced by the passage of the current, or it may be heated by the passage of the current alone. Of this last class there are two subdivisions according as the current is conveyed directly through the body or is produced in it by induction.

The intense heat of the electric furnace, 3500° C or more, has made it possible to fuse silica and to produce therefrom utensils of great use in the laboratory; has permitted the reduction of the most refractory ores, notably those of aluminum; has enabled the chemist to manufacture graphite, silicon, etc.; and finally has led to the production of chemical compounds hitherto unknown.

487. Moissan's Furnace.—One of the earliest electric furnaces was that devised by Moissan. It is shown in section in Fig. 236

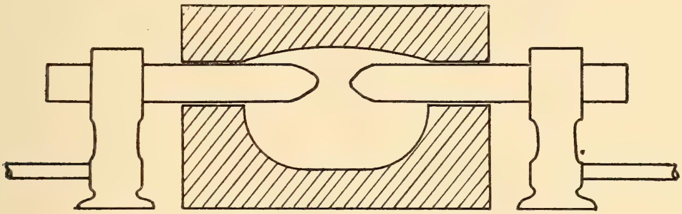


Fig. 236.

and consists of a chamber scooped in a block of lime and covered by a lid made from a second block. Lime is used since when either hot or cold it does not conduct appreciably. The carbons enter through grooves on opposite sides. The body to be heated is placed in the cavity in the lower block and the heat produced by the arc is reflected down upon it, that is, the furnace is in principle a reverberatory furnace.

Furnaces of this kind can not be made on a large scale. They are quite small and are used for fusing small amounts of refractory substances, as in the production of artificial gems.

488. Manufacture of Carborundum.—In 1890 Acheson made in a small electric furnace a crystalline substance which he supposed to be a compound of carbon and corundum, or emery, and he accordingly named it *carborundum*. It is now known to be the carbide of silicon, or SiC. It is of great hardness and has come into extensive use as an abrasive, displacing emery in the various wheels, grindstones, whetstones, polishing cloths and powders.

It is made on a large scale at Niagara Falls. The furnaces, built of brick without mortar, are some fifteen feet long by seven feet wide and high. At each end (Fig. 237) there are built into the wall heavy copper terminals to each of which are attached the electrodes proper, sixty carbon rods, three inches in diameter and two feet long. These electrodes are connected by a core of crushed coke about nine feet long and two feet in diameter. Around this core there is packed about ten tons of an intimate mixture of 34% coke, 54% sand, 10% sawdust, and 2% salt. The salt acts as a flux; the sawdust keeps the mass porous. An alternating current of 4000 amperes at 185 volts is turned on. This, as will

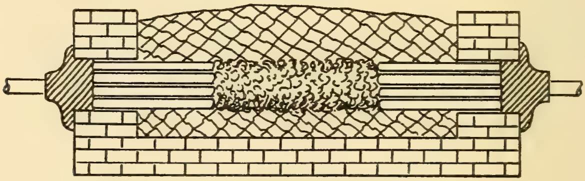


Fig. 237.

be shown in the next chapter, represents about 1000 horsepower. In a short while a large amount of carbon monoxide is produced and burns as it emerges from the crevices between the bricks. In twelve hours the furnace becomes red hot but the current continues to flow for twenty-four hours before it is turned off. When it has cooled sufficiently the furnace is dismantled. The interior core of coke is found to be converted into graphite. This is surrounded by a sixteen inch layer of iridescent purplish crystals of carborundum. Outside of this layer there are slag-like clinkers.

In a somewhat similar manner calcium carbide, CaC_2, is made by heating a mixture of lime and powdered coke, the reaction being

$$CaO + 3C = CaC_2 + CO$$

Calcium carbide is used for the production of acetylene gas for illuminating purposes.

489. Manufacture of Aluminum.—Although very widely distributed, the ores of aluminum are most refractory and until recently their reduction was one of the difficult processes in metallurgy. The metal is now obtained from *bauxite,* a mineral containing over sixty per cent of aluminum oxide, Al_2O_3. Alone,

this is practically infusible but dissolves readily in fused *cryolite,* a double fluoride of aluminum and sodium. The current is passed through this fused mass, aluminum is released at the cathode and oxygen at the anode. The aluminum being liquid settles to the bottom and is drawn off from time to time, fresh supplies of bauxite being continually added. The cryolite is not affected. The action in this case being electrolytic as well as thermal, direct current must be used. Aluminum which ten years ago sold for eight dollars a pound can now be produced with profit at twenty-five cents.

490. Electric Iron Furnaces.—There is an increasing use of electric furnaces for the treatment of pig iron by a process similar to the ordinary open-hearth process. The fused metal is one of the electrodes, the other consists of large carbons which penetrate the dome of the furnace. The arc plays between these carbons and the metal beneath. Suitable linings are used and the proper ingredients are added to the molten metal to remove the sulphur, phosphorus and other objectionable substances. Such furnaces are now made of a capacity of fifteen tons.

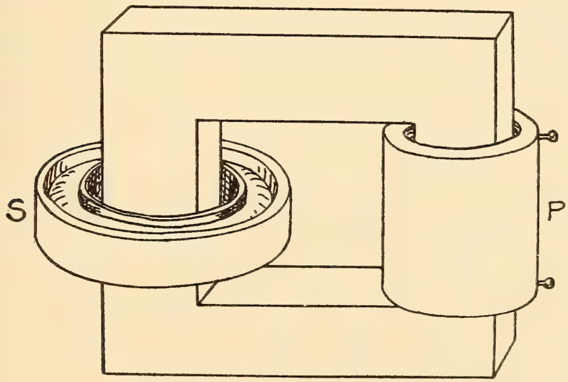


Fig. 238.

491. The Induction Furnace.—The induction furnace, recently introduced for the manufacture of high-grade steel, is a special application of the principle of the transformer. It is shown diagrammatically in Fig. 238. *P* is the primary and *S* is an annular trough of non-conducting fire-brick. Into this trough is placed the metal which is to be treated and this mass of steel constitutes

a short-circuited secondary of a single turn. The alternating current in P is stepped down in S to a current of large amperage sufficient to bring the steel to a molten state. At the proper time the required amount of spiegeleisen or other material is added. These furnaces have been made large enough to handle ten tons of steel at a charge.

CHAPTER 36.

ELECTRIC POWER.

492. Power Defined.—If a certain hoisting engine raises a weight from the ground to the top of a building in two minutes, and a second engine raises the same weight the same height in one minute, the work in each case is the same but the second engine does its work twice as rapidly as the first and is therefore said to be twice as powerful. *Power* may be defined as *the rate of doing work.* It would ordinarily, therefore, be measured in foot-pounds per second.

493. Horse-Power.—About one hundred and fifty years ago, the mine owners in Cornwall employed horses to operate the pumps which kept their mines free from water. As the mines sunk deeper, the difficulty and expense of removing the water increased so that many were abandoned as no longer profitable. It was at this time that Watt perfected his steam engine and began to introduce it in the mines. The miners knew how many horses were required to lift so much water but had no notion of the capabilities of the new-fangled engine; they therefore required that before purchasing an engine they should be told how many horses it could supplant. In order to be able to furnish this information, Watt carried out a series of tests with the powerful horses used in the London breweries, as a result of which he concluded that such a horse working eight hours a day could perform work at a rate equivalent to raising 33,000 pounds one foot per minute. This figure has ever since been accepted as the measure of a horse-power. The unit of time is, however, commonly taken as one second, the corresponding foot-pounds being 550.

In electrical measurements, it is desirable to express this in absolute units. Remembering that the pound is about 445,000 dynes (Par. 11), and that the foot $=30.48$ centimeters, the horse-power is in round numbers 7,460,000,000 (or 746×10^7) ergs per second.

494. Expression for Electric Power.—There are a number of ways in which an expression for electric power may be deduced.

It is superfluous to say that in every case the results must be the same, yet, several of these methods will now be explained, for each presents the matter from a slightly different view-point and the student will thus get a broader grasp of the subject. We shall begin with the simplest.

(a) In Par. 477 we saw that the heat developed by a current of strength I flowing for t seconds through a resistance R is I^2Rt. This represents energy expended, or work, and if divided by t it will give the rate at which the work is done, or the power (Par. 492). Hence, the power developed by a current I in heating a resistance R is I^2R.

This last expression may be factored as follows: $I^2R = I \times IR$. But IR is the drop of potential E between the two points A and B (Fig. 234), hence for I^2R we may write IE, or the power expended in heating any portion of an electric circuit is measured by the product of the current flowing in the circuit by the difference in potential between the ends of the portion.

(b) In Par. 358 it was proven that the work done by a current I flowing around a coil is IN, N being the change in the number of lines of force embraced by the coil. If this work be done in time t, the power $= IN/t$. But (Par. 425) $N/t = E$, and the foregoing expression also reduces to IE, or, as above, the power developed in a coil, a portion of a circuit, is measured by the product of the current flowing through the circuit by the difference in potential between the ends of the coil. In this case, the heating effect is not considered.

(c) Finally, taking the most general case of a portion of a circuit of any shape whatsoever, and placing no restriction upon the nature of the work performed by the current, if E be the difference of potential between the ends of the portion and if during the time under consideration Q units are transferred around the circuit, the work done in the portion is QE (Par. 72). But $Q = I \times t$, hence the work is $I.t.E$. Dividing this by t to obtain the power, we again arrive at IE, or, in general, the power expended in any portion of an electric circuit is measured by the product of the current by the difference in potential between the ends of the portion.

495. Development of Power in a Battery.—Since the source of the energy developed in a single cell is the chemical action resulting in the consumption of the zinc by the acid (Par. 192), no matter

how a battery may be grouped, if the same amount of zinc be consumed in the same time, the same power is developed. This may be illustrated as follows: Suppose N cells, each of an E. M. F. of e volts and an internal resistance of r ohms be grouped in series with an external circuit of negligible resistance. The E. M. F. of the battery is Ne, the current is $Ne/Nr = e/r$, and if z be the zinc consumed in one cell per second, the total amount consumed per second is Nz.

If these same cells be grouped in parallel, the E. M. F. of the battery is e, the current through each cell is e/r, the total current is Ne/r and the total consumption of zinc per second is again Nz. The power developed in the two groupings should therefore be the same. In the first case it is $Ne \times e/r$ or Ne^2/r; in the second case it is $e \times Ne/r$ or again Ne^2/r.

496. Units of Electrical Power.—From Par. 494 the expression for electrical power is

$$P = IE$$

If in this, I be one absolute unit of current and E be one absolute unit of E. M. F., P becomes one absolute unit of electric power. This unit has received no name but represents the expenditure of energy at the rate of one erg per second.

If in the same expression, we make I one ampere and E one volt, we again have $P = 1$. This unit, the practical unit of electric power, is called the *watt*. Since the ampere is 10^{-1} absolute units of current and the volt is 10^8 absolute units of E. M. F. (Par. 427), the watt $= IE = 10^{-1} \times 10^8 = 10^7$ absolute units of power, or ten million ergs per second.

We saw in Par. 493 that the horse-power was 746×10^7 ergs per second. The horse-power is therefore 746 watts. The commercial unit of electric power is the *kilowatt*, or one thousand watts. The kilowatt is 1000/746, or just about $1\frac{1}{3}$ horse-power. The commercial unit of electric *work*, the unit by which it is bought and sold, is the kilowatt-hour.

497. Measurement of Electric Power.—Since the power expended between two points in an electric circuit is measured by the product of the current by the difference in potential between the two points, we may measure the current by an ammeter, and the difference of potential by a voltmeter, and by multiplication obtain the watts. As an illustration, suppose we wish to determine

the consumption of power in the 16 candle-power, 100 volt lamp, *AB*, Fig. 239. Connections are made as shown. The ammeter reads the current *I* flowing through *AB*, and the voltmeter reads the difference of potential *E* between *A* and *B*. The product of these two readings gives the watts consumed. If, for example, the current be one-half ampere and the difference of potential between *A* and *B* be 100 volts, the power is 50 watts. It requires,

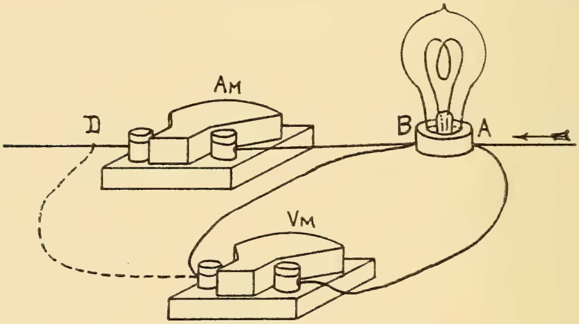


Fig. 239.

therefore, a kilowatt to run 20 such lamps, or about one horse-power to run 15.

If, in the above example, the ammeter be read while the voltmeter is connected up, a slight error will be committed, for examination of the figure will show that the ammeter reads not the current through the lamp but the sum of the currents through both the lamp and the voltmeter. If the resistance of the voltmeter be 15,000 ohms (Par. 458), the current through it is $\frac{100}{15000}$ or $\frac{1}{150}$ ampere. The current through the lamp is therefore really $\frac{1}{2}-\frac{1}{150}=\frac{74}{150}$ ampere, and the power consumed in the lamp is $100\times\frac{74}{150}=49\frac{1}{3}$ watts instead of 50, indicating an error of $1\frac{1}{3}$ per cent. This may be reduced by connecting the voltmeter, as shown by the dotted line *D*, in shunt with both the lamp and the ammeter, or by reading the ammeter before the voltmeter circuit is closed. In a similar manner it may be shown that if the current be large and the difference of potential between *A* and *B* be small, the connection as shown in the figure is the best.

The determination of power from the reading of two separate instruments does not give correct results when applied to alternating current circuits. This fact cannot be explained until the subject of alternating currents is reached.

498. Measurement of Power by Electro-Dynamometer.—By making a slight change in the connections of an electro-dynamometer, it is possible to use that instrument to measure electric power. For example, suppose we wish to measure the power expended in an incandescent lamp. Connections are made as shown

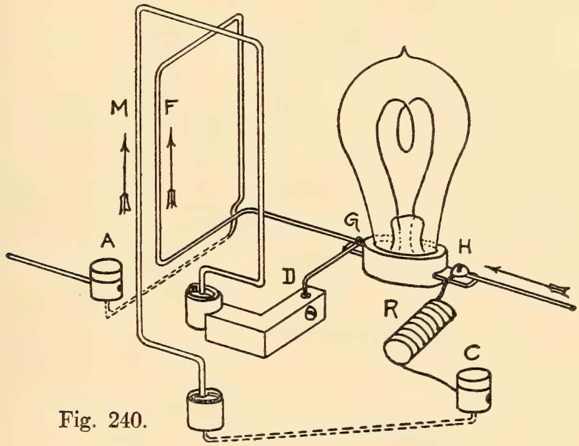


Fig. 240.

diagrammatically in Fig. 240. *F* represents the heavy wire fixed-coil of a Siemen's electro-dynamometer (see Fig. 171). The end of this coil connected to the terminal *A* is left undisturbed. The other end, which was fastened to the metal bracket at *D*, is disconnected and attached to *G*, one terminal of the lamp. The main current now enters at *H*, passes through the lamp and the coil *F* and out by *A*. A current is shunted off at *H*, passes through a resistance *R* of several thousand ohms, thence to the terminal *C*, thence around the movable coil *M* to the bracket *D* and finally reunites with the main current at *G*.

In Par. 383 it was shown that the force exerted between the two coils carrying currents is

$$f = K.I.I'$$

K being a constant and *I* and *I′* being the currents in the respective coils. If *E* be the difference of potential between *H* and *G*, and if *R* be the resistance of the shunt circuit *HRCMDG*, then the current through the movable coil is $I' = E/R$. Substituting this for I' in the expression above, we have

$$f = \frac{K}{R}(IE)$$

whence, since *K* and *R* are

constants, we see that the force exerted between the two coils of the instrument is *proportional* to IE, the watts consumed between H and G. In the same paragraph it is shown that this force is proportional to the angle of torsion, that is, to the angle through which the milled head of the dynamometer is turned in order to bring the pointer of the movable coil back to the zero, whence the wattage between the points H and G is also proportional to this angle.

As pointed out in Par. 497, an error is committed in this measurement unless the shunt current (the voltage current) be so small as to be negligible. On this account, the resistance R is made very large.

499. Indicating Wattmeter.—Instruments from which may be read direct the power developed between two points in a circuit

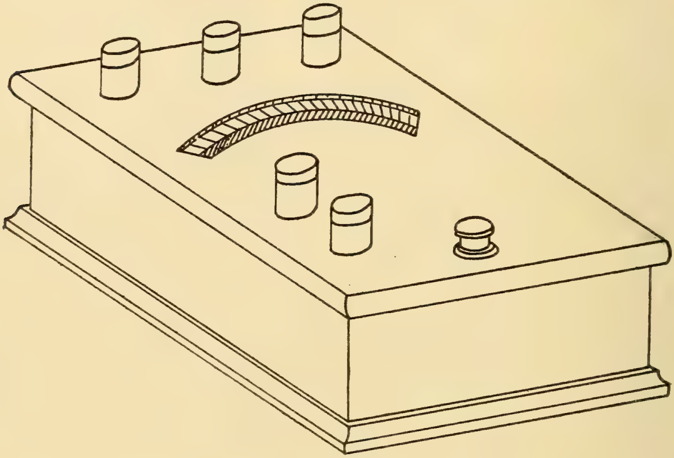


Fig. 241.

are called *wattmeters*. They are of two general classes, the first giving the value of the power at any instant and called *indicating wattmeters;* the second summing up or integrating these instantaneous values and called *integrating wattmeters*.

Indicating wattmeters operate on the principle of the electro-dynamometer described in the preceding paragraph, but are usually so arranged as to avoid the errors pointed out in Par. 497. Fig. 241 represents the external appearance of the Weston wattmeter. Across the top there are three terminals, the outer ones being used

for the voltage current and one of them being marked +. The central terminal is used in a certain process of calibration, not necessary to describe here. On the left side are the two terminals for the main current, one of these also being marked +. At the bottom is a button switch which closes the voltage circuit. This instrument may be used with either direct or alternating currents. When in use, if the main current be brought in at the plus terminal, the voltage current must enter by the plus terminal of the top row; if the main current be brought in at the negative terminal the voltage current must enter by the negative terminal of the top row.

The internal arrangement of the instrument is quite similar to that of the D. C. A. C. voltmeter as described and figured in Par. 470. Fig. 242 represents diagrammatically the connections made

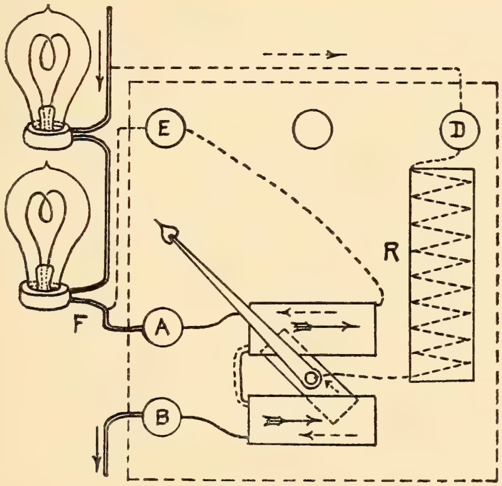


Fig. 242.

to read the power consumed in two incandescent lamps in series. The main current enters at *A*, passes around the two fixed coils in the direction shown by the heavy arrows, and leaves by *B*. It does *not* pass through the movable coil. The voltage current is shunted off at *C*, enters at *D*, passes through the large resistance *R*, which is wrapped so as to be free from inductance, thence to the movable coil around which it flows as shown by the broken arrow, then around the fixed coils but opposite in direction to the main current, thence out by *E*, and rejoins the main current at *F*.

The current through the fixed coils, as has already been pointed out (Par. 497), is greater than the current through the lamps since it consists of that current plus the shunt current. To correct for this, the shunt current is carried around the fixed coils in opposite direction to and making as many turns as the main current.

500. Integrating Wattmeter.—A consumer of electrical power is charged on an equitable basis when he pays in proportion to the work performed for him by the current. He must therefore pay, not for the power alone, but for the product of the power and the time during which it has been supplied, for since power is the rate of doing work $= w/t$ (Par. 492), work is equal to power $\times$ time. Electrical power is therefore sold not by the watt, but by the watt-hour, or more usually by the kilowatt-hour (Par. 496). The wattmeter described in the preceding paragraph indicates the instantaneous values of the power but takes no account of the time element. Instruments which sum up the successive amounts of work performed by the current are called *integrating wattmeters*. Their principle is simple but can not be fully explained at this point. One form consists of a coil which revolves continuously as long as the current flows through it, the rate of revolution at any instant varying directly with the power, and therefore the total number of revolutions varying with the total amount of work performed. These revolutions are recorded by an arrangement like that used in cyclometers but the dials are graduated to read kilowatt-hours direct. The instrument is therefore analogous to a gas-meter which indicates at any instant the total amount of gas which has flowed through it up to that time but does not indicate the amount actually flowing through.

501. Electrical Transmission of Power.—The two prime sources of power utilized at present are water and steam. Of these, water power is much the cheaper.

The difference in level, upon which water power largely depends, may be natural, as in the case of falls, or may be artificially produced by the erection of dams. In either case, unoccupied localities suitable for the development of such power are rapidly becoming scarce. In the immediate vicinity of these falls and dams, the available sites for power plants are usually restricted. By means of shafting, belting, cables, etc., the power developed

by these plants may be transmitted a few hundred feet. Beyond this limited zone, recourse must be had to steam power.

In the majority of steam plants, coal is the fuel used and this has to be transported from the mines to the plants. On the average, the cost of transportation is greater than the cost of the coal itself, therefore, steam plants located near the mouth of a coal mine have a great advantage over those at a distance.

From the foregoing, the need of a method of cheap transmission of power to a distance is evident. This problem is solved by electricity, the mechanical power developed by the plant being transformed into electrical power, sent out over the line to the desired spot and there transformed back into mechanical power.

502. Considerations Affecting Electrical Transmission of Power.—It was shown above (Par. 494) that the electrical power between two points in a circuit is measured by IE, the product of the current by the difference of potential between the points. These two quantities may therefore vary reciprocally and the power remain constant. This principle is of the utmost importance in the electrical transmission of power. When a current flows through a conductor, a portion of the power is spent in heating the conductor, the power so spent being I^2R (Par. 494), or varying as the *square* of the current. To avoid this waste, the current should be kept as small as possible. From what has been shown above, we can reduce the current and still transmit the same power provided the voltage is varied inversely with the current. An example will bring this out more clearly.

Suppose an electric generator operated by a water wheel is producing ten amperes at a pressure of two hundred volts, or developing a power of 2000 watts, and is transmitting power over a No. 3, B. and S., copper wire to a factory at a distance of five miles. For round numbers, the resistance of this wire may be taken as one ohm per mile. The I^2R loss due to the resistance of the wire is $100 \times 10 = 1000$ watts, that is, fifty per cent of the power generated is lost in the wire. If this same generator turned out one ampere at a pressure of 2000 volts, it would still develop the same power, 2000 watts, but in this case the I^2R loss would be only 10 watts, or only one-half of one per cent of the total power. Furthermore, if the fifty per cent loss be permissible, a No. 15 wire may be used with the 2000 volt current and the loss still be kept within the limit. Since the No. 15 wire weighs 52 pounds per mile as com-

pared to 838 pounds for the No. 3 wire, there would result a saving of 7860 pounds of copper costing about $1000.

The secret of electrical transmission of power to a distance is therefore the employment of high potential currents. As will be shown in Part V, high potential alternating currents are much more easily generated and transformed up and down than are corresponding direct currents, for which reasons, in the transmission of power to a distance, alternating currents are used almost exclusively. Voltages as high as 20,000 and 30,000 are frequently employed, and in a few cases 150,000 has been reached and power has been transmitted upwards of three hundred miles. With these very high voltages, the difficulty of obtaining proper insulation for the line increases greatly. The wires must be spaced widely apart on the cross arms of the poles and special forms of porcelain insulators must be used. In rainy weather, the loss from leakage becomes excessive. Finally, the element of danger to life assumes serious proportions.

CHAPTER 37.

ELECTRIC LIGHTING.

503. The Electric Light.—In Chapter 35 we examined the heating effect of the electric current. If a body be raised to a sufficiently high temperature it will emit light. The production of light by electricity is therefore only a particular case of heating.

There are at present three distinct classes of electric lights. These are:

(a) The incandescent lamp. The current is passed through a conducting solid which is raised to incandescence. No combustion takes place.

(b) The arc lamp. The current is passed across the gap between two electrodes whose tips are thereby heated to incandescence. A portion of one of the electrodes is volatilized and the resulting vapor conducts the current across the gap. Combustion takes place, but simply because air cannot be excluded.

(c) The luminous vapor lamp. The current passed through rarefied gases or vapors contained in glass tubes causes these vapors to glow. No combustion takes place.

504. The Incandescent Lamp.—The incandescent lamp does not differ in principle from the fuze described in Par. 483. The earlier forms consisted of a bare platinum wire which was made white-hot by the passage of the current. These failed because the platinum was necessarily near its melting point and a slight increase in the current would cause it to give way; moreover, the cost of the platinum was excessive, and for these reasons the incandescent lamp did not become a commercial success until the development by Edison of the carbon filament. Carbon is infusible and, although a conductor, is a poor enough conductor to permit the filaments to be made of sufficient size for strength and yet preserve the resistance required for the development of the heating effect. If, however, carbon be heated in the presence of oxygen it is soon consumed. The filaments must therefore be enclosed in a vacuous glass bulb.

505. The Carbon Filament.—The first successful carbon filaments were made from bamboo. Later on, they were made from a compact paper which was cut into thread-like strips. They have also been made from cotton thread. They are now manufactured from a pure cotton fibre which is dissolved into a glue-like liquid by a solution of zinc chloride. This is forced through small holes in a die and emerges in rather soft endless threads, a little over one-fiftieth of an inch in diameter, which are caught in a vessel containing alcohol. The alcohol dehydrates and hardens the threads, which are then washed free of the zinc chloride, coiled up and dried. They now resemble fiddle strings. These are cut up into the proper length, given the required shape by being wrapped upon a form and are then embedded in pulverized carbon in a covered crucible and carbonized at a high temperature. After cooling, they are attached to holders, placed in a vessel in which they are surrounded by the vapor of gasoline, and heated white hot by a current. This process is called "flashing." The gasoline is decomposed and deposits a semi-metallic film of gas coke on the filaments. This renders them stronger, more uniform in resistance, and of a steely black color. The diameter has now shrunk to .0035 inch.

An additional process recently introduced, consists in placing the filaments, both before and after flashing, in an electric furnace and raising them to a still higher temperature by which they are partially graphitized. Filaments so treated are said to be "metallized," and their light-giving efficiency is much increased.

506. Manufacture of the Lamps.—The current enters and leaves the glass bulb through two wires fused into a small glass tube or stem which is inserted into the bulb and fused to it. The portion of these "leading-in wires" which passes through the glass of the stem (*A* and *B*, Fig. 243) must be of platinum. The coefficients of expansion of glass and of platinum are about the same and they therefore expand and contract together. With other metals, the glass would either be cracked by the greater expansion of the wire or the vacuum would be destroyed by the shrinking of the metal away from the glass. Copper wires are fastened to the outer ends of *A* and *B* and the filament is attached to the other ends by means of a carbon paste. One of the copper wires is soldered to the brass shell which carries the screw threads of the lamp base. The bottom of this shell is closed by a glass or porcelain button in the center

of which is a brass contact, pierced with a small hole. The remaining copper wire is drawn through this hole and soldered to the contact. The shell is fastened to the bulb by a cement or by plaster of Paris. Lamp sockets are so arranged that when a bulb is screwed in, the required connections are made.

A small tube is left at E. This is now attached to an air pump and most of the air is withdrawn from the bulb. When a good vacuum has been obtained, a current is sent through the lamp. This drives out the gases which have been occluded in the carbon filament. The last traces of oxygen are removed by igniting a small amount of phosphorus inserted for that purpose at E, and E is then sealed by a blow-pipe flame.

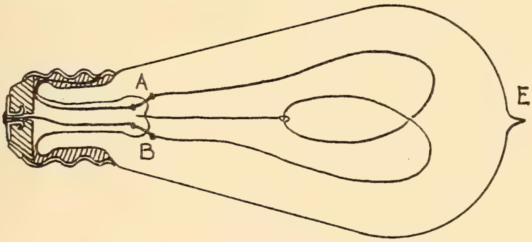


Fig. 243.

In lamps with long and slender filaments, the filaments are liable to be broken by excessive vibration, or when hot may droop, touch the bulb and crack it. To remedy this they are often supported at their middle point by a short wire, one end of which is fused into the tip of the glass stem on the interior of the lamp. Such filaments are said to be "anchored."

Incandescent lamps are run at constant voltages. Since the heating effect, on which the light-giving effect depends, varies as $I^2R = IE$ (Par. 477), and since E is constant, the lighting effect is increased by increasing the current. This is done by *decreasing* the resistance of the lamp, that is, by making the filament shorter and stouter.

507. Recent Incandescent Lamps.—As has just been stated, the light-giving effect of an incandescent lamp increases with the temperature. It is therefore desirable to heat the filament as highly as possible. As the temperature of the ordinary carbon filament increases, so does the brilliancy of the light it emits, but the life of the lamp is very much shortened thereby and it is not found practicable to exceed a temperature of 1350° C.

We saw (Par. 504) that in the early lamps attempts were made to use platinum filaments. Platinum, which fuses at 1775° C, was the most infusible metal which could then be obtained yet had to be abandoned because the filaments melted. There are known, however, certain rare metals whose fusing points are much higher than that of platinum. Among these are osmium, tantalum and tungsten, this last fusing at 3200° C. Their rarity, the difficulties of their metallurgy, and their consequent cost prohibited their use. These metals may now be obtained and are successfully used in incandescent lamps. Their conductivity being so much greater than that of carbon, in order to secure the necessary resistance they must be drawn into extremely fine wires. When they have been drawn down so that they look almost as slender as a spider's web, their resistance is still too small and can be in-

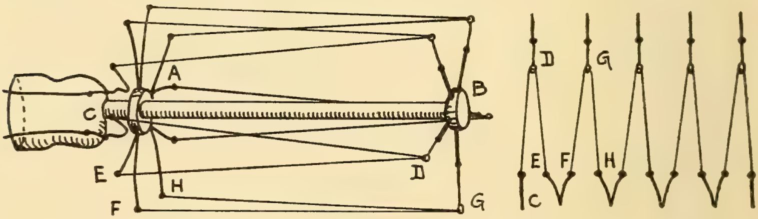


Fig. 244.

creased only by taking longer portions for filaments, about twenty inches on an average. Even with this length, it is stated that as many as 20,000 may be made from a single pound of tantalum, and this although the specific gravity of tantalum is greater than that of lead. To insert these long filaments into the lamp bulb, they must be folded back and forth a number of times and having very little rigidity when cold and becoming soft when heated, they must be supported at several points. The expansion and contraction of a twenty-inch filament, especially if it be attached to supports at intermediate points, is very liable to break it, for which reason it is found better to cut the filament into four or five pieces and to connect these pieces in series. Even in this case, especial provision must be made to allow for the expansion and contraction. Fig. 244 shows diagrammatically the arrangement of the filament in a tungsten lamp. The leading-in wires pass through a glass stem just as in the carbon lamp. To this stem and in prolongation of it there is fused a slender glass rod expanded into a button at

A and at *B*, points about two inches apart. Into the button *A* there are fused four V-shaped pieces of wire, the vertices of the V's being embedded in the glass so that the free ends radiate like the spokes of a wheel. Into the button *B* there are fused five equidistant straight pieces of wire. These are shorter than the pieces in *A*, but are brought out to the same length by an attached piece of the filament wire, this last terminating in a small circular loop. A piece of the filament is attached to the terminal *C*, the free end is then threaded through the loop *D* and brought back and attached to *E*. A second piece is attached to *F*, carried through the loop *G* and fastened to *H*, and so on around the axis. A development of these connections is shown at the right of Fig. 244, whence it is seen that the successive pieces of filament are in series. The flexible ends of those arms which radiate from *B* allow for the expansion and contraction of the filaments which they support. These lamps produce a very fine white light with a smaller expenditure of energy than in the case of the carbon lamp.

508. The Nernst Lamp.—The oxides of certain of the rarer metals, yttrium, thorium, zirconium, are infusible and if highly heated emit a very bright light. It is on this account that these oxides are used in the mantles of the Welsbach burner. When cold, they do not conduct electricity but if heated to about 700° C they become conductors and if a current be now passed through them they may be heated to a point where they glow with great brilliancy. This property is utilized in the Nernst lamp. The light is emitted from a *glower*, a little rod of these oxides about two centimeters (three-quarters of an inch) long and one millimeter in diameter. The light-giving power of a lamp is increased by using more than one glower. The lamp must be provided with an auxiliary arrangement by which (a) the glower is heated up to the conducting point and (b) the current is then switched from the heater to the glower.

Fig. 245 shows diagrammatically the operation of the lamp. *A* is an armature, bent at an angle and pivoted as shown. Its shape causes its lower end to hang out and make contact at *C*. *H* is the heater, a slender porcelain tube around which is wrapped a coil of very fine platinum wire which, for protection, is embedded in a white cement paste. *M* is an electro-magnet with an L-shaped core. The current enters at *D*, travels down *A*, passes through the contact *C*, around *H* and out by *E*. The passage of the current

through H heats it and in less than half a minute the glower G has been raised to a conducting temperature. The current entering at D may now pass around M, through the resistance B, through G and out. M becomes magnetized, the armature A is attracted and the contact at C is broken. The full current now passes through G. Owing to the method of operation of the current shifter, these lamps are restricted to a vertical position.

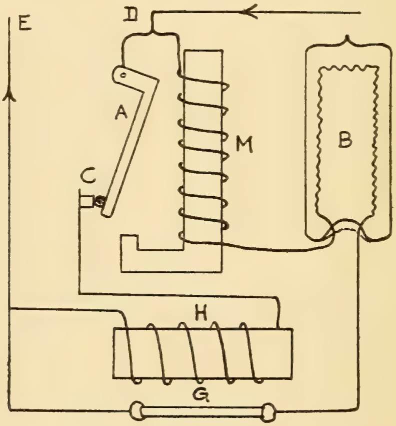


Fig. 245.

As the temperature of G rises, its resistance decreases. This would permit a larger current to flow through G and its temperature would rise still higher, and so on, until the glower would be melted. This rise of current, however, is controlled by the resistance B, a fine iron wire which, to prevent oxidation, is sealed up in a glass tube in an atmosphere of nitrogen. It is adjusted to permit the passage of the required current at the voltage for which the lamp is intended. The resistance of iron increases rapidly with the temperature and an increase of seven per cent in the current will double the resistance of B. Variations in the voltage do not, therefore, cause proportional variations in the current through the lamp. A resistance such as B, which steadies or prevents undue fluctuations in the current, is commonly called a "ballasting coil" or simply "ballast."

Since the glower is composed of oxides, it is not necessary to seal it up in a bulb. It is, however, usually surrounded by a glass globe. Doubtless on account of electrolytic action, the

life of a glower is less with direct current than with alternating current.

509. Candle-Power.—Lamps are rated according to the intensity of the light which they emit under normal conditions, as 4, 8, 10, 16, 32, 50, and 100 *candle-power*. The British standard candle is defined as a spermaceti candle, seven-eighths of an inch in diameter, weighing one-sixth of a pound, and burning at the rate of 120 grains per hour. The German standard, the Hefner unit, or the *hefner*, is the light emitted by a lamp of prescribed dimensions burning amyl acetate. The hefner is about .88 of a candle-power. In actual measurements of candle-power, use is made of secondary standards, incandescent lamps whose candle-power has been determined by comparison with the primary units. The standards in use in this country are determined from the hefner.

In many electric lamps, the light emitted in certain directions is greater than that emitted in others. Such lamps are frequently rated according to their *mean spherical candle-power*, that is, the candle-power if the total light emitted were spread uniformly over the surface of a sphere with the lamp as a center.

510. Photometry.—Measurement of candle-power is made by *photometers*. Various kinds of these instruments are described in detail in the electrical handbooks. In brief, they consist of an arrangement by which a beam from the standard falls side by side on a screen with a beam from the light being measured. One of the lights is shifted back and forth until the illumination on the adjoining surfaces is the same. When this equality of illumination has been attained, then, since the intensity of illumination varies inversely as the square of the distance from the source, the candle-power of the lamps are to each other directly as the squares of their respective distances from the screen.

511. Life of Incandescent Lamp.—The life of an ordinary 16 candle-power incandescent lamp may exceed 2000 hours. However, the candle-power of a lamp, although slightly above normal for the first fifty hours, decreases steadily thereafter, and it is laid down as a rule that the *smashing point* of the lamp is reached when its candle-power has fallen to 80 per cent of its rated value. This, on an average, is after about 600 hours' use. The useful life depends greatly upon the accuracy with which the voltage is

regulated. It is stated that an increase of three per cent in the voltage will shorten the life of a lamp one-half. On the other hand, a decrease of ten per cent in the voltage reduces the candle-power 47 per cent.

512. Efficiency.—The efficiency of an incandescent lamp should be measured by the light produced by the expenditure of a certain amount of power, that is, by the candle-power per watt. In practice however, a custom the reverse of this has arisen and the efficiency of a lamp is given by stating the number of watts required to produce one candle-power. In this case, the greater the number of watts, the less the efficiency of the lamp. The hot resistance of an ordinary 110 volt, 16 candle-power lamp is 220 ohms. The current through the lamp is therefore one-half ampere, and the power consumed is $110 \times 1/2 = 55$ watts. The wattage per candle-power is therefore $55/16 = 3.1$. By increasing the voltage, more light is produced and the efficiency may be made 2.7 watts per candle-power, but in this case the life of the lamp is very much shortened (Par. 511).

The efficiency of the Nernst lamp is 1.75 watts per candle-power and that of the tungsten lamp is 1.5 watts or even less.

513. Control of Light.—An objection to the incandescent lamp is that it can not easily be turned down. We shall see later that if a large number of closely-grouped lamps, such as are used in illuminating the stage of a theatre, be run by alternating current, it is possible to turn them down simultaneously by a simple piece of apparatus (Par. 621), but it is not practicable to apply this to individual lamps. It is theoretically possible to insert in series with a lamp a variable resistance, a rheostat (Par. 302), by which the current, and consequently the light, may be controlled, but the cost and the necessary bulk of such arrangement prohibit its use.

514. Grouping of Incandescent Lamps.—Assuming that in transmitting electrical power from the generator to the spot where the power is to be used the principles outlined in Par. 502 have been observed, in utilizing this power for purposes of illumination, the lamps may be grouped either in series or in parallel, though the latter arrangement is by far the commoner of the two. Among the considerations which lead to the selection of one grouping in preference to the other, the principal are the distances by which

the individual lamps are separated and the nature of the current, whether direct or alternating.

If the lamps are to be located close together, as in the illumination of the rooms of a building, the parallel arrangement should be followed. A striking advantage of this arrangement is the independence of the several lamps and the automatic adjustment of the current to suit the demands made upon it. The following will make this clear.

In Fig. 246, G represents a generator constructed, as will be explained in Part V, so as to maintain a constant difference of poten-

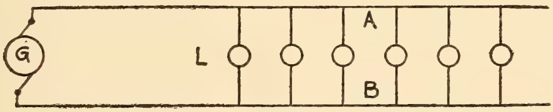


Fig. 246.

tial between the mains A and B. L represents a number of lamps arranged in parallel between these mains. Suppose the resistance of a lamp to be 220 ohms, and the difference of potential between A and B to be 110 volts. If one lamp be turned on, the current through it will be $I = E/R = 110/220 = 1/2$ ampere. If four lamps be turned on, the resistance between A and B is reduced to $220/4 = 55$ ohms and the current is now $110/55 = 2$ amperes, but since there are four paths, one-fourth of the total current, or one-half ampere, flows through each so that each lamp gets its proper current. So long as the difference of potential between A and B is maintained, each lamp when turned on will receive its proper current, and whether it be turned off or on will not interfere with the remaining lamps.

There are still other parallel arrangements, such as the three-wire system, the five-wire system, etc., in which more than two mains are used, but explanation of these is deferred until the machines supplying the currents for these systems have been described.

If the lamps are to be widely scattered, as in street illumination, they should be arranged in series and supplied by a *constant current* generator. At the Military Academy the roads are lighted by incandescent lamps, each requiring three amperes at 50 volts, and arranged in series, 50 in a circuit. The generator must, therefore, supply three amperes at a pressure of 2500 volts. Were these

lamps arranged in parallel, the mains would have to carry, for a portion of their length at least, a current of 150 amperes.

Since the lamps are in series, should one burn out, the remainder would ordinarily be extinguished. To avoid this, an arrangement shown diagrammatically in Fig. 247 is employed. From the lamp socket proper there extend downward two brass springs *C* and *D*, shaped so that they press tightly together like a pair of spring tweezers. They are kept from actual contact by a thin sheet *E* of mica, or of similar insulating material, which is inserted between them. When in position, these upper springs make contact with corresponding springs *A* and *B*, by which the current is brought in and taken out. Should the lamp burn out, breaking the circuit, the voltage between *C* and *D*, which up to this time had been 50, immediately mounts to 2500. This is sufficient to pierce the sheet of mica *E*, burn it out, and re-establish the circuit.

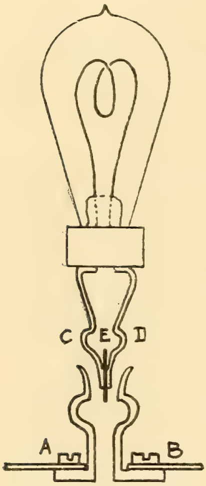


Fig. 247.

515. The Arc Lamp.—The electric arc was described in Par. 485 and later its use in the electric furnace was explained. It was also pointed out that not until the comparatively recent development of machinery for supplying the necessary current did it become possible to utilize it. It was discovered by Davy in 1808. By means of a battery of 2000 cells and with charcoal electrodes he produced an arc four inches long and of very great brilliancy. Thirty-five years later Foucault substituted the more compact gas coke for the charcoal used by Davy. Carbon is still the principal material used, although certain other substances have recently been introduced (Par. 523).

Arc lamps may be grouped in series or in parallel, the same considerations governing as explained in the preceding paragraph. Since they are most largely used for external illumination, and also since they require a much larger current than does the incandescent lamp, they are usually arranged in series.

516. The Carbons.—The carbons for use in the arc lights are made with the greatest care. They are made from lampblack, or from gas coke, or from a similar coke produced in refining certain

petroleum products. These forms of carbon are ground to a very fine powder, passed through a bolting cloth like that used in the manufacture of flour, and intimately mixed with granulated pitch which is warmed enough to cause the ingredients to adhere. The mixture is then cooled and again ground to a fine powder and passed through the bolting cloth. The resulting meal is formed into rods, either by being compressed between steel molds by hydraulic pressure or by being forced through a die and emerging in a continuous piece which is cut up into the required lengths. The rods are then placed in layers in a furnace, the layers being separated and covered by sand, and they are then heated and maintained at a high temperature for from ten days to two weeks. In this process a good many are spoiled by warping. The carbons thus prepared are frequently copper-plated. The coating of copper strengthens the rods, prevents chipping and the formation of dust, and adds about one-fifth to the life of the carbon, but its main object is to obtain a better electrical contact. The molded carbons are the most largely used but, mainly because of the remains of the web along the sides, they are not exactly cylindrical and can not be used in certain forms of arc lamps described later (Par. 521). The pressed carbons are perfectly cylindrical and when necessary can also be made in the form of a tube for the manufacture of *cored* carbons. The average arc light carbons are one-half inch in diameter and vary from six to twelve or more inches in length. Their average resistance is 0.15 ohm per foot. Carbons for search lights may be as much as two inches in diameter.

517. Requirements of Arc Lamp Mechanism.—The mechanism of an arc lamp must automatically perform the following functions:

(a) When the current is turned on, it must bring the carbons into contact.

(b) It must then "strike" the arc by separating the carbons the proper distance.

(c) As the carbons consume away, it must feed them together.

(d) If the carbons approach too close, it must separate them.

(e) If the arc goes out it must restrike it.

(f) In a series arrangement, if the carbon burns out or breaks, a cut-out switch must operate to shunt the current by the disabled lamp.

When it is realized that the mechanical and electrical arrangements by which the foregoing objects are attained must differ according as the lamps are connected in series or in parallel, and also must differ according as direct or alternating current is to be used, it will be seen that the kinds of lamps are very numerous. We can do no more, therefore, than outline the principle of operation of a few typical forms.

518. The Clutch.—In all ordinary direct current arc lamps, the positive carbon is the upper one. There are two reasons for this. The first and principal is because eighty-five per cent of the light produced by the arc is emitted from the crater at the tip of the positive carbon and therefore this must be above so as to throw its illumination downwards. The second is because the positive carbon is consumed more than twice as rapidly as the negative, or in open arcs at the rate of about one and a half inches per hour and by placing it above it is in the best position to be fed by gravity. These considerations do not apply to alternating current lamps, nor to certain projectors and search lights. In this last class it is desirable that the crater should face the reflector and lie in its focus; the carbons are accordingly often placed horizontally, or one horizontal and the other vertical, and both may be fed automatically or by hand. The arrangement by which the upper carbon is lifted and held at the proper distance from the lower and by which it is allowed to slide down as it burns away, is called the *clutch*. There are many forms of clutches. Some operate like the tongs used in hoisting stones and close when they are raised but open when they are lowered. A very simple form is shown in section in Fig. 248. This consists of a metal plate *A* pierced with a circular hole slightly larger in diameter than the carbon holder which passes through it. One end of this plate fits loosely in the jaws *B* of the lifting apparatus. As *B* rises, the plate *A* is canted and thus grasps the rod. When *B* is lowered, *A* strikes the stop *C* and is brought to a horizontal position, thus releasing the carbon which slips down.

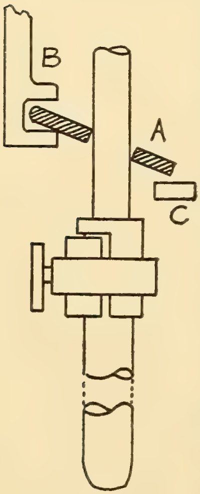


Fig. 248.

519. Constant Potential Arc Lamp.—As stated above, arc lamps may be run in series or in parallel. The series arrangement is by far the more common, but the parallel grouping is also frequently employed, especially for interior illumination. In this case the lamps are connected across mains between which a constant difference of potential is maintained. One of these lamps is shown diagrammatically in Fig. 249. With the carbons in contact, when the switch *S* is closed the current enters at *A*, passes through the resistance *R*, thence through the solenoid *C* to the upper carbon, down this to the lower carbon and out by *B*. The current passing through *C* causes it to suck up the plunger and, through the clutch, to raise the upper carbon and thus strike the arc. As the carbons burn away, the arc gets longer and its resistance increases. This reduces the current, and the lifting power of *C* grows less until finally it can no longer support the plunger and the carbon and they fall. The clutch strikes the stop and releases the carbon which slides down, shortening the arc. This increases the current and the plunger is again drawn up, and so on.

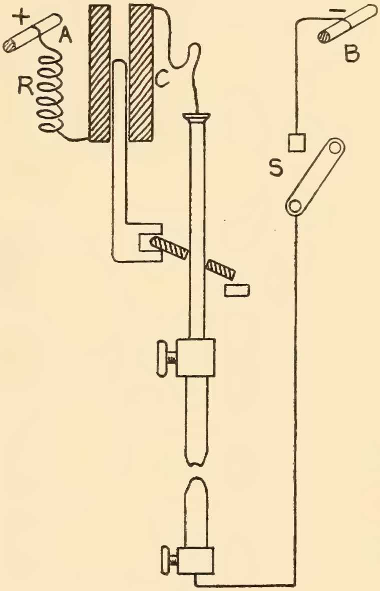


Fig. 249.

Without the resistance *R*, the result of closing the switch with the carbons in contact would be in the nature of a short circuit (Par. 306). This resistance steadies the current by preventing violent fluctuations and it is therefore a "ballast" as described in Par. 508.

520. Constant Current Arc Lamp.—For operating arc lamps in series, the generator and its regulator are designed so as to furnish a constant current, therefore, whether the arc be long or short the current is the same. On this account, resistance in series with the lamp is not required. Furthermore, the arrangement described in the preceding paragraph could not be used, for

the pull of the solenoid upon its plunger being constant, the carbon would not feed. For such lamps the so-called "differential" mechanism is employed. This is shown diagrammatically in Fig. 250. With the carbons in contact, the opening of the switch S causes the current entering at A to pass around the solenoid to the point C, thence to the upper carbon, thence to the lower and out by B. The passage of this current actuates the clutch and strikes the arc. To cause the carbon to feed, a differential coil is taken off at the point C and connected at D, that is, it is in shunt with the arc. This coil is of many turns of fine wire and is wrapped in opposite direction to, and inside of the first, but for clearness is represented in the diagram as being below. The two coils being wrapped in opposite directions, the pull upon the solenoid plunger is due to the difference of the ampere turns in the two. With the carbons in contact, the difference of potential between E and D is very little, therefore, a very small current flows through the differential coil. As the carbons draw farther and farther apart, the resistance, and consequently the difference of potential, between E and D increases. This causes an increasing current to flow through the differential coil and weakens more and more the pull on the plunger. A point is finally reached when the plunger drops and the carbon feeds.

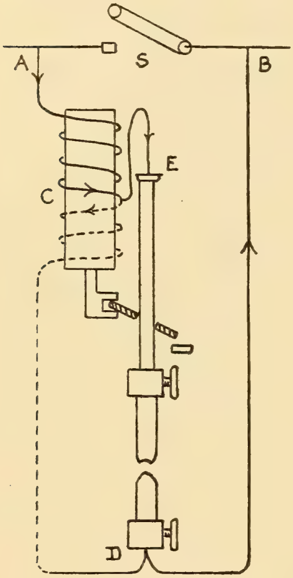


Fig. 250.

521. The Enclosed Arc.—The wasting away of the carbons in the ordinary arc lamp is mainly due to the combination of the white hot carbon vapor with the oxygen of the air. It is not practicable to enclose the carbons in air-tight globes but in recent years there has been introduced a form of arc lamp in which the arc is surrounded by a globe so fitted that the admission of air is reduced to a minimum, and in these the life of the carbons is very greatly prolonged, the consumption being reduced from 1.5 inches per hour to less than one-tenth of an inch. In addition to the sav-

ing in carbons, there is a very great saving in labor since the lamps, instead of having to be "trimmed" or supplied with fresh carbons daily, average over 100 hours and may be run as long as 200 hours without attention. Other advantages are a steadier light and absence of the hissing noise of the open arcs. The principal channel for the admission of air to the arc is the space around the carbon since this latter must be free to be moved by the lamp mechanism. To reduce this, the carbons must fit the opening very accurately, for which reason, as already mentioned (Par. 516), pressed carbons are used instead of the molded.

522. The Flaming Arc.—With the common arc light, the carbons are from one-sixteenth to less than a quarter of an inch apart and the greater part of the light is emitted from the incandescent carbons, although the maximum heat is developed within the arc itself (Par. 485). If it were possible to suspend within this arc a non-combustible solid, like the mantle of the Welsbach burner, it would be heated to incandescence and the heat energy of the arc would be converted into light energy. This object is partially realized in the so-called *flaming arcs*. In these, the positive carbon is either impregnated with certain salts of calcium or of magnesium or has a core filled with these salts. The vapor produced when these salts are volatilized is highly heated and emits a powerful reddish yellow light, and since it conducts it also permits the carbons to be separated by upwards of an inch. They need not be raised to such a high temperature as in the common arc lamps and therefore their life is longer. The efficiency of these lamps is at least three times that of the common form.

Instead of the carbons being in the same vertical line, they are sometimes arranged both pointing downward like the letter V, the arc being at the vertex. In this way, neither carbon screens the other and both tips throw their light down. There is a tendency, however, for the arc to ascend between the carbons. This is corrected by arranging a magnetic field, similar to the magnetic blow-out (Par. 485), but only strong enough to keep the arc down at the tips of the carbons.

An additional advantage of this arrangement is that the slag formed by the fusion of the impregnating salts drops off and does not clog the tips of the carbons with a non-conducting glassy material.

523. The Magnetite Arc Lamp.—The magnetite arc lamp, but recently developed and used with direct current only, resembles the flaming arc lamp in that the chief source of light is the arc which is an inch or more in length. It differs from other arc lamps in that little or no light is given off by the electrodes, also that the maximum amount of light is developed at the negative end of the arc. The positive electrode is of copper and is of such size that the heat developed is conducted away so that the electrode is not consumed. The negative electrode is a thin steel tube, the size and shape of an ordinary carbon. It is packed with a mixture of powdered magnetite, Fe_3O_4, and oxides of chromium and titanium. The magnetic oxide renders the electrode a conductor, the remaining oxides not conducting until they have been heated. The oxide of titanium imparts the luminosity to the arc; the oxide of chromium increases the life of the electrode. An eight-inch electrode in such a lamp with a current of 4 amperes at a pressure of 80 volts will burn for upwards of 200 hours. Since the constituents of the electrode are oxides, there is no combustion and the arc is not enclosed. These oxides, however, are volatilized and condense immediately beyond the limits of the arc in a reddish soot which if not removed soon covers globes, reflectors, etc. It is therefore necessary in these lamps to provide some form of chimney with a strong draught by which this deposit is carried off.

524. Efficiency of Arc Lights.—The efficiency of an arc light is much greater than that of an incandescent lamp. The common arc lamp, carrying a current of about 10 amperes at a pressure of about 50 volts, develops 2000 candle-power in the zone of maximum luminosity, or, in round numbers, one candle-power per 0.25 watt. The mean spherical candle-power (Par. 509) is, however, considerably less than 2000. The larger search lights, taking 200 amperes at 60 volts, develop nearly eight candle-power per watt, but it must be noted that there is a lack of agreement and much uncertainty as to the measurement of the candle-power of these powerful lights.

525. Luminous Vapor Lamps.—Suppose a high voltage, such as that produced by an induction coil, be applied to two platinum wires sealed into the opposite ends of a glass tube, and suppose that at the same time an air pump be set to work to exhaust the air from the tube. If the wires be not too far apart, sparks will

pass between them, but as the air is exhausted, these sparks lose their definiteness and finally take the form of an effulgence or glow completely filling the tube. The color of this glow varies with the nature of the gas enclosed in the tube. For air, it is rosy pink; for nitrogen, yellow; for carbon dioxide, white. At this stage the rarefied gas has great conductivity. If the exhaustion of the tube be continued, the conductivity decreases, the luminous column begins to break up in striae and finally disappears. When the pressure has been reduced to about one-millionth of an atmosphere, the glass itself begins to phosphoresce. Beyond this, the resistance becomes so great that no current can be sent through the tube. There is therefore a stage of rarefaction in which gases conduct electricity and in doing so emit light, and these effects diminish if the pressure be increased or decreased from what it is at this stage. Explanation of this will be given later; for the time being it will suffice to say that when highly rarefied these gases ionize and therefore conduct (Par. 276). If the exhaustion be complete, there are no ions left and consequently a vacuum is a non-conductor.

The foregoing is the principle of the luminous vapor lamps, two of which we shall now describe. Their luminous efficiency is very high, for while in the ordinary carbon filament lamp less than one per cent of the total energy expended is developed as light, in these luminous vapor lamps twenty per cent or more is so developed. They have not yet been made in small units but are rather used for general illumination of large spaces.

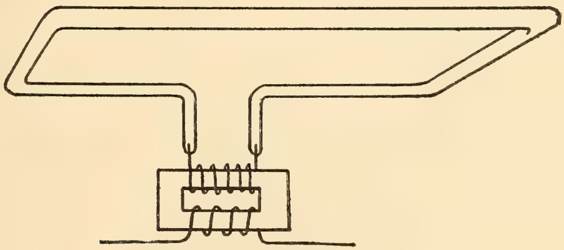


Fig. 251.

526. The Moore Light.—The apparatus for producing this light, shown diagrammatically in Fig. 251, takes the form of an exhausted glass tube one and three-quarters inches in diameter and of any length up to 200 feet. It is usually suspended along the

ceiling of the room to be illuminated. When in operation, it emits a soft, diffused light, without flickering or unsteadiness, the color varying, as stated in the preceding paragraph, according to the gas contained in the tube. To produce a light of fifteen candle-power per running foot, about 70 volts per foot are required, the corresponding current being about one-third of an ampere. By increasing the voltage, the candle-power can be raised to a maximum of thirty per foot. A tube 100 feet long requires 7150 volts. This high voltage is obtained from an alternating current by means of a simple step up transformer, as shown in the figure above.

As the lamp is used, the gas in the tube appears to be consumed and the rarefaction increases. This causes the resistance to increase. It therefore becomes necessary to introduce from time to time minute amounts of gas, and a simple and effective automatic valve has been devised for this purpose.

527. The Cooper Hewitt Mercury Vapor Lamp.—If in a glass tube, otherwise vacuous, there be introduced a small amount of mercury, the vacuous space would quickly become filled with the vapor of mercury (Par. 277). An electric current passed through this vapor would cause it to glow with a greenish light. This arrangement would not differ in principle from the Moore light, just described. In investigating it, however, Cooper Hewitt discovered some remarkable properties. Thus, at the outset, its resistance is so great as to require several thousand volts to start the current through it. This resistance seems to be confined to the surface of the negative electrode, and is temporarily destroyed by the passage of a current. Although several thousand volts are required to start the current through a tube twenty inches long, when once started it may be maintained by a pressure of 50 volts, provided it does not fall below one ampere. If it falls below this, the negative-electrode resistance re-asserts itself, the current ceases, and the high voltage is required to start the current again.

Various forms of this lamp have been devised, all alike in principle but differing in the arrangements for starting. A common form is shown in Fig. 252. This particular lamp, designed for use in a 100 volt circuit, and taking a current of three and a half amperes, consists of a one-inch glass tube, AB, 45 inches long and shaped as shown. It is supported by a frame CD, which carries the lead wires and which hangs from the suspension bar E. The

canopy *F* contains the various coils and electro-magnets used in connection with the lamp. The tube is exhausted to a pressure of one millimeter. The positive electrode *A* is of iron, a metal to which mercury does not adhere, and the negative electrode *B* is a small puddle of mercury.

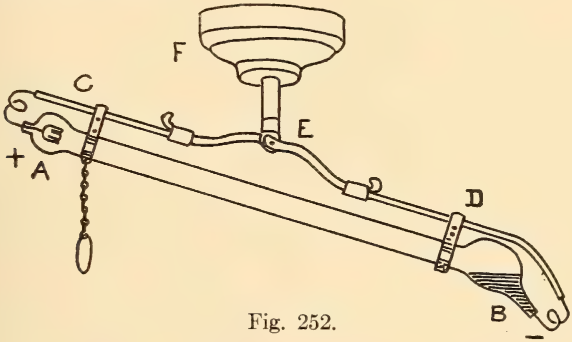


Fig. 252.

To start the lamp, the ring attached to *A* is pulled down, the lamp and frame rotating about the point *E* until *A* is slightly below the level of *B*. The mercury in *B* flows down the tube and makes contact at *A*. This little stream of mercury between *A* and *B* would act as a short circuit were it not for a ballasting coil (Par. 508) in the canopy *F*. The ring is now released, the lamp tips back to its original position and the mercury runs back into *B*. In doing so, the thread of mercury breaks at some point producing a flash-like arc, volatilizing some of the metal and ionizing the vapor so that the lamp starts. This voltage at break is aided by an inductance coil in series. In the smaller sizes of lamps, this tipping is done by electro-magnets. Several other starting devices are in use.

This form of lamp can be used with direct current only, but others are made for use with alternating currents. The principle of these latter will be explained when the subject of the mercury arc rectifier is reached.

The efficiency of the light is high, being 0.64 watt per candle-power. It is rich in actinic rays and especially valuable for photography, blue printing, etc., but has one very grave objection. It is devoid of red rays and red objects placed in it appear purple or black. It imparts to persons a peculiarly ghastly appearance and can not be used where colors are to be shown in their proper relation. No way has yet been discovered of adding the needed red.

CHAPTER 38.

THERMO-ELECTRICS.

528. Seebeck's Discoveries.—In 1821, in investigating Volta's contact series (Par. 187), Seebeck discovered that in a circuit composed of two metals, if one of the junctions be at a different temperature from the other, an E. M. F. and current will be produced. Fig. 253 represents a circuit composed of a strip of copper and one

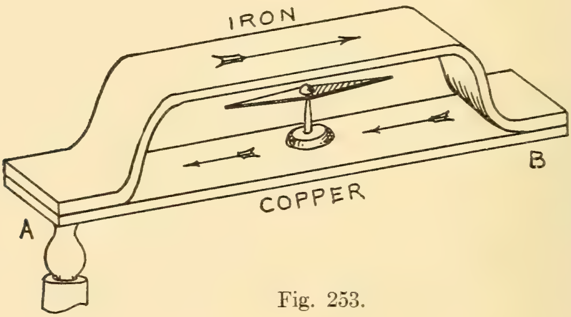


Fig. 253.

of iron which are joined at the points *A* and *B*. The strips may be welded, or soldered, or simply pressed together. If the junction *A* be heated so that its temperature is higher than that of *B*, a current will flow around the circuit in the direction indicated by the arrows, that is, at the cool junction it will flow from the iron to the copper, and at the hot junction, from the copper to the iron. The needle placed within the circuit will indicate this current. The two metals constitute a *thermo-couple*, and the E. M. F. produced is called the *thermo-electric electro-motive force*. Seebeck found further that this E. M. F. varied (a) with the metals used and (b) with the difference of temperature of the junctions, and he was able to arrange the following *thermo-electric series* in which, in a thermo-couple composed of any two, the current at the cold junction flows from the metal higher on the list to the metal which is lower.

Thermo-Electric Series.

Antimony	Tin
Iron	Lead
Zinc	Copper
Silver	Platinum
Gold	Bismuth

In accordance with these observations, thermo-couples are usually made of antimony and bismuth, though certain metallic sulphides may also be used. The E. M. F. produced is very feeble. Even for an antimony-bismuth couple, it is only about one ten-thousandth of a volt per degree Centigrade, or if one junction of such a couple be placed in boiling water, the other in melting ice, the E. M. F. will be about one-hundredth of a volt.

529. Thermo-Electric Inversion.—In 1823 Cumming added to the discoveries of Seebeck by showing that the thermo-electric

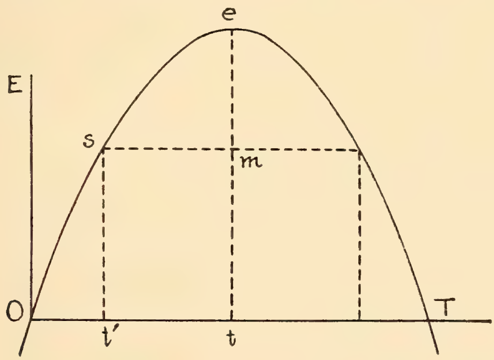


Fig. 254.

E. M. F. varied not only with the difference of temperature of the two junctions but also with their actual temperatures. Thus, if one junction of the copper-iron couple shown in Fig. 253 be kept at a constant temperature and the other be heated so that its temperature increases at a uniform rate, the E. M. F. will at first also increase uniformly but finally will slacken and will reach a maximum at 275° C, after which it will decrease. This is shown graphically by the curve in Fig. 254, in which the abscissae represent temperatures and the ordinates the corresponding E. M. F. The temperature *Ot*, at which the E. M. F. *te* is a maximum, is called the *neutral temperature* and varies for each different pair of

metals. If the temperature of the junctions be equally distant from t, the E. M. F. is zero. Thus at $OT = 2 \times Ot$, the E. M. F. is zero and beyond T it is negative, hence the current is reversed and OT is called the temperature of inversion. Had the constant temperature of one junction been Ot' instead of O, the maximum E. M. F. would have been me, the neutral temperature remaining unchanged. This thermo-electric curve has been shown by Lord Kelvin to be a parabola.

530. The Peltier Effect.—From what has just been seen, if one junction of an antimony-bismuth thermo-couple be heated, as

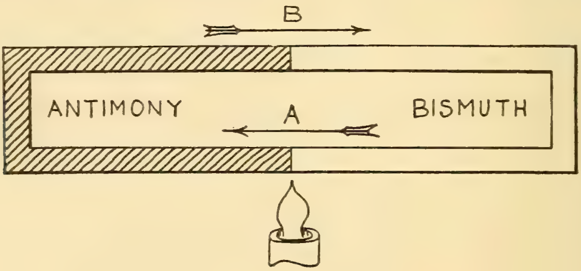


Fig. 255.

shown in Fig. 255, a current will flow around the circuit as indicated by the arrows, that is, flowing at the cold junction B from the antimony to the bismuth.

If the source of heat be now removed, the current will still continue to flow so long as the junction A is at a higher temperature than the junction B. The only conceivable source of this current is the heat energy at A, and since this heat energy is converted into electrical energy, there must be at that point an absorption and disappearance of heat. Also, since the actual current through the junction B is opposite in direction to the current which would have been produced by the absorption of heat at that point, the logical inference is that heat is developed at B. The correctness of this inference was shown by Peltier in 1834. A bar of antimony and one of bismuth were placed crosswise as shown in Fig. 256 and were soldered together. Between the ends C and B were connected a galvanometer G and a key S. Between A and D were connected a battery and a key K. K was closed for a while, allowing a current to flow around the triangular circuit in the direction DEA, or passing at the junction from the bismuth to

the antimony. K was then opened and S was closed. The galvanometer immediately indicated a current from C to B, showing that the junction E had been *cooled* below the temperature of B and C by the passage of the current from the battery. The battery was now reversed so that when K was closed the current flowed in the direction AED, or from the antimony to the bismuth. After a while, K was again opened and S closed. The galvanometer now indicated a current from B to C, showing that the junction E had been *heated* above the temperature of B and C.

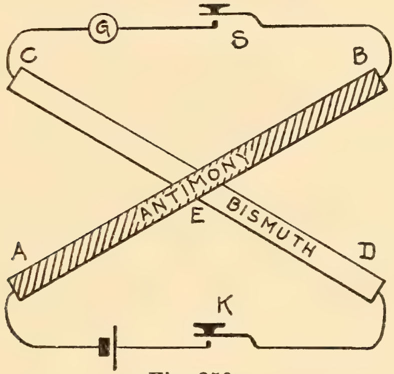


Fig. 256.

We thus see that when a current is passed across the junction of two dissimilar metals, heat is evolved if the current flows from the metal that is the higher in the thermo-electric series (Par. 528), and heat is absorbed if it flows from the metal that is the lower in this series.

This heating or cooling produced by the passage of a current across the junction of two dissimilar metals is called the *Peltier effect*, and is entirely distinct from the Joule effect discussed in Chapter 35. The Joule effect varies as the square of the current and is independent of the direction of flow; the Peltier effect varies as the first power of the current and is reversed if the direction of the flow be reversed.

In the manufacture of very delicate electrical measuring instruments, consideration must be given to these various thermo-electric effects. If in the circuit of such instruments a junction of different metals occurs, the heating effect of the current may set up thermo-electric effects which might cause appreciable error in the indications of the instrument.

531. The Thomson Effect.—Sir William Thomson (Lord Kelvin) showed that when a current flows through a homogeneous conductor which is heated at one point more than at another, heat is either developed or absorbed, depending upon the nature of the conductor and the direction of the current. Thus, in a copper wire whose center is hotter than the ends, heat is absorbed by the current as it flows towards the hot center and evolved as it flows from this center. With an iron wire, these effects are reversed, heat being developed in the first half and absorbed in the second. This *Thomson effect* has not been observed in lead and consequently lead is taken as the standard, or is made one of the elements in each thermo-couple which is tested in order to determine the thermo-electric power of the various metals.

The subject of thermo-electricity is susceptible of elaborate mathematical treatment but its importance is not now sufficient to warrant a more extended discussion. We shall therefore pass at once to a description of some of its practical applications.

532. The Thermopile.—Although, as stated above (Par. 528), the E. M. F. of a thermo-couple is very feeble, if a number of these couples arranged in the same order be connected in series and the alternate junctions be heated, the E. M. F.s will all act in the same direction and the total E. M. F. will be the sum of the separate E. M. F.s, in other words, the arrangement is similar to a battery composed of a number of cells connected in series. Such an arrangement is called a *thermopile.*

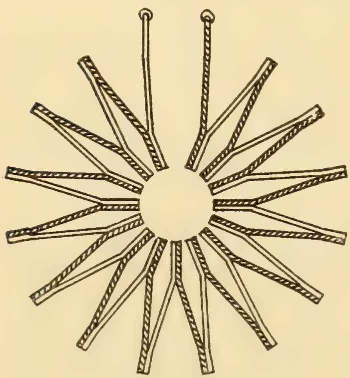


Fig. 257.

Many forms of thermopiles have been devised. For example, the couples may be grouped as shown in Fig. 257 like the spokes of a wheel radiating from a central cylindrical opening, and there may be a number of these groups placed one above the other and all connected in series. The interior cylinder may then be heated by a small furnace, by gas jets, or by hot water, the outer ends of the couples being cooled by the air.

At first sight it seems that the thermopile affords a satisfactory solution of an extremely important problem, the direct conversion

of heat energy into electrical energy without the usual intermediate steps of heating water, producing steam, utilizing the expansion of the steam to produce rotation, and by means of this rotation producing electricity as outlined in Par. 423, each of which steps is accompanied by inevitable loss of energy. Thermopiles have been constructed to furnish the small currents required in gold and silver plating, and are used in certain extremely sensitive heat-measuring instruments (Par. 533), but where electricity is to be supplied on a large scale, they are a failure. The Joule effect, the Peltier effect and the heat conductivity of the two metals all tend to raise the temperature of the cool junctions and thus decrease the E. M. F., and the couples themselves deteriorate rapidly with use. Their efficiency is very low, less than one-half of one per cent of the heat energy being converted into electrical energy.

533. The Radiometer.—There has been employed for the comparison of radiant heat from different sources, a thermopile consisting of a rectangular bundle of thermo-couples arranged in series and mounted in a frame as shown in Fig. 258. The contiguous

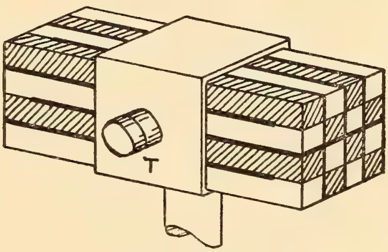


Fig. 258.

couples and the metal strips of each couple, except at the junctions, are insulated from each other by sheets of mica. The first and last strips of the series are connected to terminals T, which are attached one on each side of the frame. The pile, except the end which is to receive the radiant heat, is shielded by a protecting hood. The receiving end is coated with lampblack, the best absorbent of heat. When in use, a sensitive galvanometer is connected to the terminals, the current through the galvanometer varying directly as the difference of temperature of the hot and cold faces of the pile.

Thermometers and pyrometers have been constructed on the principle of the thermopile. In the pyrometers, the couple is composed of platinum and rhodium.

534. The Radio-Micrometer.—An extremely sensitive form of radiometer, the *radio-micrometer,* has been devised by Vernon Boys. It combines the principles of the thermo-couple and the d'Arsonval galvanometer. As shown diagrammatically in Fig. 259 it differs from the d'Arsonval galvanometer (Par. 378) only in that a quartz fibre is substituted for the phosphor-bronze suspension, and the coil consists of a single vertically-elongated loop of copper wire. To the lower ends of this loop there are soldered two small bars of antimony and bismuth and these bars are connected by a little sheet of lampblack-coated copper foil, only one-tenth of an inch square. When the copper foil is heated, the E. M. F. of the couple is very small but, since the resistance of the copper loop is also small, the current is appreciable and the loop moves in accordance with Maxwell's law (Par. 371), the deflection being observed by means of the mirror *M*. It is said that a change in the temperature of the copper foil of one-millionth of a degree will cause a deflection of one division on the scale, and that the radiant heat of a candle can be detected at a distance of two miles. Instruments of this kind, known also as *bolometers,* have been used to measure the heat radiated from the stars and to compare the heat emitted from different portions of the solar spectrum.

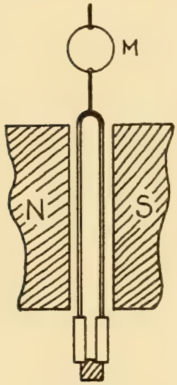


Fig. 259.

CHAPTER 39.

REMARKS ON CERTAIN ELECTRIC UNITS.

535. Two Systems of Electric Units.—There are two distinct systems of electric units; one, the *electro-static*, based upon the interaction of static charges; the other, the *electro-magnetic*, based upon the interaction of a magnetic pole and the field produced about a conductor carrying a current. The electro-magnetic units, and the derived practical units, are, on account of their suitability for practical purposes, used to the exclusion of those of the electro-static system. Nevertheless, it is desirable for the student to be acquainted with both systems and to understand the relation existing between them.

In the electro-static system, the starting point is the unit quantity, which is defined (Par. 56) as that quantity which when placed at a distance of one centimeter in air from a similar and equal quantity, repels it with a force of one dyne.

In the electro-magnetic system, the starting point is the unit pole, or (Par. 133) that pole which, when placed at a distance of one centimeter from a similar and equal pole, repels it with a force of one dyne.

536. Units of Current and Quantity.—Thus far, there does not seem to be much to choose between the two systems. In the next step, however, there is a marked difference.

In the electro-static system the unit current is that current which conveys unit quantity in unit time.

In the electro-magnetic system, the unit current can not be defined so simply. We have shown, however (Par. 353), that a current flowing in a conductor establishes about that conductor a magnetic field which varies directly with the current. Therefore, with other conditions constant, we may take the strength of the field produced as a measure of the strength of the current, and the simplest way to compare magnetic fields is to compare the forces which they exert upon the same pole. The electro-magnetic unit of current is therefore defined (Par. 355) as that current which,

flowing through one centimeter of a conductor bent into the arc of a circle whose radius is one centimeter, exerts a force of one dyne upon a unit pole placed at the center of the circle. This current, we have seen, is ten amperes.

Having thus defined the unit current, we may now define the electro-magnetic unit of quantity as that quantity conveyed by unit current in unit time. The ampere flowing for one second conveys one coulomb; the absolute unit of quantity is therefore equal to ten coulombs. It is thus seen that in the electro-static system we pass from unit quantity to unit current; on the other hand, in the electro-magnetic system, we pass from unit current to unit quantity.

By experiments and measurements based on widely different methods, it has been found that the electro-magnetic unit of quantity is about $(2.98+)(10^{10})$ times as great as the electro-static unit of quantity. For round numbers, this is taken as 3×10^{10}, or thirty billion. The coulomb, therefore, as has already been stated (Par. 56), is three billion (3×10^{9}) times as great as the electro-static unit.

537. Units of Electro-Motive Force.—In either system, unit difference of potential exists between two points when the expenditure of one erg is required to convey a unit of quantity of electricity from one to the other. The electro-magnetic unit of potential is therefore $\frac{1}{3 \times 10^{10}}$ times the electro-static unit of potential. In Par. 427 it was stated that 10^{8} absolute electro-magnetic units of potential were equal to one volt. The volt is therefore $3 \times 10^{10-8} = 3 \times 10^{2} = 300$ times as small as the electro-static unit of potential, or, as was stated in Pars. 77 and 78, the electro-static difference of potential in ergs must be multiplied by 300 to reduce it to volts.

538. Primary Electro-Magnetic Units.—The units of E. M. F., current and resistance are bound together by Ohm's law, $I = E/R$, which necessarily is true whatever units be employed, that is, whether we use the *absolute* or the *practical* units. It follows that

$$\text{absolute unit of current} = \frac{\text{absolute unit of E. M. F.}}{\text{absolute unit of resistance}}$$

If, therefore, any two of these units be fixed upon, the third follows as a matter of course; or, it suffices to define any two, and these

definitions fix the third. It was this consideration that led to the definition of resistance as a ratio, to which definition attention was called in Par. 307.

The question now arises, which two shall be selected as our primary units.

In Par. 355, the definition of the absolute unit of current was given (repeated in the preceding paragraph), and in Par. 374 it was shown how by means of the tangent galvanometer a current could be measured in absolute units. The absolute unit of current is therefore selected as one of the primary units.

Reflection will show that of the three units, resistance is the only one which could be perpetuated in a material standard, such as a given length of a certain-sized wire of a specified material. If resistance could be measured absolutely, it would naturally be selected as the second primary unit. We shall now explain how this may be done, but preliminary thereto we must develop another conception of electric resistance.

539. Dimensional Formulae.—It has been shown (Par. 10) that the fundamental units of our system are the centimeter, the gram and the second, and that all the other units are derived from these. It is therefore possible to express any derived unit in terms of length, mass and time. Such expressions are called the *dimensional formulae* of the units in question. A study of these dimensional formulae will afford a clearer conception of the nature of the units and will bring to light unexpected relations.

540. Dimensional Formulae of Electro-Magnetic Resistance.—From Ohm's law, $R = E/I$. E, the difference of potential, is measured by the work done in moving unit quantity of electricity through a difference of potential E. If to move Q units the work done is W, then to move one unit, the work is W/Q, whence

$$E = \frac{W}{Q} \qquad \text{(I)}$$

But work = force × path = $F \times L$, and $Q = I \times T$. Substituting these values in (I)

$$E = \frac{F \times L}{I \times T}$$

Substituting this for E in Ohm's law

$$R = \frac{F}{I^2} \times \frac{L}{T} \qquad \text{(II)}$$

Two poles, each of strength m, at a distance L apart exert upon each other a force $F = m^2/L^2$, whence

$$m = \sqrt{F.L^2} \qquad \text{(III)}$$

A pole of strength m placed in a magnetic field of strength H is acted upon by a force $F = m.H$, whence $H = F/m$.

The field produced at the center of a circular coil by a current I (Par. 354) is proportional to I/L, or $H = I/L$, L being the radius of the coil. Equating these two values of H and solving for m, we have $m = F.L/I$.

Substituting this value of m in (III), and solving, we have $F = I^2$, whence (II) becomes

$$R = L/T$$

But L is length and T is time, hence resistance is of the nature of a velocity.

541. Resistance Expressed as Velocity.—Why it is possible to express resistance as a velocity may be shown as follows: Let Fig.

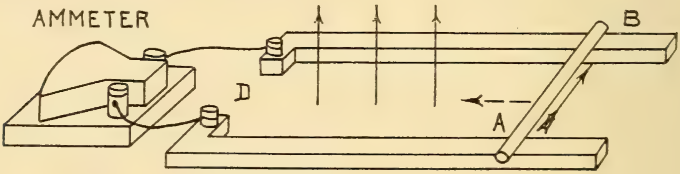


Fig. 260.

260 represent the arrangement of parallel rails and sliding cross bar which we have already described several times. Suppose the rails to be of negligible resistance, to be one centimeter apart and to embrace between them a uniform unit field. AB, moving with uniform velocity, is slid along towards D, which is at an indefinite distance to the left. If AB moves V centimeters per second it will cut V lines of force and will generate V absolute units of E. M. F., in direction from A to B (Par. 422). If the resistance of AB be R, the current through AB will be

$$I = \frac{V}{R}$$

Since the current varies directly with V, the velocity of AB, it is possible to move AB rapidly enough to make I one absolute unit of current. When I becomes 1, the above expression becomes $R = V$, or R is expressed as a velocity.

If R be one ohm, in order to drive a current of one absolute unit through AB, it must be moved with a velocity of 10^9 centimeters (ten million meters, or one earth's quadrant per second (Par. 4)).

From the foregoing, knowing the strength of the field between the rails and the velocity with which AB is moved, we could determine V. The current in the circuit could be read from an ammeter at D. Thus having V and I, the quotient of the former by the latter would give R, the resistance of AB. Practically, such a determination is impossible. AB could not be moved for a sufficient length of time with the desired rapidity; it would not, as it moved, maintain unvarying contact with the rails; and finally, the resistance of the rails is not negligible, hence the resistance of the circuit would continually decrease. However, several methods have been devised by which these difficulties are obviated and we shall now explain one, first proposed by Weber and improved by later investigators.

542. Absolute Measurement of Resistance.—In Fig. 261, AB represents a circular coil of a number of turns of wire, the ends of

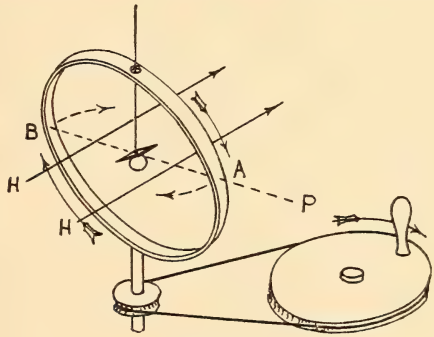


Fig. 261.

the coil being joined together. It is mounted upon a vertical axis about which it may be spun rapidly. Through an opening in the top there extends a silk fibre from which there hangs at the center of the coil a needle. The arrows H represent lines of force of the earth's field. If the coil, viewed from above, be spun in a clockwise direction, it will cut the lines H and consequently an E. M. F. will be induced. Application of the right hand rule (Par. 422) will show that as the side B moves from B to A, it will generate an E. M. F. acting upward and during the same time a downward

E. M. F. will be generated in the side A, that is, there will be induced in the coil a current, which, viewed from the point P (a point on the horizontal axis of the coil perpendicular to the meridian), will be counter-clockwise in direction. As B passes the position A, and A passes the position B, the direction of the E. M. F. in B and in A, and consequently the direction of the current in the coil, is reversed, but at this same instant the opposite face of the coil is presented to P, so that viewed from P, the current flowing around the coil is always in the same direction. This current is pulsating. It is zero when the plane of the coil is at right angles to the magnetic meridian, and it is a maximum when this plane coincides with the meridian, hence it rises and falls with every half revolution of the coil. At the instant when the plane of the coil coincides with the magnetic meridian, the instrument is in principle the same as a tangent galvanometer (Par. 373), and at all times it may be regarded as a tangent galvanometer traversed by a current whose value is a mean of the instantaneous values of the current. The suspended needle will be deflected accordingly.

The induced E. M. F. will vary directly with the rate of cutting of the lines of force embraced by the coil. The number embraced is $\pi r^2 H$, r being the mean radius of the coil. The rate at which these are cut varies with ω, the angular velocity of the coil, and with n, the number of turns in the coil. If R be the resistance of the coil, the current through it is proportional to

$$\frac{\pi . r^2 . H . n . \omega}{R}$$

The field produced at the center of the coil is (Par. 354)

$$f = I \times \frac{2\pi n}{r} = \frac{2\pi^2 r H n^2 \omega}{R}$$

If the needle be deflected through an angle δ, we have (Par. 146)

$$\frac{2\pi^2 r H n^2 \omega}{R} = H . \tan \delta$$

whence

$$R = \frac{2\pi^2 n^2 r \omega}{\tan \delta}$$

But $r\omega$ is the actual velocity of a point at the extremity of the horizontal diameter of the coil. Calling this v, we have

$$R = \frac{2\pi^2 n^2}{\tan \delta}.v$$

whence we see that the resistance of the coil is equal to the product of a velocity by a numerical factor. In the expression above, n and r are constants of the instrument and ω and δ are determined by observation. It will be noted that it is not necessary to know the strength of the needle or the intensity of the field H. If v be expressed in centimeters per second, R will be in absolute units of resistance.

In actually carrying out the above determination, many delicate refinements were observed. These are described in detail in the Report of the British Association for the Advancement of Science for the year 1864.

Resistance has been measured absolutely by several other methods.

543. The Ohm.—As a result of the experiment outlined in the preceding paragraph, the investigators became possessed of a coil of wire whose resistance in absolute units was accurately known. The absolute unit being excessively small, the next step was to select a practical unit which should be based upon this absolute unit. It has been shown (Par. 284) that the need for a unit of resistance had been felt for some time. The resistance coils made by Ohm could not be standardized. In 1860 Siemens defined as a unit of resistance a column of pure mercury one meter long and one square millimeter in cross-section, the mercury being at a temperature of 0° C. Electricians had become accustomed to this unit and the German scientists especially were loath to give it up. The practical unit of resistance, *the ohm,* was accordingly chosen so as to agree as nearly as possible with Siemen's unit, and was defined as 10^9 absolute units of resistance, or (Par. 291) as the resistance of a column of mercury, one millimeter in cross-section and 106.3 centimeters in length, at a temperature of 0° C. It was later found more convenient to retain the length of the column but to specify the quantity of mercury in terms of weight, or as 14.4521 grams.

544. The Ampere.—We have seen above (Par. 538) that the absolute unit of current had been determined from the tangent

galvanometer. It remains now to fix the practical unit of current. The existing practical standard of E. M. F. was that of the Daniell cell (Par. 206). This applied to the practical unit of resistance, the ohm, should drive through it the unit current. This current was found to be very nearly one-tenth of the absolute unit. The practical unit of current, *the ampere*, was therefore selected as exactly one-tenth of the absolute unit. Its definition has already been given (Par. 228).

545. The Volt.—The selection of the primary practical units of resistance and current also fixed *the volt*, the practical unit of E. M. F. From Ohm's law, $E=IR$. Since $I=10^{-1}$ absolute units of current and $R=10^9$ absolute units of resistance, $E=10^{-1}\times10^9$ $=10^8$. The volt was therefore defined as 10^8 absolute units of E. M. F.

546. Résumé.—The following résumé will show the thread of connection between the successive steps in the adoption of the absolute and the practical electro-magnetic units.

(a) The absolute unit of current was determined by means of the tangent galvanometer.

(b) The absolute resistance of a coil of wire was determined by rotating the coil.

(c) From this was determined the absolute unit of resistance.

(d) This was found to be about $.954\times10^{-9}$ of Siemen's mercury unit, already in use.

(e) To disturb this standard as little as possible, the practical unit of resistance, the ohm, was taken as 10^9 absolute units of resistance.

(f) The existing practical standard of E. M. F. was that of the Daniell cell (1.07 volts) and it was desirable to disturb this as little as possible.

(g) A Daniell cell applied to a circuit of one ohm drove through it a current which was very slightly greater than one-tenth of the absolute unit of current.

(h) The practical unit of current, the ampere, was taken as exactly one-tenth of the absolute unit of current.

(i) The selection of the practical units of resistance and current involved that of E. M. F., the volt, since the three units are bound together by Ohm's law. The volt is, therefore, 10^8 absolute units of E. M. F.

547. Comparison of the Dimensional Formulae in the Two Systems.—A comparison of the dimensional formulae of the units in the two systems will point to the contradictory conclusion that they do not agree. As an example, let us compare the dimensional formulae of the units of quantity.

In the electro-static system, we have from Coulomb's laws for the force exerted between two equal quantities Q (Par. 56), $F=Q^2/L^2$, whence $Q=L\sqrt{F}$. In mechanics it is shown that force = mass × acceleration, or $F=M\times L/T^2$. Substituting this value of F in the expression above, we have for the electro-static dimensional formula of quantity

$$Q=L\sqrt{M.L}/T \qquad \text{(I)}$$

In the electro-magnetic system, $Q=I\times T=(E/R)\times T$. In Par. 540 it was shown that $E=F\times L/Q$ and that $R=L/T$, whence the electro-magnetic dimensional formula of quantity is

$$Q=\sqrt{M.L} \qquad \text{(II)}$$

Comparing (I) and (II), we see at once that they are not the same, and that the ratio of (I) to (II) is L/T, a velocity.

In a similar manner may be determined the dimensional formulae of the remaining units of current, capacity, potential resistance, and inductance as given in the following table:

Unit	*Electro-static*	*Electro-magnetic*	*Ratio*
Current	$L\sqrt{M.L}/T^2$	$\sqrt{M.L}/T$	$L/T=V$
Quantity	$L\sqrt{M.L}/T$	$\sqrt{M.L}$	$L/T=V$
Capacity	L	T^2/L	$L^2/T^2=V^2$
Potential	$\sqrt{M.L}/T$	$L\sqrt{M.L}/T^2$	$T/L=1/V$
Resistance	T/L	L/T	$T^2/L^2=1/V^2$
Inductance	T^2/L	L	$T^2/L^2=1/V^2$

The V which enters all of these ratios has been determined in widely different ways by a number of observers and found to be 3×10^{10}, or thirty billion, centimeters per second. *This is the velocity of light.*

548. Explanation of Lack of Agreement.—It is on the face of it absurd that like quantities should have different dimensional formulae, and also that these formulae should contain such irrational quantities as the square root of a mass and of a length. Consideration will show that this state of affairs results from our failure to take into account in the formulae above the dielectric

coefficient K (Par. 90) in the case of the electro-static units, and the permeability μ (Par. 392) in the case of the electro-magnetic units. The medium being air, these factors are both unity and hence are of no arithmetical effect, but in omitting them we are not justified in ignoring their dimensions. What these dimensions are, we do not know, but that they account for the lack of agreement in the dimensional formulae of the two systems the following will show.

In the preceding paragraph, in determining the dimensional formula of the electro-static unit of quantity, our assumed expression for the force between two equal quantities Q should have been (Par. 90)

$$F = Q^2 / K.L^2$$

whence

$$Q = L\sqrt{K}.\sqrt{M.L}/T \qquad \text{(I)}$$

Likewise, in determining the dimensional formula of the electro-magnetic unit of quantity, the expression for the force between two equal magnetic poles should have been (Par. 133)

$$F = m^2/\mu.L^2$$

whence

$$m = L\sqrt{\mu}.\sqrt{M.L}/T$$

The force exerted by a current I, flowing in a circular coil of radius L, upon a pole m at the center of the coil is proportional to mI/L (Par. 355), whence

$$I = F.L/m$$

Substituting the value of m above and multiplying by T, we have, since $Q = I \times T$

$$Q = \sqrt{M.L}/\sqrt{\mu} \qquad \text{(II)}$$

Equating the second members of (I) and (II) and solving

$$\frac{L}{T} = \frac{1}{\sqrt{K\mu}}$$

We see then that while the dimensions of the separate factors K and μ are unknown, the reciprocal of the square root of their product is a velocity, and therefore they can not be disregarded.

This velocity, as stated in the preceding paragraph, is the velocity of light, and is also the velocity with which electric waves travel through space. As will be shown later it has an important bearing on Maxwell's electro-magnetic theory of light, which is that light is really due to the passage of electric waves through the ether.

PART V.

ELECTRO-MECHANICS.

CHAPTER 40.

DIRECT CURRENT GENERATORS.

549. Electro-Mechanics.—*Electro-Mechanics,* the subject which we are now to take up, is a more or less artificial division intended to embrace the production of electric currents by machinery, a consideration of these mechanically generated currents, and finally their employment to operate other machines.

Electricity, no matter how produced, is always the same agent and the principles which have been developed in the preceding pages suffice to explain all the facts which we shall now bring out. The currents produced by machines are, however, more or less pulsating and are often alternating, that is, they periodically (usually many times a second), change their direction. These rapid changes in the current give rise to certain phenomena which renders it desirable to consider these currents in detail.

550. Classes of Electrical Machines.—Electrical machines are primarily of two classes, *generators* and *motors.* The former, also called *dynamos,* transform mechanical energy into electrical energy and therefore deliver electrical energy to a circuit. On the other hand, motors transform electrical energy into mechanical energy and therefore receive electrical energy from a circuit.

Machines are further classed according as they are designed to deal with direct currents or with alternating currents. We shall now consider generators of the former class.

551. Coil Rotating in a Magnetic Field.—Suppose CD, Fig. 262, to be a coil in the magnetic field NS and free to rotate about the axis AB. Suppose its initial position to be, as shown in the figure, with its plane perpendicular to the lines of force of the field. It

now embraces the maximum number of these lines, and the first effect of rotation about AB, whether clockwise or counter-clockwise, will be to decrease this number. This change will develop in the coil an induced E. M. F. whose direction may be determined by application of the rule given in Par. 421. It is simpler, however, to apply the right hand rule given in Par. 422, whence we see at once that whether the rotation be clockwise or counter-clockwise, as the side C of the coil rotates 180° from the position C to the

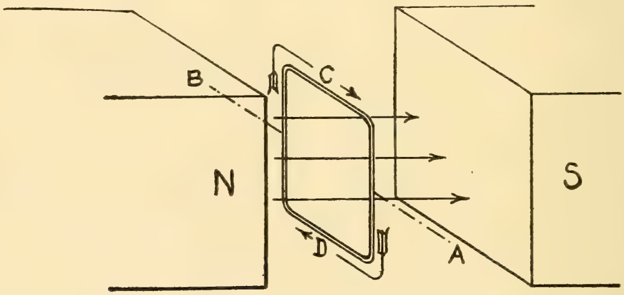


Fig. 262.

position D, there is an E. M. F. induced in C from rear to front, while as it rotates from D to C, the E. M. F. induced is from front to rear. The E. M. F. is reversed in direction whenever the coil passes through the perpendicular plane, and is zero when the coil lies in it, for which reason this plane is called the *neutral plane.*

552. Calculation of E. M. F. of Rotating Coil.—The E. M. F. induced by the rotation of a coil in a magnetic field is from Par. 426 equal to the rate of decrease of the number of lines of force embraced by the coil. If the field be uniform, this E. M. F. may be calculated as follows. Let ab, Fig. 263, be the primary position of the coil, its plane at right angles to the field. Its E. M. F. at any point, such as d, is measured by the rate of decrease at that point of the number of lines embraced.

Let the total field embraced by ab be N. Let the coil make n revolutions per second, that is, let its angular velocity be $2\pi n$. If ca, the radius of the circle described by a, be R, the actual velocity of a is $2\pi nR$ per second. At h the coil is moving at right angles across the field with a velocity which in one second would

carry it a distance hk. The total width of the field being $2R$, it would be crossed in $\frac{2R}{2\pi nR} = \frac{1}{\pi n}$ seconds, and in this time N lines of force would be cut by each side of the coil, therefore, the E.M.F. being generated at h is

$$E = \frac{2N}{1/\pi n} = 2\pi n N$$

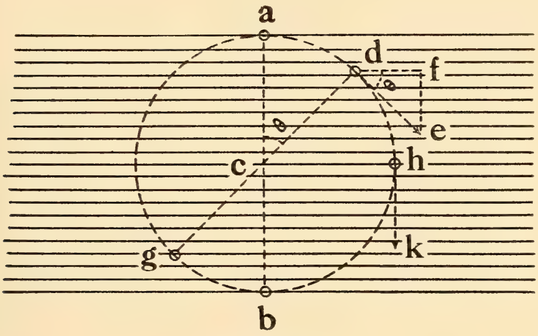


Fig. 263.

If the coil consists of S turns, the E. M. F. is $2\pi n NS$. To convert this to volts, it must be divided by 10^8 (Par. 427), whence finally

$$E = 2n\pi NS/10^8 \text{ volts.}$$

Should the coil at d continue to move for one second in the same direction and at the same rate as at d, it would move a distance $de = hk$ and in doing so would cut across the lines between f and e. If the angle dca through which the coil has turned from its primary position be θ, then $fe = de.\sin\theta$. At the same time, the other side g of the coil is cutting across the field at this same rate, the total decrease being $2de.\sin\theta$. Since $de = hk$, the E. M. F. being generated at d is

$$E = \frac{2n\pi NS}{10^8} \cdot \sin\theta$$

Or placing C for the coefficient of $\sin\theta$,

$$E = C.\sin\theta$$

For the present it is sufficient to bear in mind that the E. M. F. generated by a coil rotating in a uniform field varies as the sine of the angle through which the coil has turned from its primary position at right angles to the field, that is, from the neutral plane.

553. Production of Current by Rotating Coil.—In Fig. 264 let *CD* represent a coil rotating about the axis *XY* in the magnetic field *NS*, and suppose that instead of being a closed coil, the end *C* terminates in a ring *A*, and the end *D* in a ring *B*, these rings being attached to the axis upon which the coil rotates, but being insulated from it and from each other. In Par. 551 it was shown that as the coil rotates 180° from its present position at right

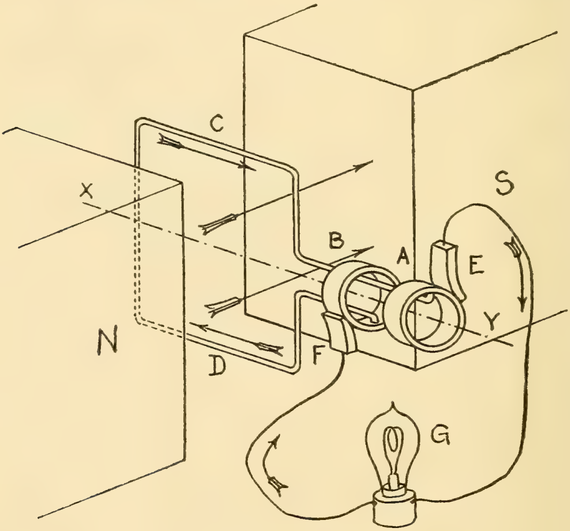


Fig. 264.

angles to the field, an E. M. F. is generated from rear to front in *C* and from front to rear in *D*. No current is produced because the circuit is broken between the rings. If now a metal strip *E* be pressed against the ring *A*, and a second strip *F* be pressed against *B*, and these strips be connected by a wire, the circuit will be completed and a current will flow through the coil and wire as indicated by the arrows. *A* and *B* are *collector rings*, *E* and *F* are *brushes*, and the wire connecting these brushes is the *external circuit*.

554. Alternating Current.—The resistance of the arrangement just described being constant, the current in the external circuit varies directly with the E. M. F. generated in the coil and this, we have seen (Par. 552), varies as the sine of the angle through which the coil has rotated from its position in the neutral plane.

Thus, at the instant shown in Fig. 264, the current in C is zero, but as C moves, a current flows towards A, reaching its maximum value when the coil has turned through 90° or has become parallel to the lines of force of the field. From this point, the current diminishes and is again zero when C has turned through 180° or has reached the position D. As C passes this point, the current again starts up, but it is now reversed, that is, it flows away from instead of towards A, and it consequently is also reversed in the external circuit. If the original direction be considered positive, this last must be considered negative. The current therefore reaches a negative maximum when C has turned through 270°, returns to zero when C reaches its primary position, and again reverses as C passes through this position.

To an E. M. F. and current which thus pass through these periodic fluctuations and reversals, the term *alternating* is applied.

555. Graphic Representation of Alternating E. M. F. and Current.—In Fig. 265 let B represent the cross-section of a coil rotating about O as a center and in a uniform field whose positive

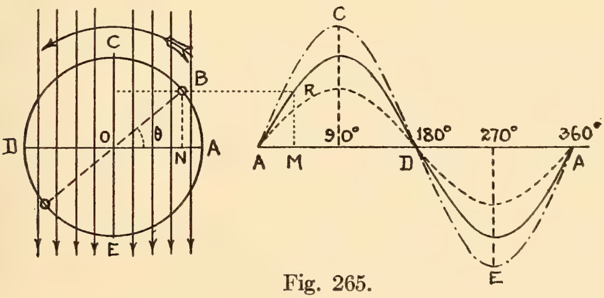


Fig. 265.

direction, as indicated by the arrows, is downwards. AD is therefore the neutral plane. Should the coil start at A, the direction of the induced E. M. F. is independent of the direction of rotation, that is, whether the coil rotates in a clockwise or in a counter-clockwise direction, the E. M. F. will act out from the plane of the paper. However, to conform to the trigonometric convention as to the direction in which angles are to be measured, we shall assume the rotation to be counter-clockwise. From Par. 552, the induced E. M. F. at any point B is proportional to BN, the sine of the angle θ through which the coil has rotated. If, therefore, we lay off on a horizontal axis, AA, distances proportional to the

angles through which the coil has turned, and at the points so determined erect ordinates upon which we lay off distances proportional to the sines of the corresponding angles, the sine or harmonic curve drawn through the extremities of these ordinates will represent the successive values of the E. M. F. For example, the point R is determined by laying off AM proportional to AB, and MR proportional (in this case equal) to NB.

The curve shows what was stated in the preceding paragraph, that is, that the E. M. F. is zero at A, rises to a maximum when the coil reaches C, decreases to zero at D where it reverses, reaches a negative maximum at E and returns to zero at A, and so on.

In the case under consideration, the E. M. F. acting towards the observer is considered positive, but this is purely a matter of convention and it is immaterial whether we regard it as positive or negative provided that the E. M. F. induced as the coil rotates from A to D be opposite in sign to that induced as it rotates from D to A. If the direction of the field be reversed, the direction of the E. M. F. is also reversed.

Since the current varies directly with the E. M. F., we may take this same sine curve as representing the current also, or we may represent the current by another sine curve of the same periodicity but of different amplitude. From Ohm's law, $I=E/R$, we see that I and E are numerically equal only when R is unity. If R be less than unity, I is numerically greater than E and would be represented by the outer broken curve in Fig. 265. If R be greater than unity, I would be represented by the inner broken curve.

Reflection will show that the abscissae of these sine curves may also be laid off on a scale of *time*, the distance AA corresponding to the *time* of one complete revolution of the coil.

556. Rectification of Alternating Current.—Fig. 266 represents the same arrangement of a coil rotating in a magnetic field as described in Par. 553, only in this case the ends of the coil terminate in the copper semicircles or segments A and B instead of in two separate rings. These segments are likewise mounted upon the shaft of the coil, insulated from it and from each other. The brush E presses against the segment A; the brush F against the segment B. For simplicity of description, suppose the rotation to be clockwise. As C moves from its present position to the position D, the induced E. M. F. acts towards A, and current will therefore enter the external circuit by the brush E and leave it

by the brush *F*. As *C*, having reached the position *D*, passes through the neutral plane, the induced E. M. F. becomes zero and immediately thereafter reverses, that is, acts *from A* and

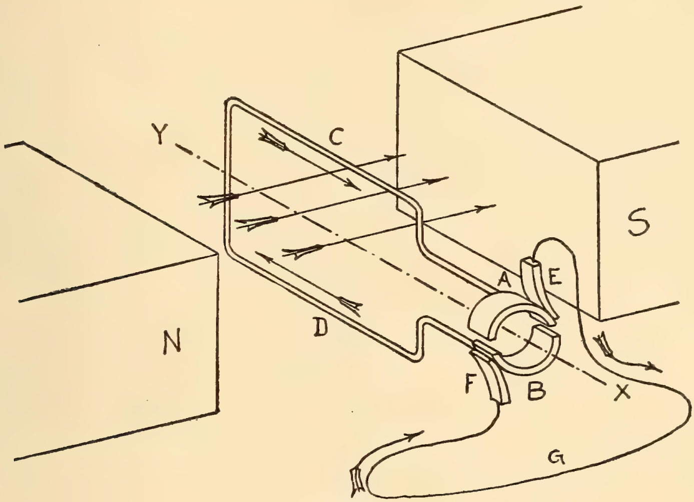


Fig. 266.

towards B. But also, as the coil passes through the neutral plane the brushes slip across the gap between the segments and *E* is now in contact with *B*, while *F* is in contact with *A*, therefore,

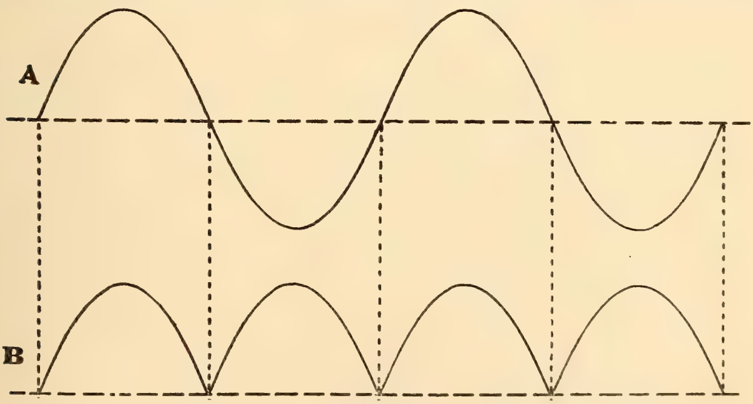


Fig. 267.

current still flows out into the external circuit through the brush *E* and the direction of the current in the external circuit remains unchanged.

This is shown graphically in Fig. 267. The sine curve *A* represents the alternating current (and E. M. F.) in the coil. *B* represents the current in the external circuit, the negative loops of the curve *A* having been reversed and made positive.

An alternating current which has thus been made unidirectional is said to be *rectified.* The *split ring,* or arrangement of copper segments by which this is brought about, is a *commutator,* and the process is called *commutation* or *rectification.* We shall see later that an alternating current may be rectified otherwise than by a commutator.

557. Increase in Number of Turns of Coil.—If the rotating coil, instead of consisting of a single turn, be composed of several

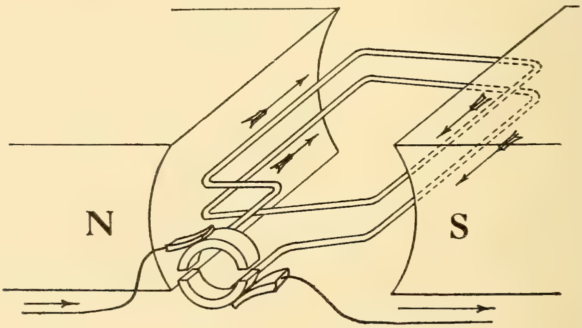


Fig. 268.

as shown in Fig. 268, an approximately equal E. M. F. will be induced in each. Examination of the figure will show that these turns being connected in series, the total E. M. F. is the sum of the separate E. M. F.s, or increases in proportion to the number of turns. The resultant E. M. F. of the coil is represented graphically by a sine curve of the same periodicity as the curves in Fig. 267 but of an amplitude greater in proportion to the number of turns.

Although the E. M. F. is thus increased by increasing the number of turns, practical considerations place a limit upon the number that may be added. Thus, the resistance of the coil increases directly with the number of turns and it is important that this resistance should be kept very small. The diameter of the wire, already large, must therefore be increased, and the wire is further enlarged by an insulating covering. In the actual

machines, the space in which these wires are wound is restricted, being usually a narrow groove or slot in the surface of a cylindrical body, and therefore the number of turns seldom exceeds six or eight.

558. Increase in Number of Coils.—For a considerable portion of the time during the rotation of the single coil described in the preceding paragraphs, the induced E. M. F. is small, and twice during each complete revolution it is zero. If there be mounted upon the same axis a second coil whose plane is at right angles to that of the first (Fig. 269), the induced E. M. F. in this second

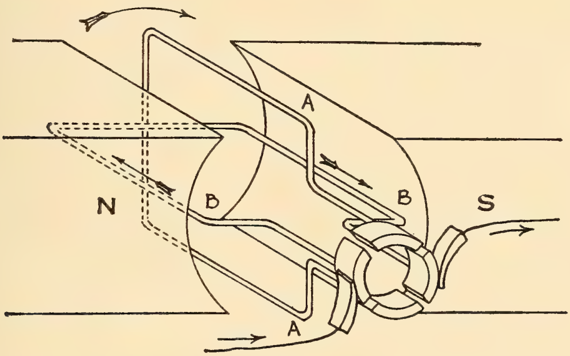


Fig. 269.

coil will be a maximum at the instant when it is zero in the first coil, and also it will be zero in the second coil when it is a maximum in the first. By a suitably arranged commutator we may always draw current from that coil whose E. M. F. is the greater, and thus avoid the periodic dropping to zero. For example, suppose the commutator to consist of four segments to which the coils are connected as indicated in the figure. At the instant represented, the E. M. F. in *AA*, the vertical coil, is zero, and that in *BB*, the horizontal coil, is a maximum, and it is this latter coil which is sending current out into the external circuit. As the coils rotate, the E. M. F. in *BB* decreases, that in *AA* increases, and these reach equality when the coils have turned through an angle of 45°. At that moment, the brushes are across the gap between the segments and in contact with both. At the next instant, the brushes are in contact with the segments connected to the *AA* coil, in which coil the E. M. F. is rising to a maximum. These changes

are shown graphically in Fig. 270. The broken and dotted curve represents the rectified E. M. F. in the *BB* coil; the broken curve represents the same in the *AA* coil. The maxima follow at intervals of 90° and midway between these maxima, as indicated by the intersection of the curves, the E. M. F.s are equal. The unbroken portion of these curves represents the E. M. F. (and

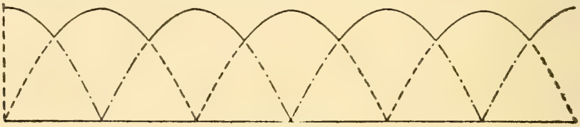


Fig. 270.

current) in the external circuit. We therefore see that by inserting the second coil we obtain a current which, while pulsating, does not drop to zero as did the current from the original coil. If still other coils be inserted between these two, we may obtain a current which fluctuates less and less, and approaches constancy as the number of coils is increased.

559. Open and Closed Coils.—Consideration of Fig. 269 will reveal the fact that except for the very brief instant when the brushes slide across the gap between the commutator segments, only one coil at a time supplies current to the external circuit. Thus, while the coil *BB* is supplying current, the coil *AA* is open at the commutator end and contributes nothing. The E. M. F. induced in these coils while the corresponding commutator segments are not in contact with the brushes is represented by the broken portions of the curves in Fig. 270. This E. M. F. is not utilized. An arrangement in this manner of the coils of a generator is called an *open-coil winding.*

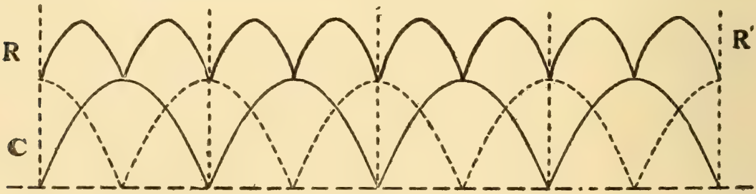


Fig. 271.

We shall shortly see (Par. 569) that there is possible another arrangement by which the various coils may be connected in series and thus instead of being idle during a portion of the rotation they all constantly contribute to a resultant E. M. F. This ar-

rangement is called a *closed-coil winding.* Points on the curve RR', Fig. 271, representing this resultant E. M. F. are obtained by adding the corresponding ordinates of the component curves. It is seen that as the number of coils is increased, not only does the resultant E. M. F. increase but also the loops in the curve RR' become greater in number and smaller in amplitude, that is, the E. M. F. becomes less pulsating and more nearly constant.

560. Essential Parts of D. C. Generator.—The essential parts of a D. C. generator are—

(a) A magnetic field.

(b) Rotating coils.

(c) A commutator.

(d) Brushes.

The coils and commutator and the shaft to which they are attached and with which they rotate are known collectively as the *armature.* The coils are usually inserted in grooves or slots in an enlarged portion of the shaft called the *armature core.* The portions of the coils on the exterior of the armature core and parallel to the axis of the shaft are called *inductors.*

561. The Field.—The magnetic field in which the armature revolves is produced by *field magnets,* which may be either permanent or electro-magnets. Permanent magnets can not be controlled nor can they be made of the size and strength required in large machines and they are therefore restricted to such small generators as those used to operate the call bell of a telephone or the sparking apparatus of a gasoline engine. In all important generators, electro-magnets are employed. It is to this class of generators that we refer in the following pages.

Whatever be the external appearance of the generator, analysis will show that the field magnets are in principle horseshoe magnets, each consisting of a yoke and two limbs, the ends of these latter being shaped to embrace between them the revolving armature. The *field coils* are wrapped about these limbs, or *magnet cores.* In the simplest form of generator, as shown in Fig. 273, there are but two magnet cores and the machine is designated as *bipolar.* If there be more than one pair of cores, the machine is *multipolar.* Whatever be the number of poles, they are alternately north and

south. Fig. 272 represents the frame of a multipolar generator of six poles. It will be seen that a similar arrangement would result by grouping around a common center six horseshoe magnets, the like poles of adjacent magnets being side by side.

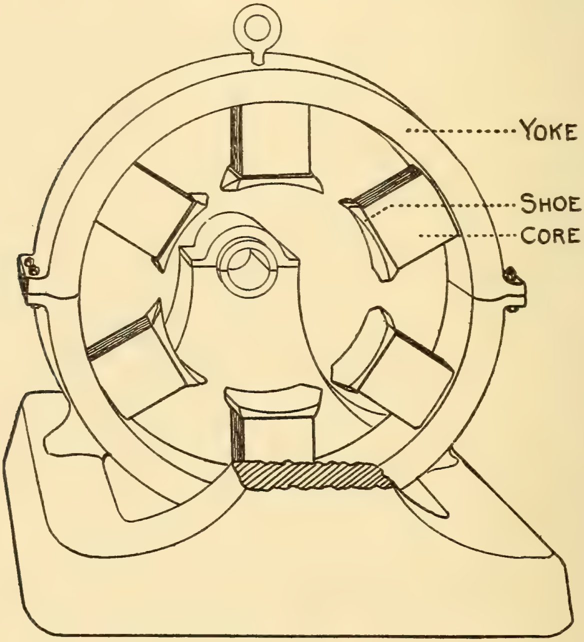


Fig. 272.

The magnet cores are made of soft annealed steel so as to be free from hysteresis. They are frequently laminated so as to avoid eddy currents. They terminate in soft iron pieces, *shoes,* which perform several functions. (a) They hold in position the field coils after these latter have been slipped over the cores. (b) They diminish the air gap between the pole faces and the armature core. (c) By the shape of their ends, or *horns,* they produce an advantageous distribution of the flux.

562. Excitation of Field Magnets.—For all D. C. generators the field magnets are *self-excited,* that is, they are excited by current from the machine itself.

Since the machine will not generate a current unless the field be excited, and since the field is excited by the current drawn from

the machine itself, it is not clear at first sight why a generator ever produces a current. If the field magnets were of perfectly pure soft iron, it is probable that no current would be produced when the generator was set in motion, but the iron is not perfectly pure and there is always some slight residual magnetism left in the cores (Par. 155), and when the machine is started, this is sufficient to produce a small current through the field coils. This strengthens the magnets which in turn increases the current, and so on, a generator on starting "building up" gradually, and frequently taking a minute or so to reach normal output. This building up may sometimes be aided by the earth's field.

563. Methods of Self-Excitation.—There are three distinct ways in which the coils of the field magnets may be wound and the exciting current passed through them so as to obtain the desired number of ampere turns. The corresponding generators are said to be *series wound, shunt wound,* and *compound wound* respectively.

In a series-wound generator, the entire current passes through the field coils. In Fig. 273, *a* represents diagrammatically a

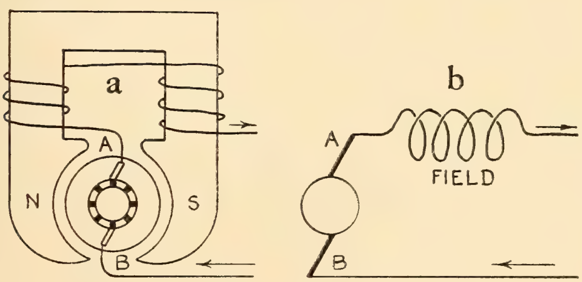


Fig. 273.

series-wound, bipolar machine. The same current which passes through the field coils flows through the external circuit, or the field coils and the external circuit are in series. A still more highly conventionalized diagram of the same machine is represented in *b*.

In a shunt-wound generator, only a portion of the entire current, from two to ten per cent, is passed through the field coils. These coils are therefore in shunt with the external circuit. In Fig. 274,

a represents a shunt-wound, bipolar machine, the shunt being indicated by the dotted line, and *b* is a more conventionalized diagram of the same machine. Since only a fraction of the entire current passes through the field coils, in order to secure the necessary ampere turns for the excitation of the magnet cores, there must be many more turns in these coils than in the case of those of a series-wound machine.

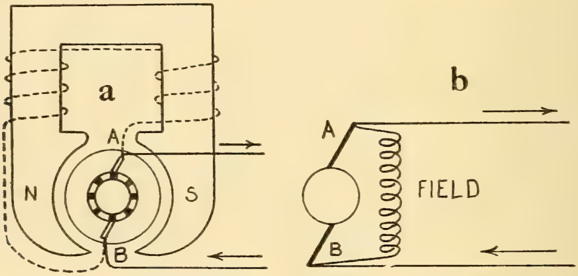


Fig. 274.

The field coils of a compound-wound machine combine series and shunt windings. Thus in Fig. 275, *a* represents a compound-wound, bipolar machine, the series winding being shown by the heavy line and the shunt winding by the dotted line. For clearness of the diagram, the windings are represented as on separate portions of the cores.

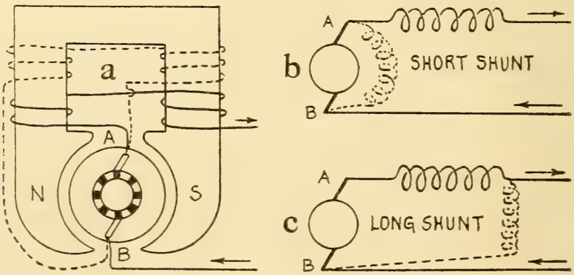


Fig. 275.

There are two varieties of the compound windings, known as *compound short shunt* and *compound long shunt.* If the shunt is taken off across the brushes *A* and *B*, as shown in *a* and more diagrammatically in *b*, it is a *short shunt.* If, as shown in *c*, one end of the shunt be taken off *beyond* the series coil, it is a *long shunt.* The diagrams *b* and *c* indicate the reason for these names. So far

as the machine itself is concerned, there is but little difference between long and short shunt, but, as will be shown in the next chapter, there is a very great difference in the three classes of machines and in the conditions under which each is to be used.

In the foregoing diagrams the yoke of the field magnets is represented as above the armature, but this is simply for clearness. While they may have any position, bipolar machines are usually mounted with the yoke horizontal and below the armature, or, less frequently, with the yoke vertical and to one side.

564. Control of Field.—In connection with this subject, reference should be made here to control of field. Since the E. M. F. developed in a generator varies with the rate of cutting of lines of force, if the field be constant, the E. M. F. can be varied only by varying the speed of rotation. Since, however, generators are

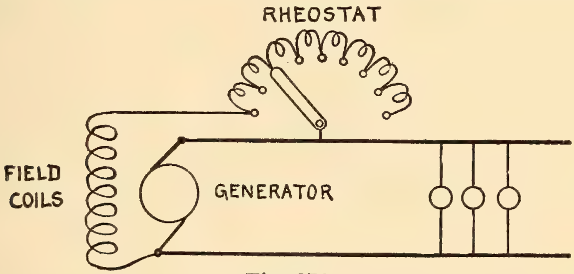


Fig. 276.

usually run at a constant speed, the E. M. F. is varied by increasing or decreasing the number of lines of force, that is, by varying the field. In a shunt-wound machine, the current through the field coils, and consequently the field, may be varied by means of a rheostat in series in the shunt circuit, as shown in Fig. 276. In a

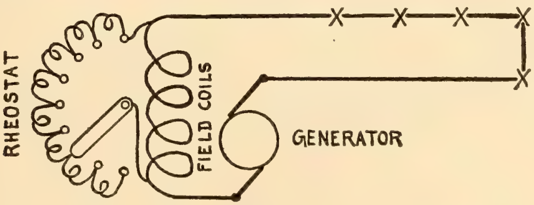


Fig. 277.

series-wound generator, the field may be varied by a rheostat in parallel with the field coils as shown in Fig. 277. The greater the current through the rheostat, the less through the field. These

field rheostats are not attached to the generator direct but are mounted upon a *switchboard,* an auxiliary piece of apparatus which will be described later (Par. 579).

565. Armature Core.—In Par. 560 we saw that the enlarged portion of the shaft to which the rotating coils are attached is called the *armature core.* This core has two separate functions to perform. (a) It serves as a rigid base of attachment for these rotating coils and is therefore cylindrical in shape. (b) As explained in Par. 145 and as shown in Fig. 182, it diminishes the air gap between the poles, thereby reducing the reluctance in the magnetic circuit and increasing the flux. It must therefore be of a highly permeable material, such as soft iron. It not only increases the flux but so directs it that the lines of force are most advantageously situated for being cut by the rotating coils. For example, in the multipolar machine shown in cross-section in Fig. 278, if the armature core were non-magnetic, the lines of force

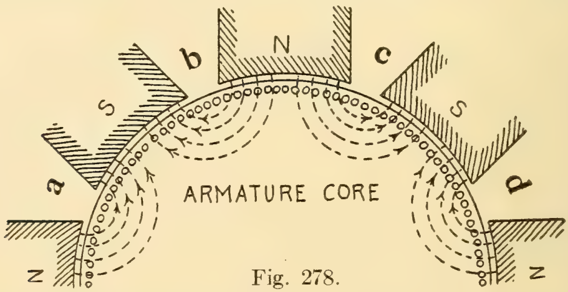


Fig. 278.

would pass directly across the gaps *abcd* and therefore would not be cut by the coils, but this core being of iron, the lines pass into it (Par. 145) as shown in the diagram and are cut by the coils as they rotate.

The core being of a magnetic substance and lying between the poles of the field magnets, it acquires polarity (Par. 119). As it rotates, this polarity shifts and to avoid hysteretic losses (Par. 399) its retentivity should be very small, that is, it should be made of soft and pure iron.

Also, since it is a conductor rotating in a magnetic field, eddy currents will be produced in it, and to reduce these it is laminated or built up of thin sheets (Par. 429).

These sheets usually take the form of punchings. For small machines they may be disc-shaped and perforated with a single

hole for assembling upon the shaft, but for large machines they are generally segments of a circle. On the outer periphery they are provided with slots in which the coils are wrapped (Fig. 279) and on the inner there are undercut grooves by which they are assembled upon a *spider* which in turn is keyed to the shaft.

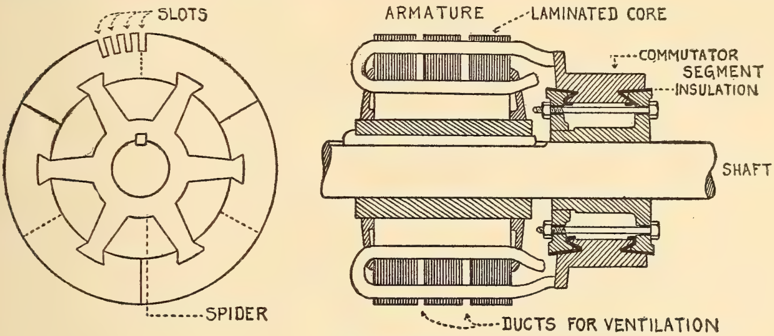


Fig. 279.

Although this lamination diminishes the eddy currents it does not entirely obviate them and to reduce their heating effect the core is not built up solid but at intervals ventilating spaces are left. The air currents enter between the spokes of the spider and emerge through these ducts.

566. Classes of Armatures.—Based upon the manner in which the coils are wrapped upon the core, there are two distinct classes of armatures, the *ring wound* and the *drum wound,* both shown diagrammatically in Fig. 280. Should the coil after passing

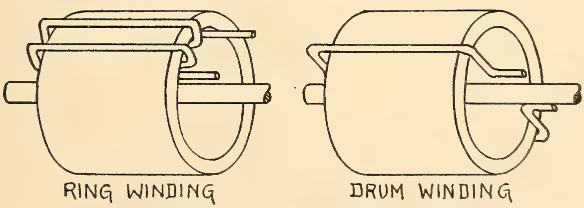


Fig. 280.

through a slot on the outer surface of the armature be threaded back through the interior of the core (Fig. 279), then again out through a slot and so on, in other words, should it be wrapped in a continuous helix around the rim of the armature, just as a wire might be wrapped around the rim of a wagon wheel to hold a tire

in position, it is a *ring winding.* On the other hand, should the coil, after passing through a slot, cross along a chord of the end of the core and return by a slot on the other side, it is a *drum winding.*

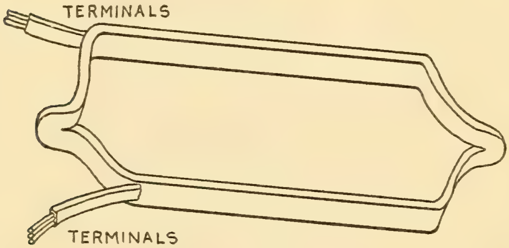


Fig. 281.

Electrically the two windings do not differ in principle but practically the drum winding is used almost to the exclusion of the ring winding.

One objection to the ring winding is that the conductor of which the coil is composed must be put on by threading it back and forth and bending it into place. This is difficult with the large copper inductors now required; moreover, any insulation about the coil would be injured in this process so that insulation has to be put on as the coil is placed in position, and it is difficult to fasten such coils rigidly.

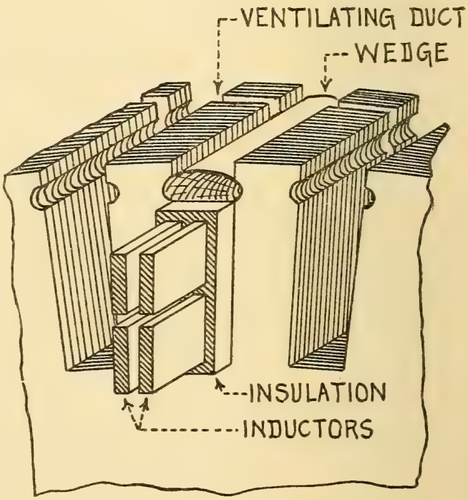


Fig. 282.

On the other hand, the coils for a drum winding being all alike may be made up on a form and of as heavy material as may be desired (Fig. 281). They are then wrapped with insulation, baked to expel moisture and varnished. Finally they are packed tightly into the armature slots and held securely in position by wooden wedges inserted as shown in Fig. 282. As an additional precaution, a certain amount of banding is usually wrapped about the armature.

567. The Commutator.—In Par. 556 we saw that the split ring, or arrangement of copper segments by which the alternating cur-

rent was rectified, is called the *commutator*. With the increase in the number of coils, the number of segments also increases and they finally reduce to relatively thin wedge-shaped copper plates of the form shown in Figs. 279 and 283. In the upright portion or *neck* of these segments there are cut mortises into which the coil terminals are soldered.

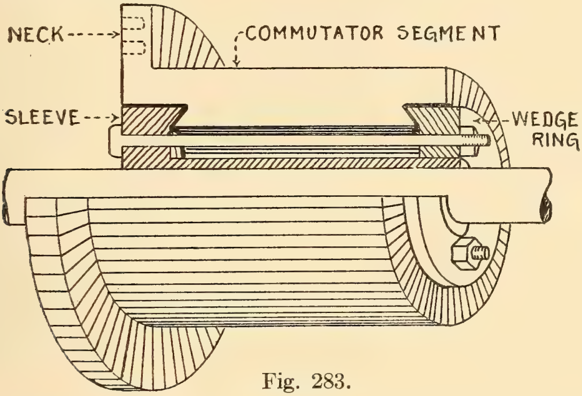


Fig. 283.

The commutator is the weakest point about the armature. Not only must the separate segments be assembled into a cylinder which is firmly attached to the armature shaft but they must also be perfectly insulated both from each other and from the shaft. The segments, separated by sheets of mica, are arranged in a cylinder, being held at one end by a *hub* or *sleeve* and at the other end by a *wedge ring*, from both of which they are insulated by a layer of a composition of mica and shellac. The sleeve and the wedge ring are drawn tightly together by means of bolts, thus binding the segments rigidly together, and these are then turned down to a perfect cylinder.

568. Brushes.—The brushes are so named because in the earlier machines they were of brass wire and resembled a stiff paint brush. In the process of evolution these took the form (still used in certain machines) of brass laminae like the leaves of a book, then were made of copper gauze compressed into prisms. They are now rectangular blocks of carbon, made somewhat in the same manner as the carbons for arc lights (Par. 516) except that there is sometimes incorporated a small amount of paraffine which acts as a lubricant. They are held in *brush holders* which are provided with springs by which the pressure of the brushes against the

commutator may be regulated. The holders in turn are secured to a *rocker frame* by which the brushes may be shifted bodily in the direction of rotation of the commutator or in contrary direction. The object of this adjustment is explained later (Par. 570). The brushes must be proportioned to the current which they are to carry and for heavy currents, instead of being of a single large carbon block, each consists of a number of smaller carbons with separate springs. These may be compared to the finger tips of a hand pressing lightly upon the commutator. Should one be momentarily jarred away from the commutator, the others preserve a flexible contact and the circuit is not broken. It will be shown later (Pars. 573 and 577) that, except for one class of drum windings, there are required as many brushes as there are poles.

569. The Ring-Wound Generator.—In the operation of a generator, the current flowing through the coils gives rise to conditions which, since they necessitate certain minor corrections and adjustments, should be thoroughly understood. On account of the

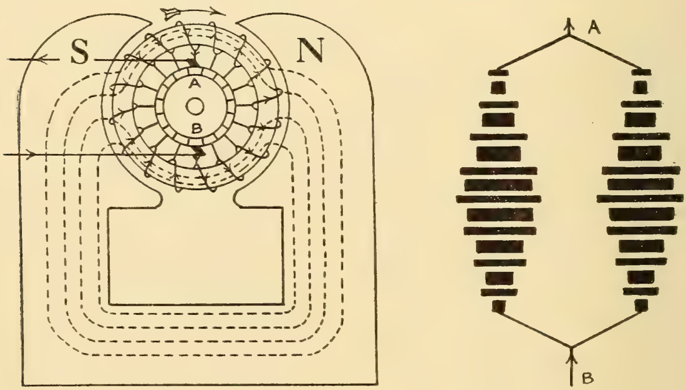


Fig. 284.

greater simplicity of the diagram, these are most readily explained by reference to the ring winding, but it must be remembered that this is selected merely for ease of explanation and that the majority of modern machines are drum wound.

Fig. 284 represents diagrammatically a bipolar, ring-wound generator. In this diagram the extremities of each turn of the winding are represented as connected to the adjacent commutator segments, but in the actual machine there may be a number of

turns between these *tapping wires* (see Fig. 286). The lines of force of the field, as shown by the dotted lines, follow around the rim of the armature core and therefore as the armature rotates, only the outer portion of the coils cuts these lines, the remaining portion being idle. These outer portions, the *inductors*, are perpendicular to the plane of the paper but, in order that they may be seen, are shown as part of the helical winding.

Assuming the rotation of the armature to be clockwise, application of the right hand rule (Par. 422) shows that the direction of the induced E. M. F. in each inductor to the right of the symmetrical plane through the axis of the armature is *from* the observer, while that in each inductor in the left half is *towards* the observer. Beginning at the bottom inductor on either side and following around to the top, the instantaneous value of the E. M.F. being generated in each, assuming the field to be uniform, is proportional to the sine of the angle through which it has turned from the symmetrical or neutral plane (Par. 552), and these inductors being portions of a continuous helix, the total E. M. F. in each half of the armature is the sum of these separate E. M. F.s. If, therefore, brushes be applied at A and at B, the two segments lying in the neutral plane, and be connected through an external circuit, A being at a higher potential than B, a current will flow out by A and returning by B will divide, one-half flowing up each side of the armature and reuniting at A. In other words, the halves are in parallel and afford two paths for the current through the armature. An analogous arrangement would be the grouping, shown at the right of Fig. 284, of sixteen cells, two in parallel and eight in series, the variation in the E. M. F. of the individual cells being indicated by the length of the lines representing the cells.

570. Armature Reaction.—The tendency of the field magnets of a generator is to magnetize the armature core by induction. As shown in Fig. 285, a north pole would be induced at N' and a south pole at S'. However, when the generator is in operation, each half of the ring core is surrounded by many ampere turns and is therefore powerfully magnetized. With clockwise rotation the current in the armature alone would produce a north pole at N'' and a south pole at S'' (Par. 404). The actual magnetization of the ring is therefore the resultant of these two, and a north pole will be found at some intermediate point as N''' and a south pole at S'''. As a result of this reaction between the original field and

the armature field, the flux will be distorted as shown and the neutral plane will no longer coincide with the symmetrical plane but will be shifted forward in the direction of rotation to some such position as *CC*. The brushes must now be shifted forward until they coincide with this plane, or are even very slightly ahead of it (Par. 572). This adjustment is made by means of the rocker frame (Par. 568). The plane in which the brushes are finally placed is called the *commutation plane* and the angle between this and the symmetrical plane is called the *angle of lead.*

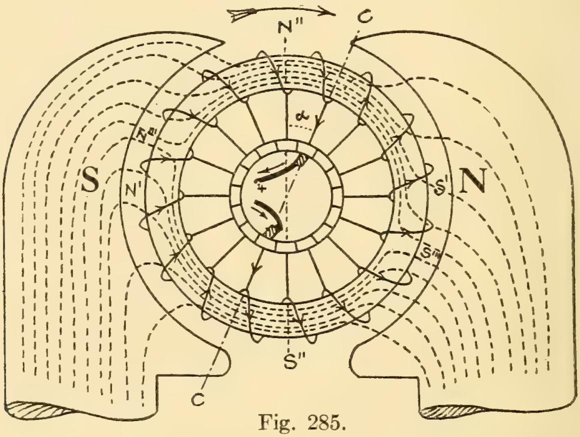


Fig. 285.

The advancing of the brushes and variations in the current through the armature may cause further shifting of the plane of commutation, but generators are now so constructed that when the brushes have once been adjusted and the machine is run under average conditions, no further movement is needed.

571. Commutation.—Fig. 286 represents diagrammatically a portion of the armature of a ring-wound generator in four successive positions. For clearness of diagram, the brush is drawn below the commutator segments. The broken and dotted vertical line represents the neutral plane. With clockwise rotation and field from left to right, currents will flow through the coils in the direction indicated by the arrows.

In position *a*, the brush is in contact with segment *G* alone. Of the total current delivered to the brush, one-half flows in from the coil *C*, the other half from the coil *B*.

In position *b*, the armature has moved until the brush, still retaining contact with *G*, has just established contact with *F*. As before, one-half of the total current flows in from *C*, but the other half, arriving from *A*, divides at *F*, a small but rapidly increasing portion flowing direct to the brush, the diminishing remainder flowing through *B* to *G* and thence to the brush. The reason why at first only a small portion flows direct from *F* to the brush is that *F* is then in contact with the brush along a narrow strip only and the resistance of this contact is considerable. However, as the armature continues to move, this resistance decreases and the current through *F* increases, that through *B* decreasing accordingly.

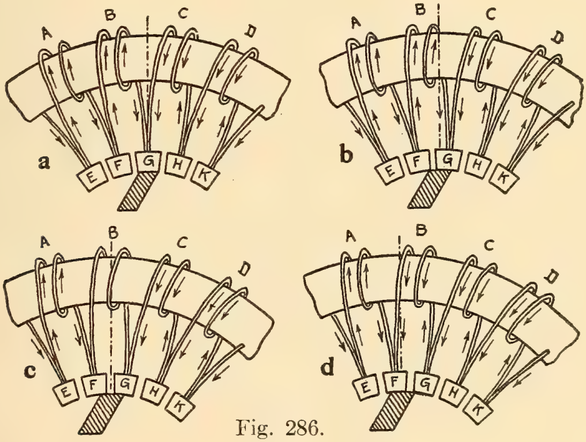


Fig. 286.

In position *c*, the brush makes equal contact with *F* and *G*, one-half of the current flows through *CG*, the other half through *AF*, and the current in *B* is zero.

In position *d*, the contact with *F* has increased and that with *G* has dwindled to a narrow line. At this instant, the full current from *A* flows through *F*, while the current from *C*, for reasons explained above, divides at *G*, a diminishing portion flowing through *G* direct to the brush, an increasing portion flowing through *B* to *F*.

When finally the brush is in contact with *F* alone, the conditions are as represented in *a*, that is, one-half of the total current flows from *A* to *F*, the other half from *B* to *F*. Originally the current in *B* flowed from left to right and it now flows from right to left,

in other words, as the successive segments slip past the brush, the current in the corresponding coils undergoes complete reversal.

When these changes of the current in the coil under the brush take place as outlined above, the commutation is said to be perfect.

572. Sparking.—There are certain conditions which interfere with the realization of perfect commutation. The armature revolves at high speed and the reversal of the current in a coil often takes place in less than one-hundredth of a second. As these coils frequently carry from fifty to one hundred amperes and are wrapped about an iron core, the self-induced E. M. F. is considerable. The effect of this E. M. F. is to oppose any change in the original direction of the current flowing in the coil, therefore, in position *d* in Fig. 286 the rise of the current from *G* into *B* is retarded and the greater part of the current from *C* is forced to flow from *G* direct into the brush. As the area of contact between the brush and *G* decreases, the current density (number of amperes per square centimeter of cross-section) may become so great as to produce injurious heating of the brush and of the commutator segments. Finally, as *G* separates from the brush, a momentary arc is produced, its heat being sufficient to volatilize a small portion of both the segment and the brush. Continuance of this "*sparking*" will injure or destroy the commutator.

If the brush be moved slightly forward in the direction of rotation of the armature (as for example under coil *C* in position *d*), the act of commutation will occur, not with a coil which is in the neutral plane but with one in which there is being induced an E. M. F. opposite in direction to the self-induced E. M. F. This induced E. M. F. therefore opposes and assists in overcoming the self-induced E. M. F. and removes this source of sparking. Generator brushes, therefore, are usually set slightly in advance of the neutral plane.

573. Multipolar Generators.—Fig. 287 represents diagrammatically a four-pole ring-wound generator. Application of the right hand rule shows that with clockwise rotation the direction of the induced E. M. F. is as represented by the arrowheads. If these be examined, it will be seen that the E. M. F. acts from the coils to the commutator in two points, *A* and *B*, and from the commutator to the coils in two other points, *C* and *D*. Therefore, if brushes be applied at these four points and be connected through

an external circuit, currents will flow out from *A* and *B*, and return by *C* and *D*. With a six-pole machine six brushes are needed and in general in ring-wound generators as many brushes are required as there are poles. Brushes of like polarity are usually connected to a common conductor, a ring, to which in turn the corresponding lead is attached.

574. Advantages of Multipolar Machines.—Multipolar machines possess some important advantages over bipolar machines and most modern machines of appreciable power are of this type.

(a) When a coil of the generator represented in Fig. 287 has rotated geometrically through 180°, it has rotated electrically

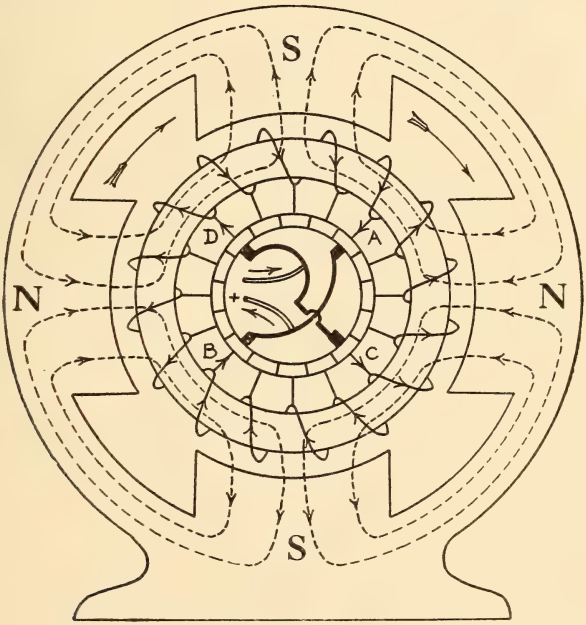


Fig. 287.

through 360°. With a six-pole machine, one-third of a revolution carries it through 360° electrically. Therefore, with the same number of lines of force from pole to pole, the same E. M. F. may be developed by a four-pole machine with an angular velocity only half as great as that of the bipolar machine. Or, if the angular velocity of the two be the same, the multipolar machine will develop the greater E. M. F.

(b) Examination of the figure will show that the current coming in to the machine divides equally between C and D and from each of these points has *two* paths to the positive brushes A and B, in other words, the current through the armature has as many paths in parallel as the machine has poles. With the same sized inductors, the resistance through the armature of a four-pole machine is only one-half of that of a bipolar machine, or, with the same total current, the inductors of the four-pole machine carry only one-half the current as those of the bipolar. This is of great importance in generators handling large currents.

Minor advantages of the multipolar machines are the more advantageous distribution of the flux and the less weight of iron required in the field magnets.

575. Drum Windings.—The distinguishing feature of the drum winding has already been given (Par. 566). Since the coils are arranged with the inductors at opposite ends of a chord of the armature core (Fig. 280), if the induced E. M. F. in one of these inductors acts from front to rear, that in the other must act from rear to front. Hence, the principle governing all drum windings is that the coils must be so wrapped that the two inductors are never simultaneously under like poles. There are a number of different windings which fulfill this condition but they all belong to one or the other of two general classes, *wave winding* and *lap winding*. These will be explained below.

576. Plane Development of Drum Winding.—There are two conventional ways of representing diagrammatically a drum winding. The first is to develop the armature by placing it

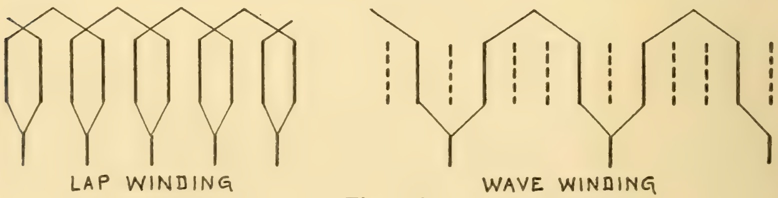


Fig. 288.

on its side and rolling it along on a plane. Fig. 288 represents in simplest form such a development of a lap winding and of a wave winding (both incomplete), and indicates the appropriateness of these names.

Fig. 289 represents a lap winding for a four-pole generator, the armature carrying sixteen inductors and eight commutator segments. The coils are composed of inductors 1 and 6, 3 and 8, 5 and 10, etc. It will be noted that in each the two inductors are under different poles. Furthermore, if we begin at inductor No. 1 and follow the winding through, it will be seen that we pass in succession through all of the inductors and finally return to the starting point; in other words, just as in the ring winding, the inductors are in series and the winding is a closed coil (Par. 559).

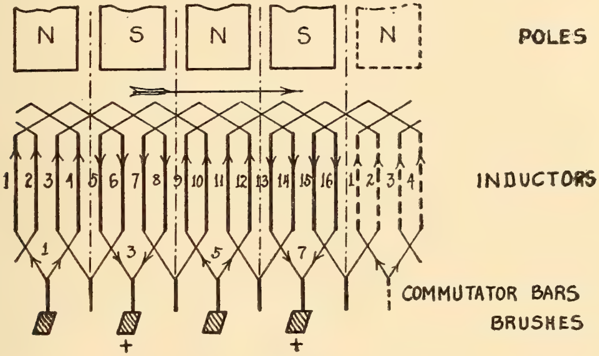


Fig. 289.

With rotation from left to right, the direction of the induced E. M. F. is as indicated by the arrowheads, and by inspection the position of the positive brushes is readily located at segments 3 and 7 and that of the negative brushes at segments 1 and 5.

577. Star Development of Drum Winding.—An objection to the foregoing diagram is that the windings are not represented as closing upon themselves. To remedy this, use is made of what may be termed a *star development*. If we should stand a barrel on end, cut all of the hoops except the one at the top, open out the staves from the bottom until the head rested upon the ground with the staves radiating like the petals of a daisy, we should have a star development of the barrel. Applying this to an armature, the commutator corresponds to the head of the barrel and the inductors to the barrel staves. The inductors and their connections are thus shown in their proper relation to the commutator segments and the windings close, the only distortion occurring in the cross connections at the back end of the armature. Such a projection

of a *lap winding* for a four-pole machine is given in Fig. 290. The heavy radial lines represent the inductors. To enable the eye to trace the back connections with least difficulty, these latter are drawn in a regular geometric pattern with salient angles.

With clockwise rotation, the direction of the induced E. M. F. is as indicated by the arrowheads. If the circuits be traced, it will be seen that there should be four brushes and that they should be located as indicated in the diagram. There are, therefore, four

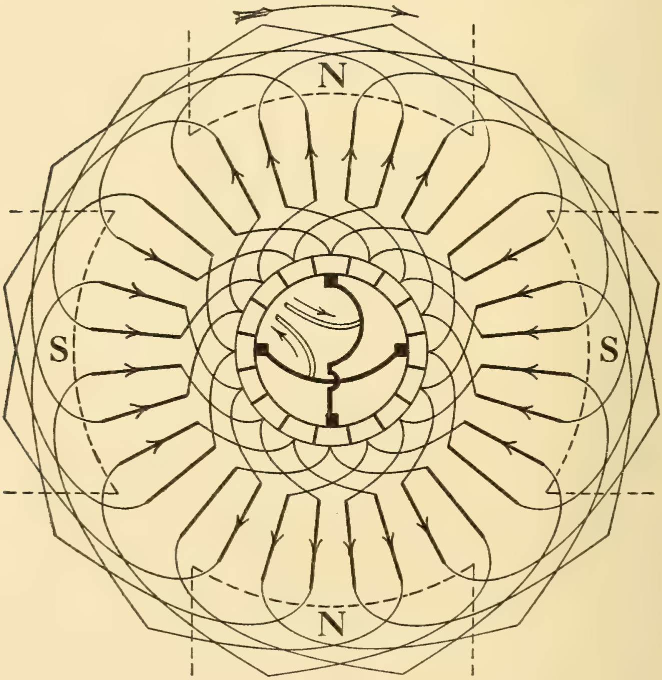


Fig. 290.

paths through the armature. For this reason, the lap winding is frequently spoken of as a *parallel winding*. It is best suited for the production of large currents at low voltage.

A star projection of a *wave winding* for a four-pole machine is shown in Fig. 291. In addition to the manner in which it is put on, this winding differs from the lap winding in several other respects, particularly in requiring but one pair of brushes. The positions of the positive and the negative brushes are shown in the

diagram. Should an additional negative brush be introduced at *c* and connected to *b*, it would be of no appreciable electrical effect, for examination of the diagram will show that *c* and *b* are already connected through the coil *cdeb* in which, at the instant shown, no E. M. F. is being induced. The inductors of a wave winding are therefore in series and there are but two paths through the armature, for which reasons wave-wound armatures are best suited for the production of small currents of high voltage.

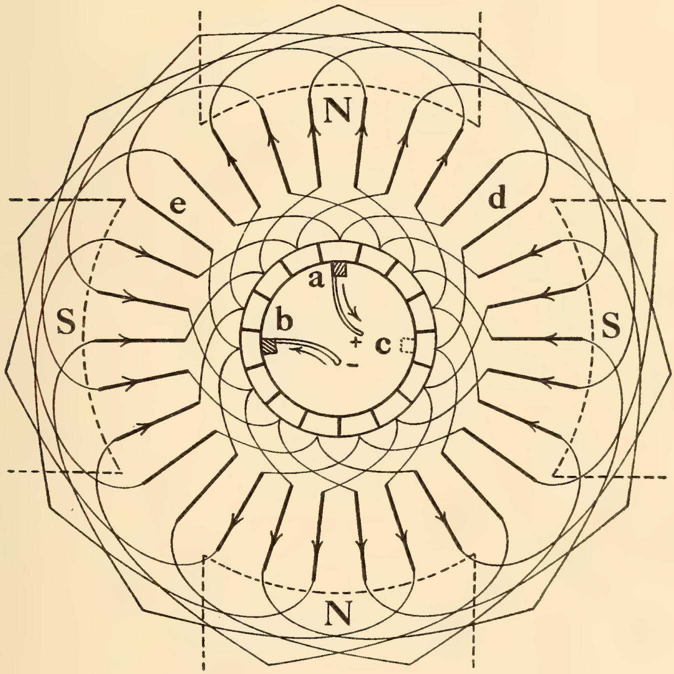


Fig. 291.

578. Calculation of E. M. F. of Generator.—The E. M. F. of a generator may be calculated as follows:

Let ϕ = flux from each pole

n = number of poles

n' = number of revolutions per second

n'' = number of paths through the armature

N = number of inductors

The number of flux lines cut by each inductor in one revolution is $n.\phi$.

The number of flux lines cut by each inductor per second is $n'.n.\phi$.

The E. M. F. generated by each inductor is $n'.n.\phi/10^8$.

But since there are N/n'' inductors in series, the total E. M. F. is $\dfrac{N.n'.n.\phi}{n''.10^8}$ volts.

579. Switchboards.—A generator may be called upon to furnish current for various uses, as, for example, for lighting, for charging a storage battery, for running a motor, etc., etc., and it may be required to do these things one at a time or in various combinations. Wires must therefore be run from the generator to the lamps, battery, machines, etc., and there must be switches in the various circuits. The generator must be supplied with a field rheostat (Par. 564) by which its E. M. F. may be adjusted, and this implies that it must also be equipped with a voltmeter by which this E. M. F. may be measured. If a storage battery is to be charged, its E. M. F. must be known before the current from the generator can be turned on (Par. 245). It is also often desirable to know the current flowing in any one of the circuits, and for this there must be ammeters. Overload switches should be inserted in the principal circuits and an underload switch must be in the charging circuit for the storage battery (Par. 415). Should an attempt be made to connect these various switches and instruments to the generator direct, the machine would be hidden in a hopeless maze of wiring. These auxiliary pieces of apparatus are therefore gathered together, taken to one side and mounted upon a *switchboard.* Wires from the machine, not exceeding three in number, are brought over in a conduit and the distribution of electrical energy takes place at the board. This distribution is usually made from two heavy, parallel copper bars, called *bus bars,* which are connected to the source of the electrical energy and which may be regarded as its enlarged terminals.

Originally of minor consideration, the switchboard has now risen to a position of importance second only to that of the machine itself and frequently rivalling it in cost. It is composed of panels of some non-conducting material, preferably marble, upon the front of which are mounted the switches and instru-

ments; the bus bars, wiring and connections being at the back. In addition to a symmetrical distribution of the apparatus, it is customary to arrange parallel wires of a circuit on direct-current switchboards so that if they be horizontal, the *upper* one is the positive wire; if they be vertical, the *right hand* one is positive.

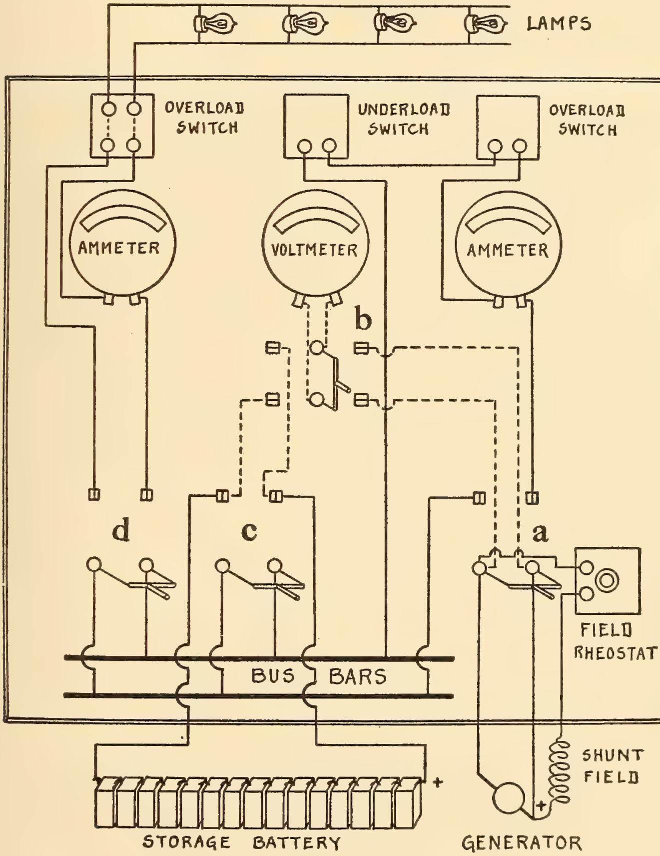


Fig. 292.

In drawings of switchboards, several conventions are observed. Wires are always drawn as right lines which are perpendicular or parallel to the lower edge of the board (Fig. 292). This is to aid the eye in tracing the circuits. If two wires cross but are not connected electrically, this fact may be indicated by a little arch in one of the wires, or they may be assumed not to make connection unless a dot be made upon the point of intersection.

580. Example of Switchboard.—A switchboard by which the current from a shunt-wound generator may be used to run a number of lamps and charge a storage battery, either separately or simultaneously, is shown in Fig. 292. The circuits are easily followed by the eye and the use of the various switches will be understood from the following:

To charge the battery:

(*a*) Close *b* to the left and read the battery voltage.

(*b*) Start the generator. Close *b* to the right and read the generator voltage. Manipulate the field rheostat until the generator voltage is about ten per cent greater than the battery voltage.

(*c*) Close *a*, *c*, and last the underload switch.

To run the lights at the same time:

Close also *d*.

To run the lights separately:

With the above arrangement open *c*.

(It will be noted that the lights are now run through the underload switch. This is not correct. An additional switch should be used by which the generator may be thrown direct on the bus bars. It is omitted in the diagram to avoid overcrowding the figure.)

The right hand ammeter reads the current from the generator.

To run the lights from the battery alone:

With all switches open, close *c* and *d*.

The left hand ammeter now reads the current from the battery.

581. Coupling of Generators; Three-Wire System.—In Par. 502 it was shown that the successful transmission of electrical power to a distance depended upon the employment of high voltage, the loss of power in the leads varying inversely as the square of this voltage. Alternating currents are easily stepped up for transmission and as easily stepped down at the point where they are to be utilized. In the case of direct currents the transformation is much more troublesome and expensive. For such currents, however, there has been devised a system by which the voltage may be doubled and thus the advantage of high voltage transmission be partly secured. This will be understood from the following explanation. It is desired to operate at a distance a number of 110 volt incandescent lamps. If two

generators, *A* and *B*, each capable of delivering 110 volts to the lamps, be connected in series as shown in Fig. 293, the voltage between the leads will be 220. If the lamps between *C* and *D* be arranged two in series, each will receive its required 110 volts,

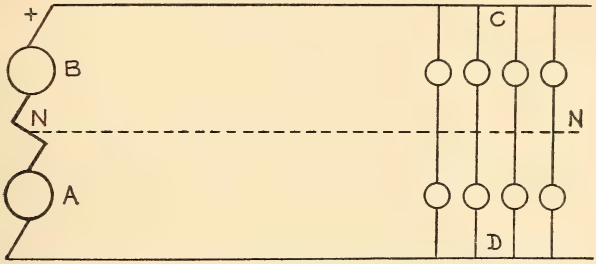


Fig. 293.

while the currents in the leads will be only one-half of that required by the same number of lamps arranged singly in parallel. The leads therefore may be reduced three-quarters in size. If now a third wire *N N*, the *neutral,* be inserted as shown in the figure, it will be possible to have a different number of lamps on the two sides. If there be more lamps above the neutral than below, the excess current flows *in* on the neutral; if there be less above, the excess current flows *out* on the neutral, in other words, the neutral needs only be sufficiently large to carry the *difference* in the currents required on the two sides. In practice, however, it is made of the same size as the other two leads. Notwithstanding the extra wire, the saving in copper in this *three-wire system* is five-eighths, or 62.5 per cent, of the amount required in a two-wire system for transmitting equal power. Against this saving must be put the cost of the extra generator (though certain special generators have been devised to supply a three-wire system from a single machine), and the extra cost of installation and of switches and switchboard appliances, so that frequently the saving is more apparent than real. In addition to this, more attention is required in regulating the two generators since with unequal loads on the two sides of the neutral, the E. M. F. of the generators must differ.

The principle involved has been applied abroad to a five-wire system.

CHAPTER 41.

GENERATOR CHARACTERISTICS.

582. Adaptation of Generator to Work Required.—Of the various proposed classifications of direct current generators, the most important is the one based upon the excitation of the field magnets (Par. 563), that is, into series, shunt and compound machines.

Each one of these classes possesses certain advantages and disadvantages which render it more suitable for some purposes and less so for others.

As an illustration, suppose we have at our disposal a series generator and a shunt generator and are required to charge a storage battery: which of the two should we use?

To prevent the storage battery from discharging back through the generator, the voltage of the latter must be kept constantly higher than that of the battery. Suppose we were to start with the series generator. Its E. M. F. can not build up until a current flows through the field coils, and no current can flow through these until the external circuit is completed. Therefore, should we simply start the generator and then switch it on to the storage battery, the battery would discharge back through the generator. We must then first build up its field by sending the current through some external circuit other than that which includes the battery and then, when the E. M. F. has reached the proper point, switch the current in on the battery.

Suppose this to have been done and that the connections are as shown diagrammatically in Fig. 294. As the battery becomes charged, its voltage rises, consequently the current sent through it by the generator grows smaller. The current through AB being smaller, the field gets weaker: the voltage of the generator consequently falls; this again causes the current to decrease; the field gets still weaker, and so on. In other words, the generator unbuilds and "drops its load," and, unless there be an underload switch in the circuit, the battery will soon discharge back.

A series-wound generator is therefore not fitted to charge a storage battery.

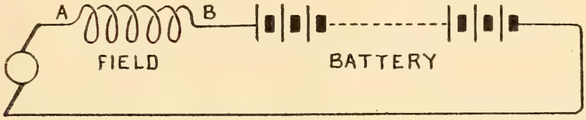


Fig. 294.

On the other hand, suppose that we employ the shunt generator and that it is connected as shown in Fig. 295. The generator is started and, the current flowing through the shunt field AB, the E. M. F. builds up rapidly. When the voltage has reached the proper point, the switch S is closed and the current is thrown in on the battery. As the battery becomes charged, its voltage rises and this counter E. M. F. cuts down the current from the generator but the effect is very different from that in the case of the series generator. As the current from the shunt generator decreases, its voltage *increases*. The explanation of this is as follows. The E. M. F. of the generator at any instant is spent

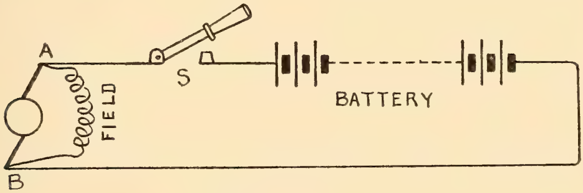


Fig. 295.

in doing two things, driving the current through the resistance of the armature coils and brush contacts (or through the internal resistance of the machine), and driving it through the resistance of the external circuit, including the overcoming of any counter E. M. F. in that circuit. This will be recognized as but another example of lost and useful volts as discussed in Par. 305. The smaller the current through the armature, the smaller the lost volts, or the internal drop Ir, and the more nearly the voltage between A and B approaches the E. M. F. of the generator. We see then that the voltage of the shunt generator always remains greater than that of the battery and that the charging can be done with safety.

583. Characteristics.—The advantages and disadvantages of the various forms of generators may be discussed in a similar manner to the foregoing. Where constancy of current is to be maintained, a series generator is under certain conditions satisfactory; where constancy of voltage is desired, a shunt or a compound generator must be employed. However, we might sometimes overlook some point in our discussion or might give undue weight to some other, therefore, the most sure method is actually to try the machine under varied conditions, keep a record of the results, tabulate and compare these. If they can be put graphically in the form of a curve, they give a clearer conception of the working of the machine. Such curves are called "*characteristics*" and much information can be derived from their study.

584. Magnetization Characteristics.—As an illustration of these characteristics, suppose that we have a generator whose field is

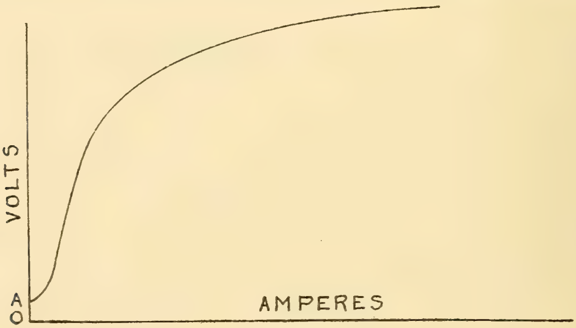


Fig. 296.

excited from a separate source, such as a storage battery. We rotate the generator at a constant speed, we excite the field by various currents and we record the strength of the exciting current and the corresponding voltage across the brushes of the generator. Plotting this data with amperes as abscissae and the corresponding volts as ordinates, we obtain a curve (Fig. 296) which is called the "*magnetization characteristic.*"

A study of this reveals (a) that with no current in the field coils there is still a small voltage, OA, due to the residual magnetism of the magnet cores (Par. 562), and (b) that as the amperes in the field coils increase regularly, the voltage at first rises rapidly

and then more slowly. Reflection will show that this curve is nothing more than the magnetization curve described and figured in Par. 393.

585. Characteristic of Series Generator.—Fig. 297 represents diagrammatically a series generator run at constant speed and

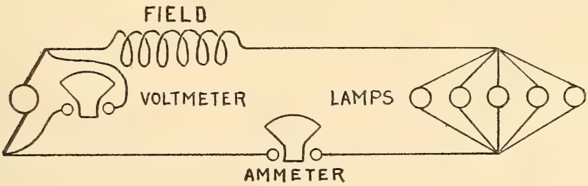


Fig. 297.

connected in circuit with a number of lamps in parallel and an ammeter. A voltmeter is connected across the brushes. By turning on lamps the resistance of the circuit is reduced and the current thereby increased. This current is measured by the ammeter and the corresponding terminal voltage is given by the voltmeter. If the amperes be laid off as abscissae and the corresponding volts as ordinates, the resulting curve, $ABMN$, Fig. 298, is the *external characteristic,* so called because, as was pointed

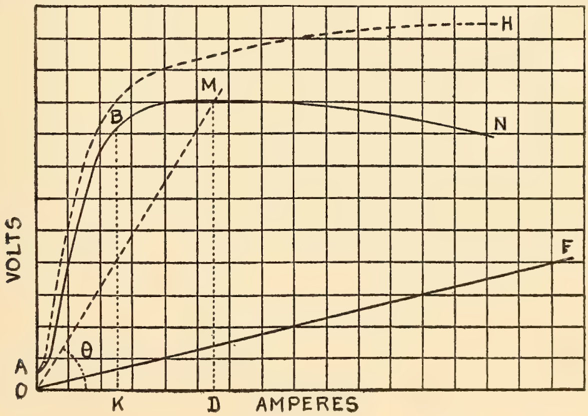


Fig. 298.

out above (Pars. 461 and 582), the voltage read by the voltmeter is not the total E. M. F. of the machine but only the IR drop over the external circuit, in other words, the useful volts. Should we wish to represent the total E. M. F., the internal drop, or lost volts Ir, must be added to the external drop.

Since r, the internal resistance of the machine, is constant, the internal drop varies directly as the current and is represented in Fig. 298 by the straight line OF. If the ordinates of OF be added to the corresponding ordinates of the curve $ABMN$, the resulting curve OH is the total E. M. F. curve or the *internal characteristic.* Were it not for the effects of armature reaction, this curve would agree with the magnetization curve described in the preceding paragraph.

Examination of the external characteristic shows that the machine should be operated with currents corresponding to the flatter portion of the curve, for if the current falls below KO, slight changes in the current produce great fluctuations in the voltage and the operation of the machine is unstable.

586. Critical Resistance.—From the figure, MD/DO is the tangent of the angle MOD, and since MD represents E. M. F. and OD represents current, $E/I = \tan\theta$. But from Ohm's law $E/I = R$, hence, at any point upon the external characteristic the corresponding external resistance is proportional to the tangent of the angle which the ordinate at that point subtends.

As the external resistance is increased, the angle θ increases and the point M moves towards B. Finally, a very slight increase in θ will cause M to drop to the origin. There is therefore for a series generator an external resistance, the *critical resistance,* beyond which the generator will not operate. Reflection will show the correctness of this conclusion since the resistance must always be small enough to permit a sufficient current to flow through the field coils and produce the necessary strength of field.

587. Characteristic of Shunt Generator.—If a shunt generator be connected up as shown in Fig. 299 and data be obtained

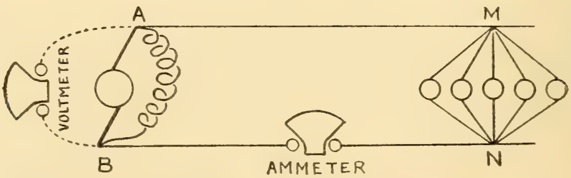


Fig. 299.

and characteristic plotted as described in Par. 585, the resulting curve (Fig. 300) will be seen to differ widely from the one obtained from the series machine. To begin with, the voltage is a maximum

when there is no current in the external circuit. As the current is increased, the voltage falls quite regularly until a final point is reached when a further decrease in the external resistance causes both the current and the voltage to drop and if all resistance be removed, the machine unbuilds entirely and the curve returns upon the origin.

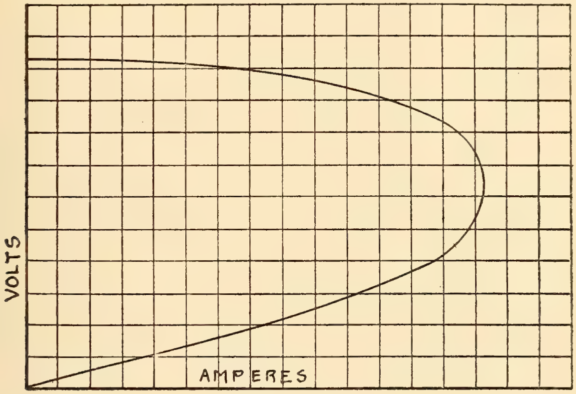


Fig. 300.

The foregoing results are brought about by two causes. First, the current through the shunt grows smaller and the field consequently weaker. Whatever decreases the difference of potential between A and B (Fig. 299) decreases the current through the field coils. With no current in the external circuit, the full E. M. F. of the machine is available for driving current through the shunt. When, however, a current flows through the armature, the available E. M. F. is the total E. M. F. diminished by the internal drop, Ir, which last varies directly with the current. At first, as the field current weakens, the voltage is not greatly affected since the field magnets are being worked on the upper part of the magnetization curve. When, however, the magnetization falls below the bend of the curve, it drops rapidly as the exciting current decreases.

Second, the field is weakened by the armature reaction. Consider the diagram (Fig. 301) of the drum-wound bipolar machine. With clockwise rotation, the brushes will be shifted from the symmetrical plane to the positions A and D (Par. 570). In the inductors in the semi-circumference $ABCD$, the current is flowing

in; in the other semi-circumference it is flowing out. The effect of the current in the inductors C to D and A to F is to produce a field in the direction of the large arrow, that is, opposite to the field of the magnets and consequently weakening that field, and this effect increases as the current through the armature increases.

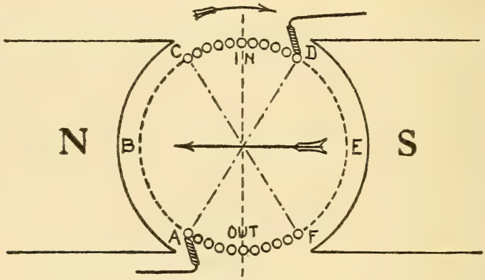


Fig. 301.

For this reason, the ampere turns between C and D and between A and F, or in the double angle of lead, are named the *demagnetizing turns.*

The *critical resistance* for a shunt generator is that resistance of the external circuit which *if reduced* will cause the machine to unbuild.

588. Compound Generator.—The properties desired of a generator vary in accordance with the use to which the current is to be put. In some circumstances constancy of current is required; in others, constancy of potential. Of these, the more important, notably in the case of electric lighting (Par. 511), is constancy of potential. Neither the series nor the shunt generator afford this desired constancy. However, we have shown above that the voltage of a series generator rises as the current is increased, while that of the shunt machine falls with this increase. The logical attempt to combine these windings in one machine so that their effects counterbalance, has led to the development of the compound generator. This compounding may be so carried out that the voltage, even under wide fluctuations in the current, remains nearly constant.

589. Overcompounding.—If in a compound machine the series coils be given a few more turns than are needed to preserve constant potential, the voltage rises with increase of current, although

not so rapidly as in the case of the simple series machine. The generator is then said to be *overcompounded.* The object of overcompounding will be understood from the following. Let G, Fig. 302, represent a compound generator supplying current to a dis-

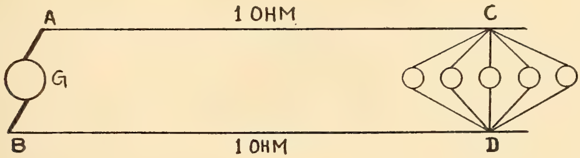


Fig. 302.

tant group of lamps CD. Suppose each lamp to require one ampere at 100 volts and suppose the resistance of the leads AC and BD to be each one ohm. When one lamp is turned on, there is a drop of one volt from A to C, and of one volt from D to B. In order therefore that the voltage between C and D shall be 100, the generator must develop between its brushes 102 volts. If all five lamps be turned on, there will be a drop of five volts from A to C, and of five from D to B; the generator must therefore develop between its brushes 110 volts. We see then that a generator is overcompounded so that a constant difference of potential may be maintained between two points *at a distance from the generator.*

CHAPTER 42.

DIRECT CURRENT MOTORS.

590. The Motor and the Generator Identical.—An electric generator, as we have already seen, is a machine to which mechanical energy is applied and from which electrical energy is drawn; on the other hand, an electric motor is a machine to which electrical energy is applied and from which mechanical energy is derived. Electrically, they are identical, and a machine which if turned by mechanical power will produce a current, will, if supplied with a current, develop mechanical power. The truth of this statement may be shown by the following simple illustration. Fig. 303 represents the arrangement, already de-

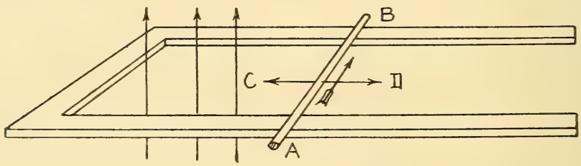


Fig. 303.

scribed several times, of a wire sliding on parallel conducting rails which include between them a magnetic field. The wire AB is a conductor in a magnetic field and if pushed in the direction C, there will be induced in it a current from A to B (Par. 422); it is therefore a generator in its simplest form. If instead of pushing the wire, a current be passed through it from A to B, it becomes a conductor carrying a current and placed in a magnetic field and experiences a force (Par. 356) which will cause it to move in the direction D (Par. 352); it is therefore also a motor.

591. Explanation of Motion.—Let AB, Fig. 304, represent a coil of wire placed in a magnetic field NS and free to revolve about the axis CD. If a current be sent through this coil it will start to rotate. The simplest explanation of this motion is that each side of the coil is a conductor carrying a current and placed

in a magnetic field and is therefore acted upon by a force which is at right angles to the field and whose strength is (Par. 356)

$$f = I \cdot H \cdot l \text{ dynes}$$

In this expression I is the current in absolute units, H is the strength of the field, or number of lines of force per square centimeter, and l is the length in centimeters of the wire at right angles

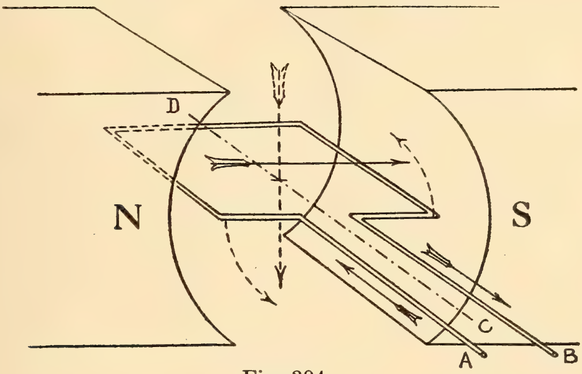


Fig. 304.

to the field. The direction of the current in one side of the coil being opposite to that in the other, the force acting upon one side is opposite to that acting upon the other; in other words, the two forces constitute a couple and rotation will be produced. Its direction may be determined by applying the left hand rule (Par. 352).

The following additional explanation of this movement is given as it involves certain conceptions which will be used in a discussion later on.

The lines of force of the field run from N to S as shown by the heavy arrow. If a current enters the coil by A and leaves by B, there will be produced within the coil a field whose direction, as shown by the broken arrow, is from above downward. In accordance with Maxwell's law (Par. 371), the coil will turn until it embraces its own field and that of the magnet; it will therefore take up a counter-clockwise rotation. The turning effect of the couple mentioned above becomes zero when the coil has revolved until it lies in the vertical plane, and is reversed when the coil passes through this plane. The coil would therefore come to rest in this position. However, by means of a suitable commutator,

as explained under generators (Par. 556), the current is reversed as the coil passes through the vertical plane; its field is therefore shifted 180° ahead and the rotation becomes continuous. Moreover, by using many coils upon the armature (Par. 558), it is always possible to have the current flowing through those in which the turning effect is at or near a maximum.

592. Power Developed by a Motor.—Power is the rate at which work is done (Par. 492), therefore

$$\text{Power} = \frac{\text{work}}{\text{time}}$$

Work is force exerted over a path, hence

$$\text{Power} = \frac{\text{force} \times \text{path}}{\text{time}}$$

$$= \text{force} \times \frac{\text{path}}{\text{time}}$$

$$= \text{force} \times \text{velocity}$$

Consider one of the inductors of the armature of a motor (Fig. 305). The force exerted upon it is (Par. 591) $f = I \cdot H \cdot l$ dynes. The same force is exerted upon the inductor diametrically opposite.

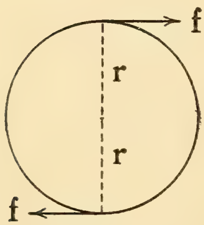


Fig. 305.

If r be the radius of the armature, in one complete revolution the inductor travels a distance $2\pi r$. In n revolutions it travels $2\pi rn$.

If these n revolutions be made in time t, the velocity with which the inductor travels is $2\pi rn/t$.

From above, power = force × velocity, hence power developed by the motor is

$$P = 2IHl \times 2\pi rn/t$$

This may be written

$$P = IHl \times 2r \times 2\pi n/t$$

But $IHl \times 2r$ = armature moment = torque, and $2\pi n/t$ = angular velocity of the armature, hence the power developed varies with the torque and with the speed of rotation of the armature.

593. Counter Electro-Motive Force.—Ignoring for the moment the cause of the movement, consider a rectangular coil, as described in Par. 591, rotating in a counter-clockwise direction in

a magnetic field. The sides of this coil are conductors moving in a magnetic field. Application of the right hand rule (Fig. 306) will show that there is induced in the coil an E. M. F. which acts in at B and out at A. The more rapid the rotation, the greater this E. M. F. (Par. 425). Comparing figures 306 and 304, we see

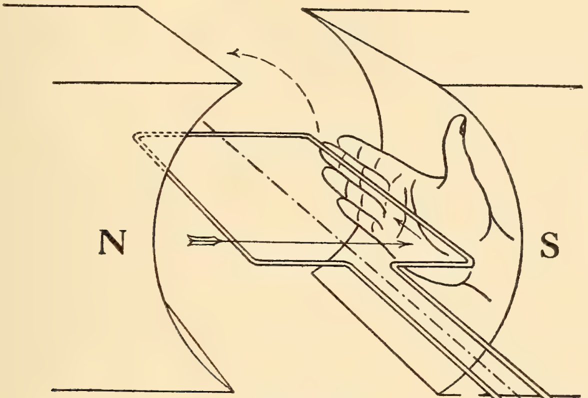


Fig. 306.

that this E. M. F. is opposed to that of the current which causes the motor to rotate; in other words, the rotation of the motor sets up an E. M. F. which opposes the current which produces the rotation. This opposing E. M. F. is called the *counter* or *back* E. M. F.

The first conspicuous effect of the counter electro-motive force developed by a motor is to cut down the current supplied. If an ammeter be connected in series with a motor and the circuit be closed, it will be noted that before the motor begins to move, the current is very large (indeed, without some special arrangement to be described later [Par. 601] it may be excessive), but as the motor speeds up, the current falls steadily.

If the E. M. F. applied to the brushes of a motor be E, and the resistance of its armature be R, the current through the armature before the motor moves is

$$I = \frac{E}{R}$$

and as R is small, I is great.

As the motor gains speed, the current becomes

$$I = \frac{E - E_B}{R}$$

or only so much as can be driven through the armature by the difference of the impressed and the back E. M. F.

594. Relation Between Counter E. M. F. and Power Developed.—Since the power which a generator delivers to the brushes of a motor is IE watts (Par. 494), and since, as shown above, I is cut down by the back E. M. F. developed and hence the power received by the motor is thereby diminished, it would seem that back E. M. F. is a defect. However, consider the following:

From above, the current which a generator supplies to a running motor is

$$I = \frac{E - E_B}{R}$$

whence $IR = E - E_B$

whence $I^2R = IE - IE_B$

whence $IE = I^2R + IE_B$

or IE, the total power delivered to the motor by the generator, is divided into two parts, one of which, I^2R, represents power lost in heating the armature coils (Par. 494); the other, IE_B, represents the useful power of the motor. Hence, the useful power of a motor is directly proportional to the back E. M. F. which it develops.

From the foregoing, the useful power of a motor varies with the product of the two factors I and E_B. In Par. 592 it was shown that this power also varies with the product of two other factors, the torque and the speed of rotation. The torque, $I \cdot H \cdot l \times 2r$, if the field H be constant, varies directly with the current I, consequently, the remaining factor, E_B, the counter E. M. F., varies directly with the speed of rotation. This might have been anticipated since we have shown above that the counter E. M. F. varies with the rate at which the lines of force of the field are cut.

Fig. 307.

595. Reading of Voltmeter Across Seat of Counter E. M. F.—There is sometimes some confusion in the mind of a beginner as to the reading of a voltmeter shunted around a seat of counter E. M. F. The correct reading is always the sum of the counter

E. M. F. and of the regular IR drop over the resistance between the two points. As an illustration, let G, Fig. 307, represent a generator connected up in circuit with a motor M across whose brushes a voltmeter is shunted, Let the E. M. F. of the generator be 100 volts and suppose its resistance to be negligible. Let the resistance of the leads be one ohm and that of the motor be one ohm. Suppose that the generator is started but that the motor is held fast and not allowed to rotate. The current is $I = E/R = 100/2 = 50$ amperes. The drop over the leads is $IR = 50$ volts and that across the motor is $Ir = 50$ volts, which is the reading of the voltmeter. Suppose now that the motor is released and speeds up, producing a back E. M. F. of 90 volts. The current is now $I = \frac{E - E_B}{R} = \frac{100 - 90}{2} = 5$ amperes, or is reduced to one-tenth of what it was originally. The IR drop over the leads is only 5 volts; the reading of the voltmeter therefore is $100 - 5 = 95$ volts, that is 90 for the back E. M. F. and 5 for the Ir drop across the armature.

596. Efficiency of Motors.—A generator delivers to the brushes of a motor a current I of voltage E. The resistance across the brushes is R. The motor rotates and by belts or gearing or otherwise turns out mechanical power. The ratio of the power turned out by the motor to the power delivered to its brushes by the generator is the measure of the motor's efficiency. Thus, if the generator supplies ten horse-power to the motor and the motor turns out nine horse-power, its efficiency is 9/10 or 90 per cent.

The power delivered to the motor is IE watts (Par. 494); the useful power turned out by the motor is IE_B watts (Par. 594); the efficiency of the motor is therefore measured by IE_B/IE or by E_B/E; that is, the nearer the counter E. M. F. approaches the applied E. M. F., the greater the efficiency of the motor.

The foregoing may be shown graphically as follows. The current through the motor when the latter is running is (Par. 593)

$$I = \frac{E - E_B}{R}$$

Substituting this value of I in the above expressions, we obtain for the power delivered to the motor

$$\frac{E(E - E_B)}{R}$$

and for the power turned out by the motor

$$\frac{E_B(E-E_B)}{R}$$

whence the efficiency is

$$\frac{E_B(E-E_B)}{E(E-E_B)}$$

Upon rectangular axes (Fig. 308) lay off $OA = OB$ proportional to E_B, and $OD = OF$ proportional to E. Complete the squares. The area of the rectangle $ADJG$, since it is proportional to $E_B(E-E_B)$, is proportional to the power developed by the motor. The area of the rectangle $BJKF$, since it is proportional to $E(E-E_B)$, is proportional to the power delivered to the motor. The ratio of the first of these rectangles to the second measures the efficiency of the motor. The rectangle $BJKF$ is greater than $ADJG$ by the area of the square $JGKH$. The efficiency of the motor approaches unity as this square diminishes, which it does as OA increases, that is, the efficiency of the motor increases as the counter E. M. F. increases.

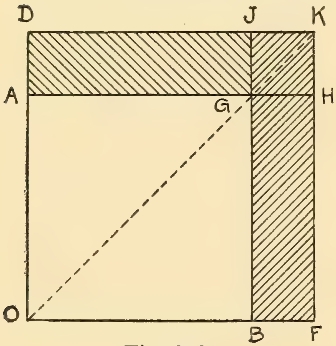


Fig. 308.

It must be noted, however, that as the counter E. M. F. $OA = OB$, increases, the current through the motor decreases, and the rectangles representing the power applied and the power turned out both diminish, therefore, so long as the motor develops appreciable power, its efficiency is never perfect.

597. Maximum Output of Power.—Maximum efficiency must not be confused with maximum output of power. From the preceding paragraph, the power turned out by the motor is

$$z = \frac{E_B(E-E_B)}{R} \text{ watts}$$

The first differential coefficient with respect to E_B is

$$\frac{dz}{dE_B} = \frac{1}{R}(E-2E_B)$$

Placing this equal to zero and solving for EB

$$E_B = \frac{1}{2}E$$

or the power turned out by a motor is a maximum when the counter E. M. F. is equal to one-half the impressed E. M. F. In this case the efficiency is only one-half; that is, there is a loss of one-half of the power delivered to the motor.

Reference to the conclusion drawn in Par. 340 will show that in a battery also when the power developed is a maximum, the loss is one-half.

598. Classes of Direct-Current Motors.—There are three classes of direct-current motors, the series, the shunt and the compound. The majority belong to the first two of these classes. In structure they are, with a few minor changes, the same as the corresponding generators. Thus, the requirement of being able to reverse the direction of rotation at will involves the setting of the brushes at right angles to the commutator surface instead of inclined thereto. So also in the operation of a motor, the armature reaction causes the brushes to be shifted *backward* from the neutral plane instead of *forward* as in the case of the generators.

As a rule, motors are operated on constant potential circuits, the voltage between the mains being constant.

599. Shunt Motors.—The shunt motor possesses certain advantages over the other forms which render it by far the most desirable for most mechanical purposes. Chief among these is its ability, as shown below, to make automatic adjustment for fluctuations in the load thrown upon it and in spite of these fluctuations to vary but little in speed.

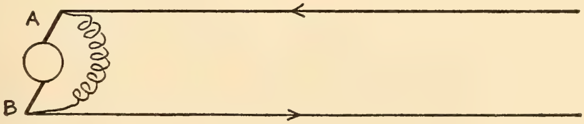


Fig. 309.

Fig. 309 represents in simplest diagrammatic form a shunt motor. The difference of potential between A and B being constant, as stated above, the current through the field coil AB is constant.

The force on the several inductors of the armature is (Par. 592)

$$f = I\ .\ H\ .\ l \text{ dynes}$$

In this expression H and l are constant, hence the torque varies directly with the current through the armature. In order therefore to vary the torque for different loads, this current must vary.

The current through the armature is (Par. 593)

$$I = \frac{E - E_B}{R}$$

From this we see that the current can be increased by increasing E, decreasing E_B, or decreasing R. Now E is the voltage between the mains, which we have seen above is constant, and R is the armature resistance, which is fixed when the machine is built. The only remedy therefore is to decrease E_B, the back E. M. F. This back E. M. F. varies with the rate at which the lines of force of the field are cut (Par. 594), that is, it varies directly with the speed of rotation.

When the load upon a shunt motor is suddenly increased, the speed will be observed to decrease slightly. This does not mean that the machine is weakening. On the contrary, by slowing down, the back E. M. F. is decreased, the current and hence the torque increase.

A numerical example will bring this out clearly. If in the above expression for the current we make $E = 110$, $E_B = 100$ and $R = 1$, we get $I = 10$ amperes. If we make $E_B = 90$, I becomes 20 amperes, hence a reduction of one-tenth in the speed of rotation doubles the torque on the armature.

Since the power developed by the motor is IE_B (Par. 594), it may be asked whether the increase in I were counterbalanced by the decrease in E_B, for if they varied reciprocally, the power, IE_B, might remain constant and nothing would be gained. However, I increases in a more rapid ratio than E_B decreases. In the numerical example above, with $E_B = 100$, the power is 1000 watts; with $E_B = 90$, the power is 1800 watts.

The valuable characteristic of the shunt motor therefore is that by slight variations in speed it adjusts itself automatically for wide variations in the load. Even should the load be suddenly entirely taken off, the motor will not "race," or speed up dangerously.

600. Control of Speed of Shunt Motors.—The speed at which a shunt motor runs under a certain load may be controlled in one of two ways. The first and most frequently employed method is by varying the strength of the field. There is inserted in the field circuit a rheostat by which the current through the field coils may be varied. By increasing the resistance in this circuit, the field

H is weakened. This causes E_B to diminish and the current through the armature consequently increases. The torque, $IHl \times 2r$ (Par. 592), is thus increased and the machine speeds up. It is true that the torque depends also upon H, but we have shown in the preceding paragraph that I increases more rapidly than H decreases. This increase of speed also follows from the fact that if the field be weakened, the armature must revolve faster in order to cut the same number of lines of force in the same time and thus develop the same power.

The second method is to insert between the motor and one of the mains, as shown in Fig. 310, a rheostat R. The field H is not

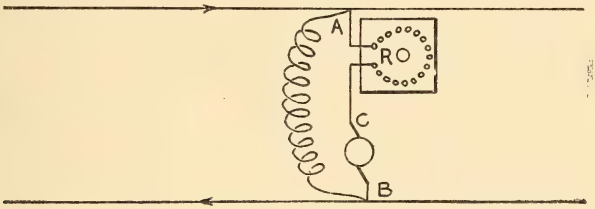


Fig. 310.

affected by this, but the voltage applied to the armature is the total voltage between A and B less the drop over the rheostat. By varying the resistance in R, and hence the drop across the rheostat, the voltage between C and B, and hence the current through the armature, may be varied. Since the torque, $IHl \times 2r$, H remaining constant, varies directly with I, a decrease in the current decreases the speed of rotation.

From the foregoing it is seen that the speed of a shunt motor may be increased (a) by decreasing the current through the field coils, or (b) by increasing the current through the armature.

It should be remarked that control by rheostat is objectionable. The power consumed in heating the coils of the rheostat represents pure waste which, where power is purchased, must be paid for just as if it were doing useful work. The waste in the second method above, since a larger current passes through the rheostat, is much greater than that in the first method.

601. Starting-Box for Shunt Motors.—It was stated above, (Par. 593), that the full voltage can not without serious risk be turned on a motor at rest. It is customary to use a *starting-box*, a form of rheostat by which, as the back E. M. F. rises, the ap-

plied E. M. F. may be gradually increased. The starting-box for a series motor does not differ sufficiently from an ordinary rheostat to warrant a special description. The starting-box for a shunt motor possesses certain features which require explanation.

Although for these motors the full voltage can not be applied at first to the armature, it can with perfect safety be applied to the field coils. This enables the field to attain its full strength H at once, and although the current I through the armature be small, the torque is great enough to cause the machine to gather headway rapidly.

Fig. 311 represents diagrammatically a form of starting-box largely used. It is a box-shaped frame with lattice-work sides for

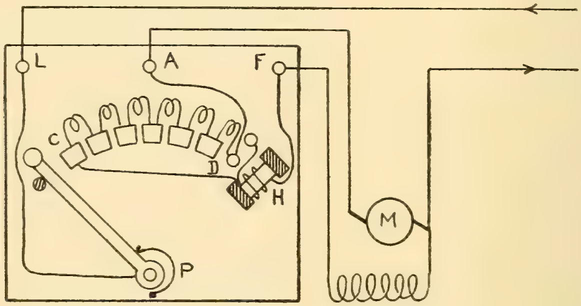


Fig. 311.

ventilation and contains a number of resistance coils in series between a set of contacts arranged along the arc of a circle on the marble cover of the box. The wire of the coils must be of sufficient size to carry the current required by the motor, and therefore to secure the necessary resistance they have to be long. An iron arm, pivoted at P, can be swept along over the contacts. At the pivot of this arm there is a spring which, when the arm is released, throws it back to the safety position. When the arm is placed on the first contact C, the current from the positive main comes in by L, thence to P, thence up the arm to C where it divides, a part passing through all the resistance coils to D, thence to A, thence to the armature of the motor and thence to the negative main, and the other part passing through the coil H, thence to F, thence through the field coils to the negative main. At starting, therefore, the current through the armature is cut down by the entire resistance of the coils from C to D, while the

field is of full strength. As the armature begins to revolve it generates a back E. M. F. and it becomes safe to apply more voltage. The arm is therefore rotated to the right and gradually cuts out the resistance in the armature circuit.

When the arm is hard over to the right, the entire resistance is out of the armature circuit and the arm is held by the electro-magnet *H*. The object of this magnet is the following. Should the circuit be broken or the power be turned off while the motor is in operation, the arm of the rheostat should be automatically returned to the safety position, otherwise the break might be repaired or the power be turned on again with the arm in its full load position and the armature coils be overheated or even burned out. When a break occurs, the magnet loses its power and the spring at *P* throws the arm back to the safety position. This arrangement is called a "no voltage release."

Again, should by any accident the current through the field coils be greatly reduced or entirely cut off leaving only the residual magnetism of the field magnets, the motor, from what has been shown in the preceding paragraph, would speed up dangerously, or, if this did not occur, would not generate sufficient back E. M. F. to keep the current through the armature down to safe limits. Therefore, in this case also the rheostat arm should be automatically thrown back to the safety position.

It will be noted that with the arm hard over to the right, the current which actuates the electro-magnet *H* is the field current and is taken off by the upper one of the contacts at *D*. Should a break occur in the field circuit, this magnet releases the arm which is thrown back by the spring. This arrangement is called a "no field release."

These starting-boxes frequently include an overload switch in addition to the two releases described above.

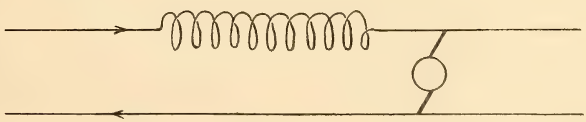


Fig. 312.

602. Series Motors.—In a series motor, shown diagrammatically in Fig. 312, the same current passes through both the field coils and the armature. As was seen in the discussion of the

magnetization curve (Par. 393), at first and when remote from saturation, the field H increases nearly in proportion to the exciting current, hence, at starting, the torque of a series motor, $IHl \times 2r$, varies practically as the square of the current. These motors are therefore especially valuable where great torque is needed at starting, for example in trolley cars, hoists, etc.

603. Speed of Series Motors.—The speed of series motors varies inversely with the load and for each particular load there is a corresponding speed. This renders them unsuitable for many kinds of machines which require a constant speed under varying loads, but well adapted for street railways where the speed is of necessity constantly varied.

Consider a generator supplying a series motor M (Fig. 313). The power developed by the motor must be equal to that sup-

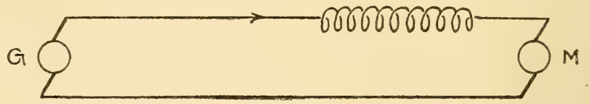


Fig. 313.

plied by the generator, less the heat loss. This last is small, hence the back E. M. F. must be nearly equal to the impressed E. M. F. As the back E. M. F. increases, the current through the motor, and hence the current through the field coils, grows smaller. The field grows correspondingly weaker and to maintain the back E. M. F. the speed of the motor must increase. This tendency to race under diminished loads is an objectionable feature of a series motor.

604. Change of Direction of Rotation.—It may sometimes be desirable to change the direction of rotation of a motor. Suppose a, Fig. 314, to represent a shunt motor, the current flowing as

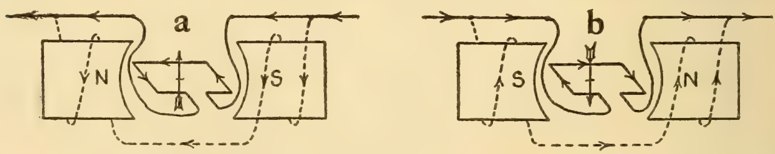


Fig. 314.

indicated. The lines of force of the coil will run upwards and the rotation will therefore be clockwise. If the direction of the current in the mains be reversed, as shown in b, the lines of force of

the coil will run downward, but the polarity of the field magnets is also reversed, and the rotation will as before be clockwise. Hence, reversing the current in the mains *does not change* the direction of rotation. If, however, the direction of the current be reversed in either the field or the armature, but not in both, the direction of rotation will be changed.

605. Motor Generators.—Alternating currents are readily stepped up or down in voltage by means of a transformer (Par. 431), but this method is not applicable to direct currents. Where such transformation is required, the direct current may be employed to operate a motor and this motor in turn operates a generator whose armature is so wrapped, or whose field is of such strength, as to develop a current of the desired voltage. Instead of having the motor rotate the generator by means of a belt or gearing, they may both be mounted upon a common shaft. This combination is called a *motor generator*, but electrically it is the same as two separate machines.

A step further may be taken and two sets of coils may be wrapped upon the same armature and rotate in a common field. Each set has its own commutator, current being delivered to the motor commutator and drawn from the generator commutator. Transformation is effected by varying the ratio of the number of coils or of the number of turns in the two sets of wrappings. This machine is called a *dynamotor*.

CHAPTER 43.

ALTERNATING CURRENTS.

606. Alternating E. M. F. and Current.—We have seen (Par. 552), that if a coil rotates at a uniform rate in a uniform field it will generate an E. M. F. which varies as the sine of the angle through which the coil has turned from its primary position at right angles to the field. If the coil is a closed circuit, or forms a part of such a circuit, there will be produced in it a current which will vary in the same manner. At every revolution of the coil, therefore, the E. M. F. and current pass through a complete cycle of values, positive and negative. The term *alternating* is applied to an E. M. F., or to a current, which thus undergoes these periodic reversals.

607. Why Considered Separately.—The mere fact that a current reverses its direction at regular intervals might not of itself warrant special discussion. There are, however, two properties, *induction* and *capacity,* which are common to all electric circuits and whose effects are conspicuously revealed in varying currents. Alternating currents vary continually and with such currents the above factors give rise to certain peculiar phenomena, some of which *appear to contradict* the principles which have been developed in the preceding pages. Among such we may mention

(a) The current through a circuit is not always equal to the E. M. F. divided by the resistance.

(b) The sum of the partial drops between two points is not always the same as the total drop.

(c) The sum of the currents in the branches of a divided circuit is not always equal to the total current.

(d) Finally, there may be a flow of current in a broken circuit.

In the following pages it will be shown that these contradictions are only apparent and that Ohm's law is as true of alternating currents as it is of direct. In order, however, to be able to explain these peculiarities, the subject of alternating currents must be considered in detail. We shall therefore begin with certain preliminary definitions and principles.

608. Cycle, Period and Frequency.—In Par. 555 it was shown that an alternating E. M. F. and current can be represented graphically by a sine curve (Fig. 315), the ordinates corresponding to the instantaneous values (values at any instant) of the E. M. F. or current and the abscissae to the angle through which the coil

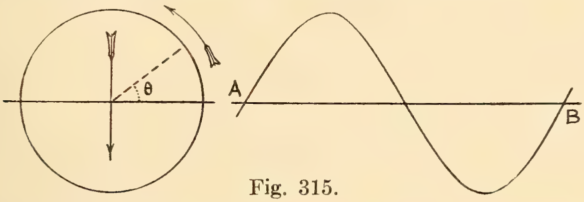


Fig. 315.

has rotated, or, if the scale of time be used, to the time elapsed since the coil moved from its primary position in the neutral plane.

If E_m be the maximum instantaneous value of the E. M. F. and if the abscissae represent the angle through which the coil has rotated, the equation of the E. M. F. curve is

$$E = E_m \cdot \sin \theta$$

If the abscissae represent elapsed time, the equation is

$$E = E_m \cdot \sin \omega t$$

in which ω is the angular velocity of the coil and t is the time in seconds since the coil lay in the neutral plane.

With every revolution of the coil, the portion of the curve between A and B (Fig. 315) is repeated, and the complete set of values, positive and negative, between A and B is therefore called a *cycle.* The more rapid the motion of the coil, the greater the number of cycles in a given time. The lengths of time of one cycle is called a *period* and the number of cycles per second is the *frequency.* The word "revolution," as used above, must be interpreted in an electric sense. Thus, in a four pole generator one revolution of the armature corresponds to *two* electric revolutions.

An additional term, sometimes encountered in books treating of this subject, is *alternation,* an alternation being a reversal of direction of E. M. F. or current. There are therefore two alternations per cycle. The number of alternations is usually given as so many per minute. It is recommended that the use of this term be discarded.

609. Phase.—For purposes of descriptive location, a cycle is considered to be divided into 360 degrees. Any point of the cycle is designated as a certain *phase*, as, for example, the thirty degree phase, etc.

Fig. 316 represents diagrammatically a ring-wound, bipolar, alternating current generator. Consider in either half of the

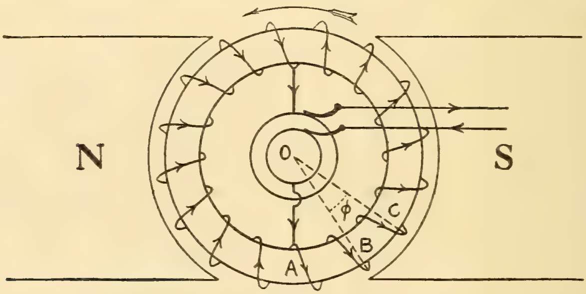


Fig. 316.

armature any two adjacent coils, as, for example, *B* and *C*. In each an E. M. F. is being induced and since in every complete revolution of the armature each coil travels around the same path and returns to its starting point, the cycle, the period and the frequency must be the same for each. At the instant shown however, the E. M. F. being induced in *C* is proportional to the sine

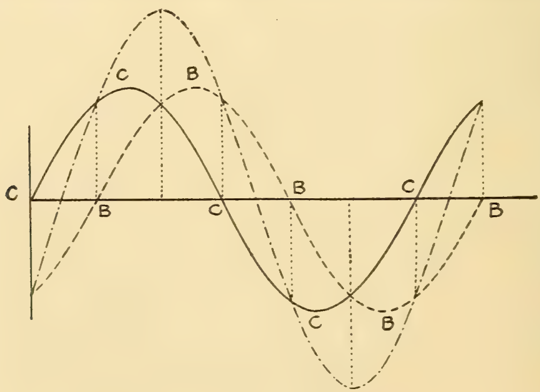


Fig. 317.

of the angle *COA*, while that being induced in *B* is proportional to the sine of *BOA*, and will not reach the value of that now in *C* until sufficient time has elapsed for *B* to move through the angle $\phi = BOC$. The E. M. F. in *C* therefore has reached a value which

will not be reached by that in B for a time corresponding to the angle ϕ. This is shown graphically in Fig. 317. The sine curve $CCCCC$ represents the E. M. F. of the coil C; the sine curve $BBBBB$ represents the E. M. F. of the coil B.

Two sine curves whose periods are the same and which reach their maximum and minimum values simultaneously (see Fig. 265) are said to be *in phase,* otherwise they are said to *differ in phase.* The *phase difference* may be expressed in time but more frequently in angular measure. Thus, the curves in Fig. 317 differ in phase by the angle ϕ which is represented by the horizontal distance CB. If the phase difference is 90°, the curves are said to be *in quadrature;* if it be 180°, they are *in opposition.*

It will be shown shortly that an alternating current generally differs in phase from its corresponding E. M. F. If the current reaches a maximum value *after* the E. M. F. has passed through its maximum, the current is said to *lag;* on the other hand, if it reaches its maximum *in advance of* the E. M. F., it is said to *lead.* In these cases, the corresponding phase difference is spoken of as the *angle of lag* or as the *angle of lead.*

610. Vector Diagrams.—Let the vector OA (Fig. 318), whose length represents the maximum value E_m of an alternating E. M. F. (or current), rotate about the point O in a counter-clockwise direction and at the same uniform angular velocity ω as the armature. The instantaneous value of the E. M. F. (or current) is represented by the line AB, for $AB = E_m \cdot \sin \omega t$. But DO, the projection of OA upon the vertical axis, is equal to AB, hence, when the vector makes the phase angle with the horizontal axis, the corresponding instantaneous value of the E. M. F. (or current) is represented by the projection of the vector upon the vertical axis.

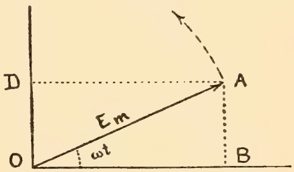


Fig. 318.

611. Composition of Alternating E. M. F.s.—During the rotation of the armature of the generator shown in Fig. 316, the coils in series combine in producing a resultant E. M. F. Thus, in Fig. 317 the broken and dotted curve is the resultant E. M. F. curve obtained by adding the ordinates representing the corresponding simultaneous values of the E. M. F. in the separate coils. By an

application of trigonometry, it can be shown that this resultant curve is also a sine curve and is of the same periodicity as the component curves, although differing from them in phase. The trigonometric process is somewhat tedious and it is thought that the following explanation will be more easily followed. In Fig. 319, the vectors OB and OC represent the maximum values of the E. M. F. in the coils B and C of Fig. 316, and ϕ is the angle of phase difference. The instantaneous value of the E. M. F. in B is, from the preceding paragraph, OB'; the instantaneous value of the E. M. F. in C is OC', and the resultant E. M. F. is the sum of OB' and OC'. Complete the parallelogram $CDBO$ and project its diagonal OD upon the vertical axis. $C'D'$ is equal to OB', hence OD' is equal to the sum of OB' and OC', or is the desired resultant. Therefore, the resultant E. M. F. of the coils B and C is always given by the projection upon the vertical axis of the vector OD, the diagonal of a parallelogram of which the adjacent sides represent the maximum values of the E. M. F. in the corresponding coils and the included angle represents the difference in phase. The length of OD represents the maximum value of the resultant E. M. F. Since ϕ, the difference in phase, is constant, the vector OD does not vary in length or in position relative to OB and OC. Its projections are therefore the ordinates of a sine curve of the same periodicity as the E. M. F. curves of the separate coils.

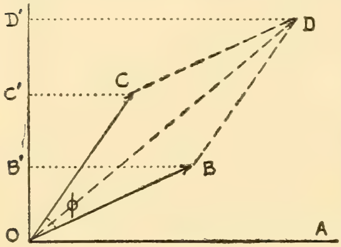


Fig. 319.

From the foregoing we see that alternating E. M. F.s which differ in phase are not compounded by simple addition but in a similar manner to that employed in the parallelogram of forces in mechanics.

612. Value of an Alternating Current.—During each cycle, an alternating current passes through the entire range of values from zero to the positive maximum, thence through zero to the minimum (negative maximum), thence back to zero. Which of all these values should be taken as a measure of the current? The logical agreement is reached that such a current is equal to that direct current which performs the same amount of work in the same length of time. Of the three classes of work which a current

may perform (Par. 444), only one, the heating effect, is independent of the direction of the current, and this is accordingly selected as the basis of comparison.

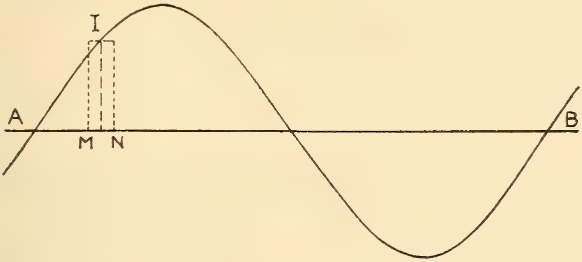


Fig. 320.

Let the curve AB (Fig. 320) represent an alternating current produced by a coil rotating with an angular velocity ω. If the maximum value of the current be I_m, the equation of this curve is

$$I = I_m \,.\, \sin \omega t \tag{I}$$

Consider any ordinate of this curve as I. The instantaneous value of the power being developed at this point is I^2R (Par. 494), R being the resistance of the circuit through which the current is flowing. Let MN represent a minute interval of time dt. Since work = power × time, the work done by I during this interval is

$$dw = I^2R \,.\, dt$$

Substituting in this the value of I from (I)

$$dw = I_m^2R \,.\, \sin^2\omega t \,.\, dt$$

The integral of this between the proper limits will give the total work performed by the current during the cycle.

$$w = I_m^2 R \int \sin^2\omega t \,.\, dt$$

$$= \frac{I_m^2 R}{\omega} \int \sin^2\omega t. \; (\omega \, dt)$$

$$= \frac{I_m^2 R}{\omega} \left(-\tfrac{1}{2} \cos \omega t \,.\, \sin \omega t + \tfrac{1}{2}\, \omega t\right) + \text{a constant}$$

Taking this between the limits $\omega t = 0$ and $\omega t = 2\pi$

$$w = I_m^2 R \,.\, \frac{\pi}{\omega} \tag{II}$$

The work performed by a direct current flowing through the same resistance for the same length of time is

$$w = I^2Rt$$

Since $\omega t = 2\pi$, t, the time of one cycle $= \frac{2\pi}{\omega}$

Substituting, we have

$$w = I^2R \cdot \frac{2\pi}{\omega} \tag{III}$$

Equating the second members of (II) and (III)

$$I^2R \cdot \frac{2\pi}{\omega} = I_m^2 R \cdot \frac{\pi}{\omega}$$

Hence $I = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$

that is, the alternating current is equivalent to a direct current whose value is only .707 of the maximum value of the alternating current. This may be otherwise expressed by saying that the *effective* or *virtual value* of the alternating current is only .707 of its maximum value. The same relation exists between the effective and the maximum voltage of an alternating current, and ammeters and voltmeters for use with such currents are graduated to read the *virtual amperes* and *volts* respectively.

613. Second Deduction.—The foregoing deduction may be made without the use of the calculus, as follows:

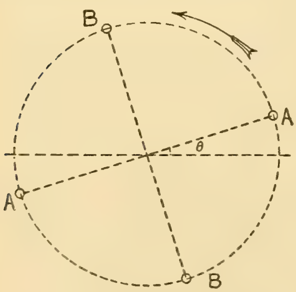


Fig. 321.

Let AA and BB (Fig. 321) be two coils at right angles to each other, both rotating at a uniform rate in a uniform field and each sending current through a resistance R. In one complete revolution the work done by the currents from both is twice the work done by the current from one. The current from A being $I_m \sin\theta$, that from B is $I_m \cos\theta$. The power developed at any instant by the current from A is $I_m^2 \sin^2\theta\, R$; that developed at the same instant by the current from B is $I_m^2 \cos^2\theta\, R$. The total instantaneous power is the sum of these two, or

$$I_m^2 (\sin^2\theta + \cos^2\theta)\ R = I_m^2 R$$

The total work done during the time t of one complete revolution is $I_m^2 Rt$, hence the work done in this time by the current from one coil is $\frac{1}{2} I_m^2 Rt$. A direct current I flowing for a time t through the resistance R does work $I^2 Rt$.

$$I^2 Rt = \tfrac{1}{2} I_m^2 Rt$$

Hence

$$I = \frac{I_m}{\sqrt{2}} \text{ as before.}$$

614. Self-Induction.—Self-induction was explained in detail in Pars. 432–436 and it was shown that its characteristic effect is to oppose any change in a current-produced field and that it does this by setting up a counter E. M. F. which opposes any change in the current in the circuit involved. Since alternating currents are *always changing,* it is in dealing with such currents that the consideration of induction assumes the greatest importance.

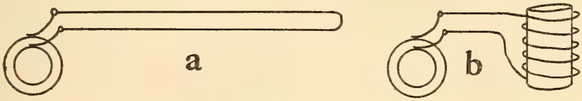


Fig. 322.

If an alternating E. M. F. be applied to a circuit of a simple loop of wire (Fig. 322 *a*), the effect of induction may be so slight as to be negligible and the current may be considered to follow Ohm's law.

If the same piece of wire be wrapped into a coil of 100 turns and the E. M. F. be applied so as to produce the same number of lines of force in the field in the same time as before, these are now cut one hundred times instead of once and the effect of induction is one hundred times as great.

Finally, if there be inserted in this coil a soft iron core (Fig. 322 *b*) and the E. M. F. be applied, the same change of current will produce about 2000 times as many lines of force (Par. 394) and the effect of induction will be 200,000 times as great as in the first case. These examples show that self-induction is developed by the cutting of the lines of force in the embraced field rather than by changes in the current in the embracing circuit.

615. Inductance.—Self-induction is measured by the cutting of lines of force produced when the current in the circuit is varied one unit. The practical unit, the *henry,* is the self-induction of

that circuit in which a change of one ampere produces a cutting of 10^8 lines of force. When the self-induction of a circuit is expressed numerically, as so many henrys, it is called *inductance.* The inductance of a given circuit is constant provided the circuit is distant from magnetic bodies. If it be not distant from such bodies, owing to their saturation, the field does not vary uniformly with the current.

Although the *inductance* is thus constant, the *counter E. M. F.* which it is instrumental in producing, and whose effect is so important, is not at all constant but varies with the rate of change of the current (Par. 432) and has therefore a different value at every different phase and for every different frequency employed. This will be shown more clearly later on.

616. Inductance and Resistance.—Inductance and resistance agree in that they oppose the flow of current in a circuit, but here the similarity ends. The following will bring out the difference between the two.

(a) The resistance of a circuit is constant and does not vary with changes in the current. The inductance of a circuit appears only when the current is changing and the counter E. M. F. which it sets up is proportional to the *rate* of this change.

(b) Resistance does not vary with the geometric form of the circuit nor with the proximity of magnetic bodies. Inductance depends essentially upon these factors.

(c) The energy spent in overcoming resistance is lost in the form of heat. That spent in overcoming the induction counter E. M. F. (Par. 359) is periodically absorbed in the field about the conductor as the current rises and is restored to the circuit as the current falls. As an analogy, the energy spent upon a fly-wheel does two things: (1) it overcomes the friction of the bearings and is thus lost as heat, and (2) it is absorbed by the wheel which, after the power is shut off, continues to turn and thus restores the absorbed energy.

All circuits contain resistance, inductance and capacity, but one or more may be so small as to be negligible. For the sake of simplicity we shall first consider a circuit in which the capacity may be disregarded.

617. Alternating E. M. F. in a Circuit Having Resistance and Inductance.—The instantaneous value of the current produced

in a coil rotating at a uniform rate in a uniform field is (Par. 612)

$$I = I_m \,.\, \sin \omega t$$

In this expression, ω is the angular velocity of the moving coil, whence ωt is the angular distance through which the coil rotates in t seconds. In one revolution the coil turns through the angle 2π. If the frequency be f, that is, if the coil makes f revolutions per second, the angular distance through which it travels in one second is $2\pi f$ and in t seconds is $2\pi ft$. We may therefore substitute $2\pi ft$ for ωt in the above expression, whence

$$I = I_m \,.\, \sin 2\pi ft$$

If the resistance of the circuit be R, the E. M. F. required to drive the current I through this resistance is, from Ohm's law, $E_R = IR$, or, substituting the above value of I

$$E_R = R(I_m \,.\, \sin 2\pi ft)$$

This E. M. F., which is variously called the *active*, the *effective*, or the *power* E. M. F., reaches its maximum value $I_m R$ when $\sin 2\pi ft = 1$, that is, at the 90° and the 270° phases, or at B and D (Fig. 323), and may be represented by the sine curve $AFCGE$, the corresponding current being in phase with it and being represented by the sine curve $ASCTE$.

Should there be in the circuit an inductance of L henry, there will be produced a counter E. M. F. whose value is (Par. 434)

$$E_B = -L\frac{dI}{dt}$$

Since from above

$$I = I_m \,.\, \sin 2\pi ft$$

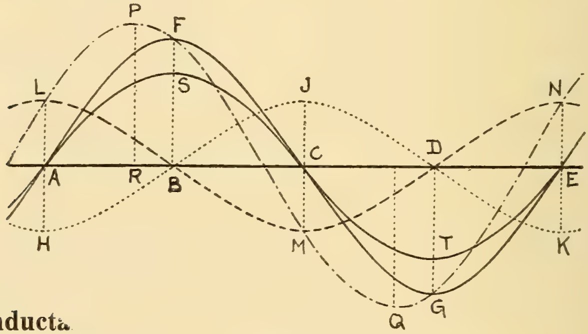
$$\frac{dI}{dt} = I_m \,.\, 2\pi f \,.\, \cos 2\pi ft$$

whence

$$E_B = -L \,.\, I_m \,.\, 2\pi f \,.\, \cos 2\pi ft$$

This counter E. M. F. may therefore be represented by a sine curve. It reaches its maximum value $I_m \,.\, 2\pi fL$ when $\cos 2\pi ft = 1$, that is, at the 0° and the 180° phases, or at A, C and E (Fig. 323). It is therefore in quadrature with the E. M. F. represented by the curve $AFCGE$. Also, since this E. M. F. opposes any change in the existing current, it is positive as the latter falls and negative as the latter rises. It is a maximum when the current passes

through zero, since at this moment the rate of change of the current is greatest, and it is zero when the current is a maximum, either positive or negative. It may therefore be represented by the sine curve *HBJDK*.



616. Induct

agree in that they op

Fig. 323.

the similarity ... drive the current I through the circuit, the impressed E. M. F. must be greater than that required by Ohm's law of a direct current, for it must not only be sufficient to overcome the ohmic resistance but also to counterbalance the back E. M. F. due to self-induction.

The curve *LBMDN* represents the E. M. F. required to overcome the counter E. M. F. Its ordinates are equal but of opposite sign to the corresponding ordinates of the curve *HBJDK*. The impressed E. M. F. is the resultant obtained by compounding the E. M. F. represented by the curve *AFCGE* and that represented by the curve *LBMDN*. The curve *LPFMQGN*, obtained by adding the corresponding ordinates of these two curves, represents this resultant. It will be noted that it reaches its maximum at P before the current reaches its maximum at S, that is, it leads the current by a difference of phase corresponding to RB. In alternating current circuits containing resistance and inductance alone, the current always lags behind the impressed E. M. F.

618. Graphic Construction of E. M. F. and Current Curves.—The power E. M. F., or E. M. F. required to overcome the ohmic resistance, and the E. M. F. required to counterbalance the E. M. F. of self-induction, may be compounded as just explained. They may also be compounded according to the method described in Par. 611. Thus, to find the instantaneous values of the various

E. M. F.s and current at any phase, such as x, Fig. 324. Lay off oa to represent the maximum value, I_mR, of the power E. M. F. and making with the horizontal axis an angle θ corresponding to the phase angle Ex. On this same line, since the current is in phase with this E. M. F., lay off oc to represent the maximum value I_m of the current. Lay off ob at right angles to oa (the two E. M. F.s being in quadrature) and of a length to represent the maximum value, $I_m 2\pi fL$, of the E. M. F. to overcome the E. M. F. of self-induction. The diagonal od of the parallelogram constructed upon oa and ob is the vector corresponding to the required impressed E. M. F. The projection of oa upon the

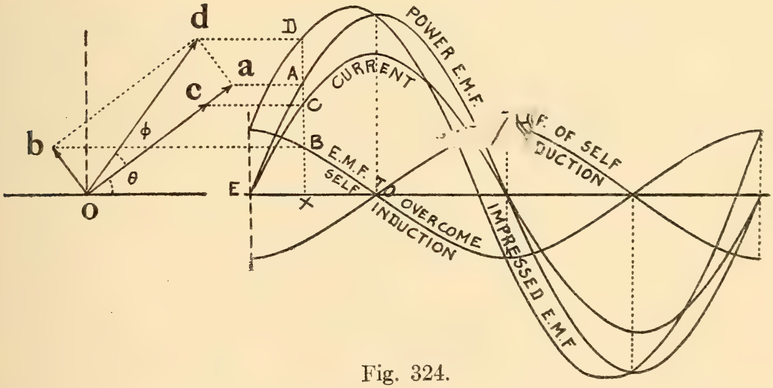


Fig. 324.

ordinate at x locates the point A of the curve of power E. M. F., that of ob locates the point B of the curve of E. M. F. to counterbalance the induced E. M. F., and that of od locates the point D of the curve of impressed E. M. F. Finally, the projection of oc upon the ordinate at x locates the point C of the current curve.

619. Inductive Reactance.—The counter E. M. F. due to self-induction varies with the *rate* at which the lines of force are cut. It therefore varies not only with the *inductance,* or number cut when the current is varied one ampere, but also with the rapidity with which the current changes. In alternating currents this is a function of the number of cycles per second, that is, of the *frequency.* In Par. 617 it was shown that this E. M. F., which is also called the *reactive E. M. F.*, is in quadrature with the power E. M. F. and that its maximum value is

$$E_B = I_m.2\pi fL$$

The factor, $2\pi fL$, is called the *inductive reactance.* It obviously varies with the frequency f and with the inductance L. It is measured in ohms, as might be inferred from the fact that when multiplied by current the product is E. M. F. By expressing it in its dimensional formula, it may be shown to be of the same dimensions (a velocity) as resistance (Par. 547). It is sometimes defined as that factor by which the maximum value of an alternating current in a circuit containing inductance is multiplied in order to obtain the maximum value of the reactive E. M. F. The reactance of a circuit for a given frequency is obtained in ohms by multiplying the inductance in henrys by 2π times the frequency.

620. Impedance.—Examination of Fig. 324 will show that od, the maximum value of the impressed E. M. F., is the hypothenuse of a right-angled triangle whose sides are $oa = I_mR$, the maximum value of the power E. M. F., and $ad = ob = I_m.2\pi fL$, the maximum value of the reactive E. M. F. It follows that (Fig. 325 a)

$$E_m^2 = (I_mR)^2 + (I_m.2\pi fL)^2$$

whence

$$I_m = \frac{E_m}{\sqrt{R^2 + (2\pi fL)^2}}$$

The resemblance of this expression to Ohm's law is obvious. The denominator of the fraction in the second member is measured in ohms since it is composed of the resistance and the re-

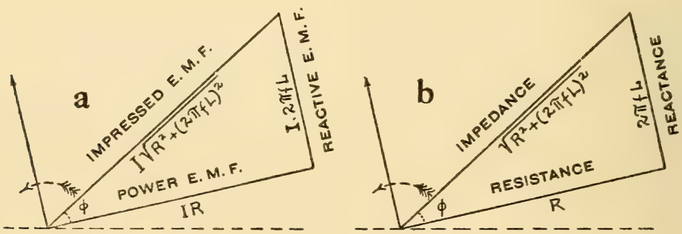


Fig. 325.

actance, both of which are measured in ohms. It is called the *impedance* since it represents the combined effect of the ohmic resistance and the reactance in impeding the flow of the current. It is sometimes defined as that factor by which the current in an alternating circuit is multiplied in order to get the corresponding impressed E. M. F. It will be noted that if $f=0$, the current becomes direct and the expression reduces to Ohm's law.

Inspection of the expression will show that the impedance is itself the hypothenuse of a right-angled triangle whose sides are the resistance and the reactance (Fig. 325 *b*).

It is also seen that the angle of lag, *doc* (Fig. 324), is given by the relation

$$\tan \phi = \frac{da}{oa} = \frac{2\pi f L}{R} = \frac{\text{reactance}}{\text{resistance}}$$

and also by the relation

$$\cos \phi = \frac{oa}{od} = \frac{E_{R}}{E_{m}} = \frac{\text{power E. M. F.}}{\text{impressed E. M. F.}}$$

or, the cosine of the angle of lag is equal to the ratio of the power E. M. F. to the impressed E. M. F. This may be otherwise expressed by saying that if the impressed E. M. F. be multiplied by the cosine of the angle of lag, the result is the E. M. F. required to [illegible] the ohmic resistance, i. e., the power E. M. F.

621. Choke Coils.—The maximum value of an alternating current in a circuit containing resistance and inductance is shown in the preceding paragraph to be

$$I_m = \frac{E_m}{\sqrt{R^2 + (2\pi f L)^2}}$$

If R, the resistance of the circuit, be small, its value may be negligible as compared to that of $2\pi f L$, the reactance, and therefore the current may depend more upon the reactance of the circuit than upon its resistance.

The reactance varies directly with the inductance and the frequency. The inductance varies with the geometric arrangement of the circuit and the proximity of magnetic bodies. The frequency in currents for commercial purposes ranges from 25 to 130. If the current is to be employed for electric lighting, the frequency should not fall below 50, otherwise there will be a perceptible vibration or flicker in the lamps.

It is possible to place in an alternating current circuit a coil of large wire, and hence a small resistance, with a soft iron core whose position may be varied at will. As the core is inserted in the coil, the reactance is increased and the current through the coil is cut down; as the core is withdrawn, the reactance is decreased and a greater current passes through.

Such an arrangement is called a *reactance coil* or a *choke coil* and is frequently used for such purposes as regulating the brilliancy of the lights in a theatre, or for controlling the current applied through a starting box to an alternating current motor. It possesses the great advantage over rheostat control in that it diminishes a current by setting up an opposing E. M. F. and hence without loss of energy, while, in the case of the rheostat, power is reduced by frittering away a portion in heat which waste must be paid for by the consumer.

622. Explanation of Operation of Choke Coil.—If a more physical conception of the operation of a choke coil be desired, it may perhaps be obtained from the following. In Par. 436 it was shown how induction retards the growth of a current. It could have been shown in a similar manner that inductance also retards the *decay* of a current, a dying current being represented by a logarithmic curve also whose ordinates are complementary to those of the curve representing the growing current. Fig. 207 shows that under the conditions given, the current in the circuit whose inductance was one henry required, after the E. M. F. was impressed, about one second to reach the value of six amperes. Suppose this to have been an alternating current of a frequency of 50. In one-two-hundredth of a second after the current started to rise, or when it had reached a value of about .03 ampere, the E. M. F. would be reversed and the current would be beaten back. It would die down as slowly as it rose and would then start to rise in the opposite direction but in one-hundredth of a second after it had been beaten down it would encounter a reversed E. M. F. and would be checked and driven back, and so on, or, figuratively, it would be a shuttle cock at the mercy of the alternating E. M. F.s. We thus see that inductance makes the changes in the current sluggish and that increase of frequency causes the rising current to be driven back more promptly.

623. Inductance and Resistance in Series.—The fact that in alternating current circuits containing inductance and resistance alone the current always lags behind the impressed E. M. F. (Par. 617) affords an explanation of some of the peculiarities of alternating currents to which reference was made in Par. 607. As an illustration, Fig. 326 represents a switch A by which an alternating E. M. F. may be thrown upon a circuit including in series a

coil *BC* with an iron core, and therefore of considerable inductance, and a rheostat whose resistance is assumed to be non-inductive, or purely ohmic. Suppose the switch to be closed and that with a voltmeter we read first the drop across the inductance *BC*, then the drop across the resistance *DE*, and finally the total drop between *B* and *E*. This total drop will be found to be less than the sum of the partial drops. The explanation is that the voltmeter takes no account of phase but indicates the virtual volts

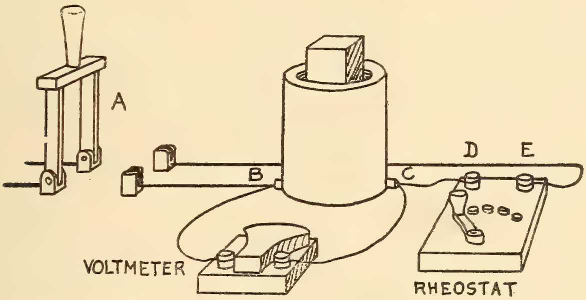


Fig. 326.

between its terminals as if the E. M. F. remained constantly at this value. The current through the circuit at any one instant is of course the same at every point, but while it is in phase with the E. M. F. across *DE*, it lags 90° behind the E. M. F. across *BC*. The maximum E. M. F. across *BC* occurs therefore one-quarter of a period *in advance of* the maximum E. M. F. across *DE*. The total drop is therefore not the sum of the partial drops, since the maximum values of these do not occur simultaneously, but is represented by the hypothenuse of a right triangle whose remaining sides are the partial drops.

624. Inductance and Resistance in Parallel.—Fig. 327 represents an inductive resistance *BC* and a non-inductive resistance

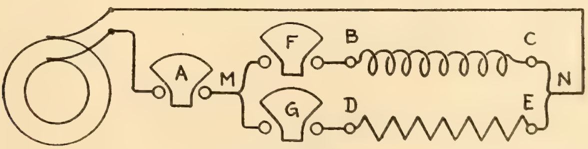


Fig. 327.

DE connected in parallel in an alternating current circuit. *A*, *F* and *G* are ammeters arranged to read the currents in the main

circuit and in the branches of the divided circuit respectively. It will be found that the sum of the currents indicated by F and G is greater than the current indicated by A. This happens because, as explained in the preceding paragraph, the ammeters take no heed of phase but indicate the virtual values of the separate currents as if these currents were constant, or as if their maxima occurred simultaneously. The drop over the two branches is always the same, being the difference of potential between M and N, but the current through DE is in phase with this E. M. F. while the current through BC lags 90°. When the current through DE is a maximum, the current through BC is still one-quarter period removed from its maximum and is correctly the difference between the current through A and that through G. The ammeter F, however, indicates the virtual value of the current through BC, as if the current were constantly of this strength. The currents through F and G are in quadrature and therefore the total current is represented by the hypothenuse of a right triangle whose remaining sides are the currents as indicated by F and G.

From the foregoing we see that the current through resistance and reactance in series is the same at every point, but the voltage across the combination is the vectorial resultant of the separate drops as given by a voltmeter. On the other hand, the voltage across resistance and reactance in parallel is the same over each, but the total current is the vectorial resultant of the separate currents as given by an ammeter.

625. Capacity.—The subject of capacity was discussed in Chapter 10 and it was there shown that the capacity of a condenser is not measured by the quantity of electricity which it can contain but by the quantity which must be imparted to it in order to raise its potential unity.

If two points between which there exists a difference of potential be connected by a conductor, there will be produced a current which will vary directly with this difference of potential and which will continue to flow so long as a difference exists. If, therefore, a source of E. M. F. be connected to the terminal of a condenser, a current will flow into the condenser so long as the potential of the source is higher than that of the condenser. This current, however, will not be of constant strength, for as the condenser becomes charged its potential rises, hence the difference of poten-

tial between it and the source grows smaller, and it is to this difference of potential that the current is proportional. We may regard the potential of the condenser as a counter E. M. F. which opposes the charging E. M. F. and thereby diminishes the current.

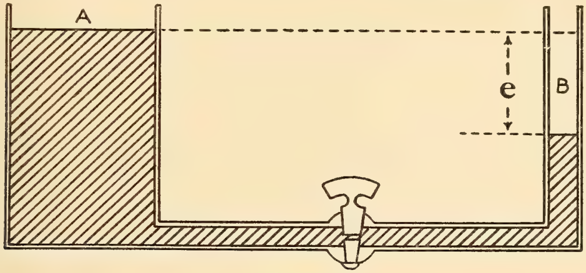


Fig. 328.

As an analogy, Fig. 328 represents a large tank *A* of water of unvarying head connected through a pipe closed by a stop cock to the smaller tank *B*. The difference of the level of the water in *A* and *B* determines whether there shall be a flow when the stop cock is opened. If at first *B* be empty, the flow is urged by the full head of water in *A* and the current is a maximum. When, however, *B* has been partly filled, as shown in the figure, its head is opposed to that of the water in *A* and the flow is determined by the diminishing difference in level *e*; therefore, as *B* fills up, the current dwindles to zero.

626. A Condenser in an Alternating Current Circuit.—Suppose a condenser to be connected in series in an alternating current circuit, as shown in Fig. 329. So long as the potential of the brush

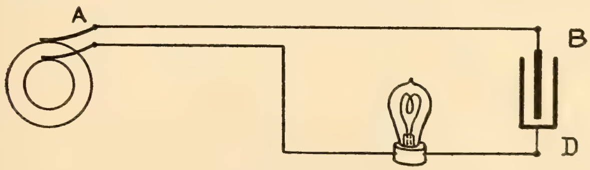


Fig. 329.

A is higher than that of the terminal *B*, a current will flow into the condenser, and when the potential of *A* is a maximum, the condenser will contain a charge *Q*, the maximum under the given conditions. As the potential of *A* diminishes, *Q* flows out and *B* is entirely discharged when the potential of *A* is zero. As the

potential of A continues to sink to a negative maximum, a charge Q flows into the coating D of the condenser and finally flows out again as the potential of A returns to zero. It is thus seen that although the circuit is broken at the condenser, a charge Q flows through the circuit four times in each cycle. If the capacity of the condenser and the frequency be sufficiently great, an incandescent lamp, connected as shown, may be made to glow by this oscillating charge.

627. E. M. F. and Current Curves in Case of Capacity.—The foregoing may be shown graphically as follows. The sine curve $EMFGH$, Fig. 330, represents the impressed E. M. F., or the potential of the brush A. The current from A to the condenser B is determined by the difference of potential between A and B and is a maximum when the potential A is increasing most rapidly. This maximum rate of increase occurs at M when the potential of A is zero. It is here that the tangent to the curve is steepest or the curve climbs up most rapidly. The current therefore, represented by the curve $JKLNO$, reaches a maximum value MK at this point. When the potential of A reaches its maximum at L, the condenser is fully charged and the current no longer flows into it. At this point the current curve is at zero. As

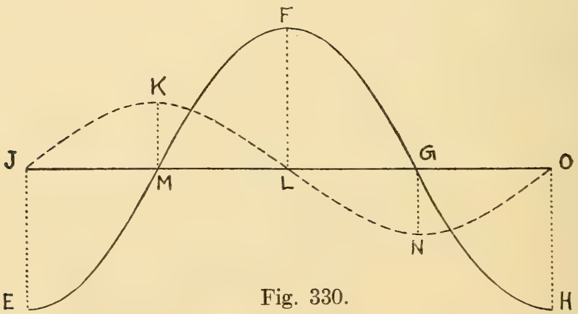


Fig. 330.

the potential of A falls, the condenser discharges, or the current is now negative. As before, the negative current is a maximum when the potential of A is falling most rapidly, and this is the case at G where the potential of A is again zero. Finally, the current is again zero at O where the potential of A is a negative maximum. It is thus seen that in the case of capacity, the current curve *leads* the E. M. F. curve and is in quadrature with it.

628. Capacity Reactance.—The instantaneous value of the E. M. F. in an alternating current circuit is

$$E = E_m \,.\, \sin \omega t$$

If this circuit contains capacity alone, the current leads the E. M. F. by 90° and is given by

$$I = I_m \,.\, \cos \omega t$$

The instantaneous value of the power developed is (Par. 494)

$$IE = I_m E_m \,.\, \sin \omega t \,.\, \cos \omega t$$

The work done in a time dt is

$$dw = I_m E_m \,.\, \sin \omega t \,.\, \cos \omega t \,.\, dt$$

Since the condenser is charged in one-fourth of a period (Par. 626) if this expression be integrated between the limits $t = O$ and $t = \frac{1}{4}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{2\omega}$ (Par. 612), it will give the work expended in charging the condenser.

Performing the integration

$$w = \frac{I_m E_m}{\omega} \cdot \frac{1}{2} (\sin^2 \omega t) + \text{a constant}$$

Taking this between the above limits

$$w = \frac{I_m E_m}{2\omega} \qquad \text{(I)}$$

But in Par. 97 it was shown that the work spent in charging a condenser of capacity K is

$$w \frac{1}{2} \cdot E_m^2 K \qquad \text{(II)}$$

Equating (I) and (II) and solving for E_m

$$E_m = I_m \cdot \frac{1}{\omega K}$$

Substituting for ω its value $2\pi f$ (Par. 617)

$$E_m = I_m \cdot \frac{1}{2\pi f K}$$

Whence also, since $E_m = E_v\sqrt{2}$ and $I_m = I_v\sqrt{2}$ (Par. 612)

$$E_v = I_v \cdot \frac{1}{2\pi f K}$$

E_v and I_v being the virtual E. M. F. and current respectively.

The factor $\frac{1}{2\pi f K}$ is called the *capacity reactance* of the circuit. It is quite analogous to the inductive reactance discussed in Par. 619. It is measured in ohms and its dimensional formula (Par. 547) shows it to be of the same dimensions, a velocity, as resistance. It is that factor by which the maximum value of an alternating current in a circuit containing capacity must be multiplied in order to obtain the value of the reactive E. M. F. due to capacity.

629. Alternating E. M. F. in Circuit Containing Resistance and Capacity.—If the circuit contains both resistance and capacity, in order to drive a current I_m through it, the impressed E. M. F. must be sufficient to overcome both the ohmic resistance and the capacity reactance. The E. M. F. to overcome the ohmic resistance is I_mR, that to overcome the capacity reactance is $I_m \cdot \frac{1}{2\pi f K}$, and these being in quadrature (Par. 627)

$$E_m^2 = (I_mR)^2 + \left(I_m \cdot \frac{1}{2\pi f K}\right)^2$$

whence

$$I_m = \frac{E_m}{\sqrt{R^2 + \left(\frac{1}{2\pi f K}\right)^2}}$$

an expression analogous to the one deduced in Par. 620, the denominator being the impedance.

It will be noted that inductive reactance varies directly with the frequency f, while capacity reactance varies inversely with this factor. Changes in the frequency therefore produce diametrically opposite results in the reactances. As the frequency *increases*, the current through an inductive circuit decreases while that through a capacity increases. On the other hand, as the frequency *decreases*, the current through an inductive circuit increases and that through a capacity decreases.

630. Alternating E. M. F. in Circuit Containing Resistance, Inductance and Capacity.—In the most general case, in order to drive a current I_m through an alternating current circuit containing resistance, inductance and capacity, the impressed E. M. F. must be sufficient to overcome the ohmic resistance and the combined reactance of the inductance and capacity. It has been shown (Par. 617) that in the case of inductance the current lags 90°; on the other hand, in the case of capacity (Par. 627) the current leads by 90°. The E. M. F.s to overcome these separate reactances therefore differ in phase by 180° and are combined by simple subtraction, hence the resultant reactance is

$$2\pi f L - \frac{1}{2\pi f K}$$

and the most general expression for the current is

$$I_m = \frac{E_m}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f K}\right)^2}}$$

631. Electric Resonance.—Fig. 331 represents an alternating current circuit in which there are connected in series a coil of resistance R and inductance L and a condenser of capacity K.

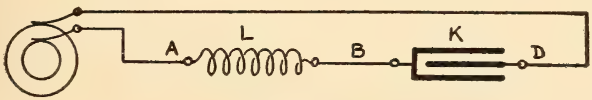


Fig. 331.

From the preceding paragraph, the current through the combination is

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f K}\right)^2}}$$

If in this expression we assign a regular series of values to f, the frequency, the remaining factors being kept constant, and plot the corresponding values of the current, it will be seen that at a certain value of f, which may be called the *critical frequency*, the

current jumps abruptly to a maximum. Inspection will show that this maximum is reached when $2\pi fL = \frac{1}{2\pi fK}$, in which case the above expression reduces to Ohm's law. In this case also $f = \frac{1}{2\pi\sqrt{LK}}$, and the periodic time $= 1/f = 2\pi\sqrt{LK}$ seconds. The above is shown in Fig. 332, the curve representing the values for

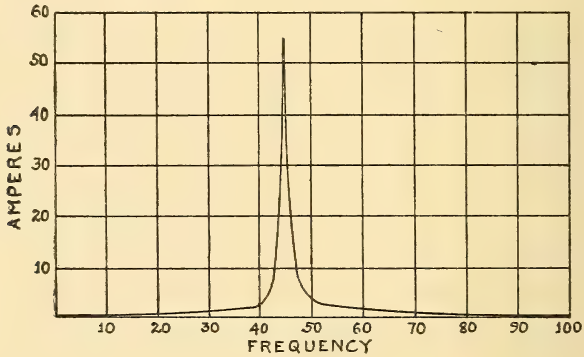


Fig. 332.

different frequencies of the current in a circuit in which $E = 110$ volts, $R = 2$ ohms, $L = 0.5$ henry and $K = 25$ microfarads. At a frequency of about 45, the current mounts suddenly to 55 amperes, while at a frequency of 5 more or 5 less it is but little greater than three amperes.

If a heavy pendulum be given a series of slight impulses, no especial effect will be produced unless these impulses be timed at the natural period of vibration of the pendulum, in which case their effect is cumulative and it may be made to swing through a wide arc.

Again, if various tuning forks be caused to vibrate near the open end of a closed organ pipe, no effect will be produced until a fork is used whose period of vibration corresponds to the natural period of vibration of the column of air within the pipe, and when this happens the column of air will vibrate in unison with the fork and the total volume of sound emitted will be greatly increased. This phenomenon is called *resonance*.

In the case of the alternating current circuit under consideration, the E. M. F. is not applied steadily but in a series of impulses

following each other at regular intervals. These impulses produce no very marked effect until the critical frequency is reached, at which time the current rises abruptly to its maximum value. From analogy, the circuit is now said to possess *electric resonance.*

Resonance exists in an alternating current circuit whenever $2\pi fL = \frac{1}{2\pi fK}$, or when the inductive reactance is exactly counterbalanced by the capacity reactance.

632. Resonance with Inductance and Capacity in Series.—When resonance exists in a circuit containing inductance and capacity in series (Fig. 331), the current follows Ohm's law and the impressed E. M. F. is simply the IR drop. The fact, however, that the inductive and the capacity reactances neutralize each other, or that their sum is zero, does not mean that they are separately zero. On the contrary, the difference of potential across the terminals of the inductance is $I.2\pi fL$ (Par. 619) and that across the terminals of the condenser is $I.\frac{1}{2\pi fK}$ (Par. 628) and these may very greatly exceed the impressed E. M. F. For example, in the numerical example given in the preceding paragraph, while the impressed E. M. F. is 110 volts, the drops across the terminals of the inductance and of the condenser are each 7778 volts.

633. Resonance with Inductance and Capacity in Parallel.—A particular case of resonance is where the inductance and capacity are in parallel as shown in Fig. 333. The current on arriving at A

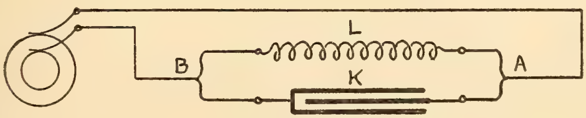


Fig. 333.

divides, but in the branch L it is retarded while in the branch K it is advanced an equal amount. The result is that the loop $AKBL$ acts as a short circuit, the current surging around it in one direction during one-half of a period and in the other direction during the remaining half. Although the current in the main circuit may be small, that in this loop may be very large. This can be shown graphically, for the current in the main circuit is the resultant of the currents in L and K (Par. 624), that is, it is the

diagonal of a parallelogram whose adjacent sides made with each other an angle of very nearly 180°.

634. Power in an Alternating Current Circuit.—In an alternating current circuit the instantaneous value of the power is the product of the corresponding simultaneous instantaneous values of the E. M. F. and the current (Par. 494). Two cases may arise: (a) the E. M. F. and current may be in phase, or, (b) they may differ in phase.

If the E. M. F. and current are in phase, at any one instant they are either both positive or both negative and therefore their product, the power, is always positive. This is shown in Fig. 334

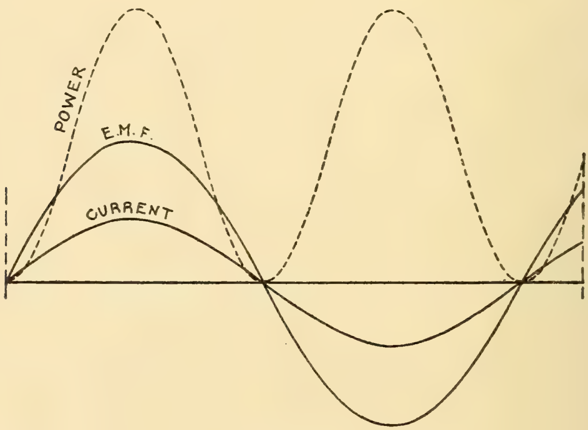


Fig. 334.

in which the broken curve representing the instantaneous values of the power lies always above the horizontal axis. The power curve is seen to be periodic and of twice the frequency of the E. M. F. and current curves. Since its ordinates represent rate of doing work and its abscissae represent time (Par. 608), the *area* included between the curve and the horizontal axis represents work performed by the current. The work is positive, for whether the current flow in or out it performs work in overcoming the resistance of the circuit.

If the E. M. F. and current differ in phase, their simultaneous values must at times differ in sign and at these times their product, the power, must be negative. The power curve, therefore, as

shown in Fig. 335, extends below the horizontal axis. The areas of the loops below this axis represent negative work, or energy imparted to the field about the circuit and restored by this field to the system (Par. 616).

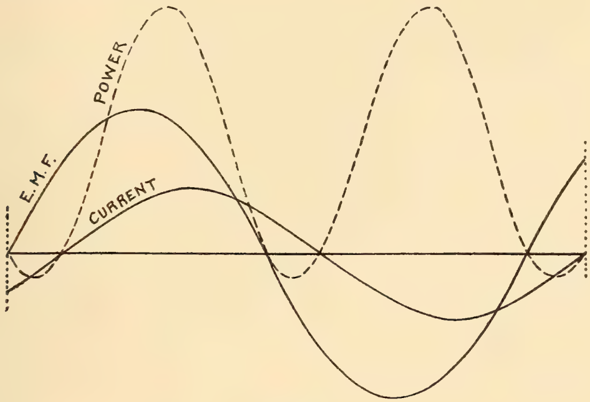


Fig. 335.

635. Power Factor.—In Par. 613 it was shown that the work done by an alternating current in one cycle is $\frac{1}{2} I_m^2 Rt$, I_m being the maximum value of the current, R the resistance of the circuit and t the time of one cycle. By dividing this by t we get the average rate of doing work, in other words, the average power, hence

$$P = \frac{1}{2} I_m^2 R \text{ watts}$$

which may be written

$$P = \left(\frac{I_m}{\sqrt{2}}\right)^2 . R$$

In the same paragraph it was shown that I_v, the virtual current, is equal to $I_m/\sqrt{2}$, hence

$$P = I_v^2 R = I_v . I_v R$$

But $I_v R$ is that component of the virtual E. M. F. which is in phase with the current, hence (Par. 620)

$$I_v R = E_v . \cos \phi$$

hence

$$P = I_v E_v . \cos \phi \text{ watts}$$

or the average power in an alternating current circuit is equal to the product of the virtual

current, the virtual E. M. F., and the cosine of the angle of lag (or lead).

The power in an alternating current circuit must be read by a wattmeter, for, except when the E. M. F. and current are in phase, it can not be determined by taking simultaneous measurements with an ammeter and a voltmeter and multiplying these readings together. The product of these readings, $I_v E_v$, is called the *apparent power*, and $\cos \phi$ is called the *power factor*, since, as shown above, it is that factor by which the apparent power must be multiplied in order to obtain the true power.

If ϕ becomes 90°, that is, if there is no resistance in the circuit so that the E. M. F. and current are in quadrature, $\cos \phi = 0$ and the power as given above reduces to zero. In this case, the area of the negative loops of the power curve (Fig. 335) equals that of the positive loops.

CHAPTER 44.

ALTERNATING CURRENT GENERATORS.

636. Alternators.—The fundamental principles of alternating current generators have already been brought out in the chapter treating of direct current generators. It was there shown that the currents generated in the revolving armatures described were all alternating and to rectify them an especial contrivance, the commutator, was required. It would therefore seem that should the commutator be discarded and collector rings (Par. 553) be substituted in its place, we would obtain an alternating current generator, or, as it is more briefly named, an *alternator*. However, we also saw that in the D. C. generators the fields were self-excited, the direct current for this purpose being drawn from the commutator. The discarding of the commutator therefore involves a change in the methods of exciting the field coils, and for this and for other reasons it is necessary to consider these machines a little more in detail.

637. Field Excitation of Alternators.—In most alternators the field coils are excited by a current drawn from a separate source, such as from a battery or from a small D. C. generator. This auxiliary generator, the *exciter*, may be operated in a number of ways. (a) It may be entirely independent. (b) It may be driven by a belt from a pulley on the shaft of the alternator. (c) It may be mounted upon an extension of this shaft. (d) It may form an integral part of the armature of the alternator itself. In this case the armature must be provided with both collector rings and commutator, the field current being drawn from the latter.

638. Compound Alternators.—As the current through an alternator increases, the internal drop also increases and consequently the voltage across the brushes diminishes. We have seen (Par. 511) how important it is in the case of electric lighting (for which alternating currents are largely used) that the voltage delivered to the lamps should be constant. To secure this constancy of potential, the voltage across the brushes must not only not fall

with increase of current but must actually rise. In direct current generators this is secured by compounding (Par. 588). This remedy is not directly applicable to alternators but there are several ways in which an approximation to it may be obtained. One of these is shown diagrammatically in Fig. 336 which represents the armature of an eight-pole alternator. The current leaving the armature windings at the coil *A*, before reaching the corresponding collector ring passes through the primary of a step down transformer *B*. The core of this transformer is attached to the armature spider or forms a part of it and therefore rotates

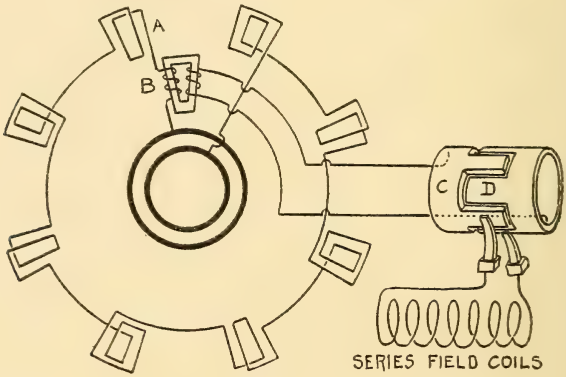


Fig. 336.

with the armature. The current from the secondary is taken to a commutator *CD* which is mounted upon the armature shaft close to the collector rings but which, for the sake of clearness, is represented in the diagram as moved off to the right and turned sidewise to the observer. This commutator has only as many segments as the alternator has poles, and the alternate segments are connected together as shown. Brushes pressing against it deliver a rectified current to the series field coils. An increase in the current in the external circuit, and hence in the primary of the transformer, causes an increase in the current through the secondary, and hence through the field coils, which in turn causes the desired rise in voltage.

It is sometimes possible to dispense with the transformer and to take the current direct from the coil *A* to the commutator. As a rule, however, alternators generate a high voltage current which, besides being dangerous, is apt to cause excessive sparking

at the commutator. For these reasons the transformer is to be preferred.

639. Alternators Usually Multipolar.—It is in general necessary that an alternator should be multipolar. This will be seen from the following. A small alternator may be driven at 1800 revolutions per minute. This speed may be exceeded by some of the turbine driven machines but is near the limit for the average small generator and much above the limit for large machines. At this rate, a point on the circumference of a twelve inch armature is travelling faster than a mile per minute. But at 1800 revolutions per minute the frequency of the current from a bipolar machine is only 30. This, we have seen (Par. 621), is too low for the operation of an incandescent lamp. Moreover, frequencies as high as 120 are often required. Since the speed of the alternator can not be increased, such frequencies can be obtained only by increasing the number of poles. In some of the larger modern alternators, the number of poles has approached one hundred.

640. Classes of Alternators.—As shown above, the classification of D. C. generators according to the method of field excitation into series, shunt and compound machines is not applicable to alternators. They may, however, be divided into two general classes; (a) those with stationary field and revolving armature, and (b) those with stationary armature and revolving field. Of this second class there is a subdivision, the *inductor alternator*, in which, although the field revolves, the exciting current passes through a single coil which is stationary (Par. 643). From an electrical standpoint, there is no call for these divisions, the principle being the same in all, but each possesses certain minor advantages and it is therefore desirable to consider them separately though briefly.

It will shortly be shown that alternators may be designed to deliver a single current or two or more separate and distinct currents which differ in phase and accordingly they are also classed as *single phase* or as *polyphase*.

641. Alternators with Revolving Armatures.—The simplest form of alternator with revolving armature is figured and explained in Par. 553. The majority are multipolar. A diagram-

matic end view of such a machine is given in Fig. 337, and in Fig. 338 a similar four-pole machine is shown as rectified, that is, the field, the armature and the collector rings are represented as having been straightened out. With clockwise rotation, the coils

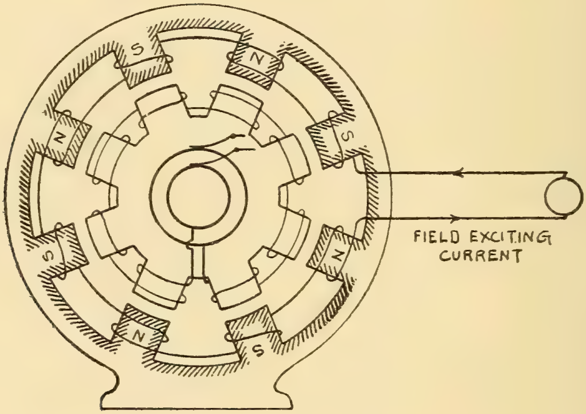


Fig. 337.

A, B, C, D move from left to right as indicated by the large arrow. Application of the rule given in Par. 421 shows that at the instant represented a clockwise E. M. F. is induced in *A* and in *C* and a counter-clockwise E. M. F. in *B* and in *D*, but since

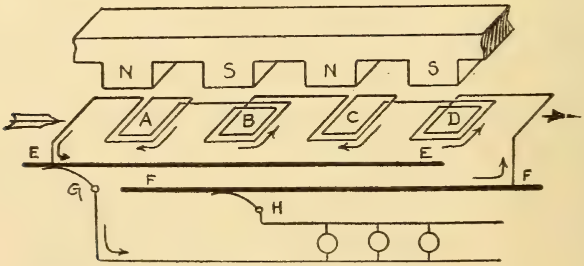


Fig. 338.

these coils alternate in the direction of their winding, they add their respective E. M. F.s. The current passes into the external circuit from the collector ring *EE* through the brush *G* and returns through the brush *H* which is in contact with the ring *FF*. The direction of this current is reversed as *A* passes beneath the center of *S*.

642. Alternators with Revolving Field.—In alternators with revolving field, the field may retain its relative position exterior to the armature, but far more frequently they interchange places and the revolving field is internal. Roughly speaking, the field core resembles the hub of a wheel whose spokes have all been sawed off to a length of two or three inches. The field coils are wrapped about these spokes, alternating in direction so as to obtain the desired polarity. The exciting current is brought in and taken out by means of a pair of *slip rings* (identical in operation with collector rings). Fig. 337 would represent such a machine if the field circuit and the external circuit were interchanged,

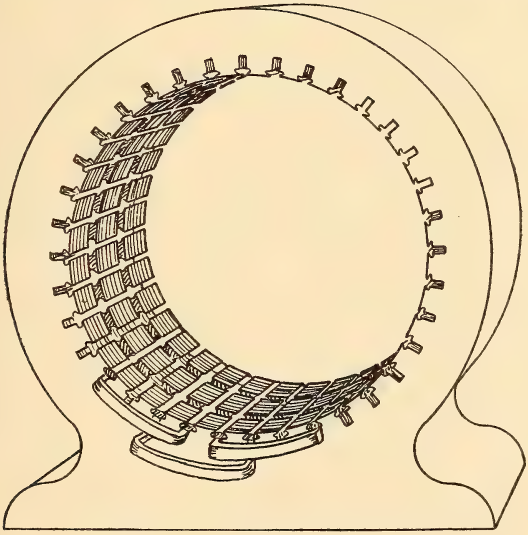


Fig. 339.

that is, if the exciting direct current were brought in through the collector rings and if the present field circuit were used as the external circuit. The armature, Fig. 339, is built up of laminated punchings, spaces being left for ventilation. The coils are placed in slots and held in position by wedges.

The great advantage of this form of alternator is that the current, which we have seen is usually of high voltage, is taken off through fixed connections, which may be insulated to any desired degree, and only the relatively small exciting current passes through the sliding contacts on the slip rings.

643. The Inductor Alternator.—The inductor alternator, shown diagrammatically in section in Fig. 340, possesses the advantage of having no sliding contacts and therefore requires no collecting rings or brushes and is free from the sparking which occurs in other machines. It consists of an *inductor*, a rotating toothed soft-iron disc around whose edge there is a deep groove. In this groove lies the annular field coil C which is fastened to the frame work and therefore does not rotate with the inductor. When a current flows through C, the inductor becomes magnetized, its faces being of opposite polarity and hence the teeth on one side being all of like polarity. The frame work which surrounds this revolving inductor has inward projections corresponding to the moving poles, and upon these projections the armature coils are wrapped. Since the poles on each side do not alternate in polarity, there is no reversal of flux through the armature coils but this flux rises and falls and thus produces an alternating current in the coils.

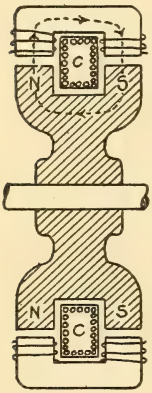


Fig. 340.

644. Polyphase Alternators.—Suppose that the ends of the two coils in Fig. 269, instead of terminating in the commutator segments as shown, should each be connected to a separate collector ring as shown in Fig. 341. There being no electrical connection between these coils, a pair of brushes C could be applied to the rings of the coil B and lead current from this coil into an external circuit. A second pair of brushes D could be applied to the rings of A and lead current from A into an entirely separate external circuit. As the armature rotates, an equal E. M. F. is generated in each coil but the currents in the respective circuits vary with the resistances of these circuits and are entirely independent of each other, in fact, the machine is electrically equivalent to two separate and distinct machines, the only connection between the two being that they generate equal E. M. F.s of equal periodicity. If the E. M. F. curves of the two coils be plotted on a common axis of time, it will be seen that their maxima occur at a constant phase difference of 90°. Since the machine thus generates two distinct currents of different phases, it is called a *two-phase* or a *di-phase* alternator.

Theoretically, other coils could be inserted midway between

those of Fig. 341 and still others between these, each with its own collector rings and each supplying a separate external circuit with current differing in phase from the currents from the other coils. Practically, the distinct windings of such alternators rarely exceed

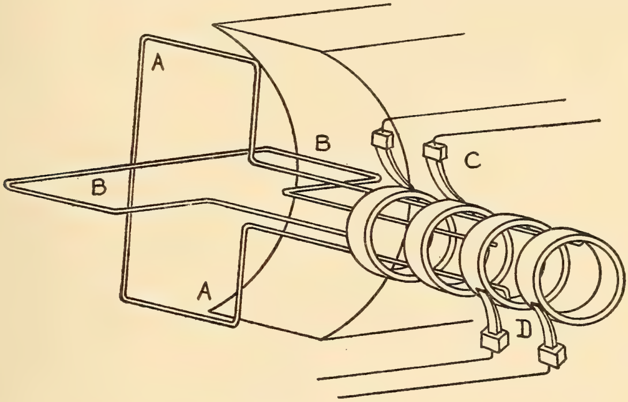


Fig. 341.

three. Those which generate more than one current are designated as *polyphase;* those which generate but one are, in contra-distinction, called *single phase.*

It can be shown that to generate these polyphase currents it is not necessary that the windings for each phase should be entirely separate. For example, as shown in Fig. 342, by tapping a ring-

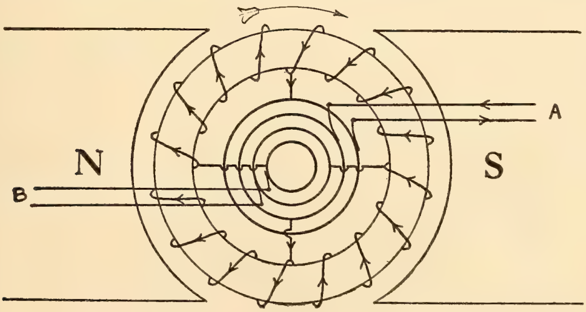


Fig. 342.

wound armature at four points 90° apart and by connecting each of the tapping wires to a collector ring, we obtain a two-phase alternator. At the instant shown in the diagram the leads A are carrying the entire current, the current in the leads B being zero since the points to which their tapping wires are connected are

momentarily at the same potential. When, however, the armature has turned through an angle of 90°, these conditions are reversed and the leads *B* will carry the entire current, while the current in *A* will be zero.

While polyphase currents are used to a limited extent in a three-wire lighting system, their principal use, as will be explained in the following chapter, is for the operation of alternating current motors.

645. Tri-Phase Alternators.—In its most general form, the armature of a tri-phase alternator carries three distinct windings spaced 120° apart and supplied with six collector rings by which currents can be distributed to three separate circuits. The E. M. F. generated by such an alternator is shown in Fig. 343, the sine waves being of equal amplitude but differing in phase by 120°.

If the resistances of the three circuits are equal, then the currents are also equal and the circuits are said to be balanced. In such a case the curves in Fig. 343 may be taken as representing

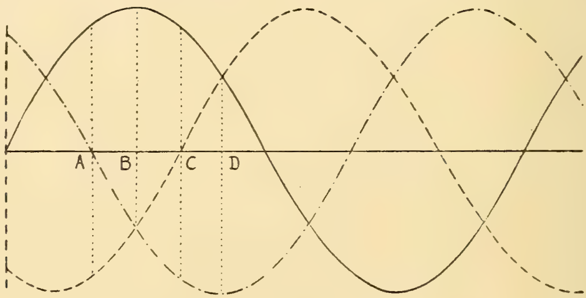


Fig. 343.

the currents also. Examination of this figure will show that at any point along the horizontal axis, the sum of the ordinates is zero. For example, at *A* and *C* where the current in one of the circuits is zero, the currents in the other two circuits are both equal and opposite, and at *B* and *D* where the current in one of the circuits is a maximum, the sum of the currents in the other two circuits is equal and opposite. It is therefore possible when the circuits are balanced to discard three collector rings and three lead wires, for whether the current goes out on one or on two wires, an equal current comes in on the remaining wires or wire. The arrangement of such a three-wire three-phase system is shown in Fig. 344.

Should the circuits not be balanced, it is still possible to reduce the number of leads and collector rings from six to four, the fourth wire serving as a common return for the excess current of the other three.

646. Tri-Phase Delta Connection.—In Fig. 344, *a* represents diagrammatically a ring wound armature tapped at three points 120° apart, each tapping wire terminating in a collector ring. These rings, *A*, *B*, *C*, for the sake of clearness are represented as

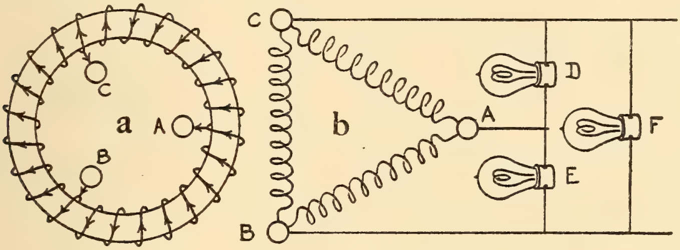


Fig. 344.

separated from their common axis. The same armature is represented in *b* in a still more highly conventionalized form, the curved portions between the tapping wires being straightened out and the rings being drawn at the vertices of the resulting triangle. This diagram also shows the three leads running from these rings and the arrangement of lamps so as to produce a balanced system. On account of the shape of the diagram this is called a Δ-*connection*, sometimes also a *mesh-grouping*. At one instant the entire current flows out on *A* and returns through the lamps *D* and *E*; at another instant it flows out on *C* and returns through *D* and *F*; at still another it flows out on *B* and returns through *E* and *F*; at all others, a varying current flows through each lamp.

At the instant represented in *a*, Fig. 344, the armature coils between *B* and *C* are sending current out by *C*, and the coils between *A* and *C* (except the few to the left of the neutral plane) are contributing to this current. The currents in these two portions of the armature windings do not reach their maxima simultaneously but the total resultant current is a maximum when these component currents are equal which is the case at *A*, the 60° phase in Fig. 343. The maximum current in the leads is therefore $2 \sin 60° = \sqrt{3}$ times the maximum current in one portion

of the armature windings. The maximum E. M. F. between any two of the leads is, however, no greater than that in one portion of the armature windings.

647. Tri-Phase Y-Connection.—Suppose that in addition to tapping the ring-wound armature in three points, as described in the preceding paragraph, we cut the winding at these points and connect the corresponding ends together as shown in Fig. 345 *a*. The current entering at *B* (at the instant represented in

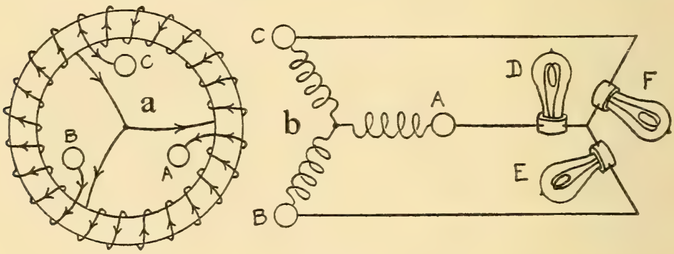


Fig. 345.

the diagram) flows to the common junction at the center where it divides, a portion going to *A*, the remainder to *C*. This arrangement, shown still more diagrammatically in *b*, is called a *Y-connection,* sometimes also a *star grouping.*

The E. M. F. of the coils between *B* and *a* is now in series with that of those between *a* and *A* and of those between *a* and *C*, excepting in both cases the few turns to the left of the neutral plane. The maximum E. M. F. between the leads of *b* is therefore the sum of the E. M. F.s in two of the three portions of the armature windings at the moment when the E. M. F. in the third portion is zero. This is represented by the double ordinate at *A* in Fig. 343. But *A* being at the 60° phase, this double ordinate is $2 \sin 60° = \sqrt{3}$, or the maximum E. M. F. between any two of the leads is $\sqrt{3}$ times the maximum E. M. F. developed in a single portion of the armature windings.

On the other hand, since at any one instant never more than two of the portions of the armature windings can combine in delivering current, and since these two portions are always in series, the maximum current in the leads is the same as the maximum current in any one of these portions.

It will be noted that in the Δ-connection the *current* is $\sqrt{3}$ times the maximum of that in the armature coils, while in the

Y-connection the *voltage* is $\sqrt{3}$ times the maximum of that in these coils. The power, IE, developed by the two arrangements is therefore the same.

648. Transformation of Direct and of Alternating Currents.—We have seen that the secret of the electrical transmission of power is the employment of currents of high potential (Par. 502). On account of freedom from trouble caused by sparking at the commutator, it is true as a general statement that an alternating current can be turned out at a higher voltage than can a direct current. Whether the current produced by a generator be direct or alternating, it is often desirable to raise its voltage still higher before sending it out on the line, and whether this be done or not, it is almost always necessary at the distant end of the line to reduce the voltage to fit the standard machines or lamps with which it is to be used. In this transmission, therefore, a current must be stepped up at the sending station and stepped down at the receiving station.

In the case of direct currents, this transformation is effected by motor generators (Par. 605). These machines are costly, their operation involves a considerable loss of power and they require as much attention as the generator itself. If power is to be distributed among scattered buildings, a motor generator and an engineer would be required in each, also space for installation of the machine. On the other hand, these changes in alternating currents are made by transformers which are relatively inexpensive, require little or no attention and have an efficiency in some cases exceeding 98 per cent. They may be placed wherever needed and occupy but little room since they are usually mounted against a wall or upon a pole like a letter box. For these reasons, for the transmission of power to a distance, the alternating current has a great advantage over the direct.

649. Transformers.—The principle of transformers was outlined in Par. 431 but they are considered here again in order that some additional facts about their use may be brought out. That they rightfully fall under the heading of the present chapter, the following will show. An alternator is a machine which induces an alternating E. M. F. by rapidly varying the magnetic flux through a coil. From this point of view, a transformer is also an alternator, the E. M. F. in the secondary being induced by the

changing flux produced in it by the primary. Moreover, since the transformer has no moving parts (and is hence sometimes called *static*), there is no loss of energy in overcoming friction, etc., and by proper design the combined losses due to magnetic leakage, eddy currents, hysteresis and resistance may be reduced to less than two per cent, so that we may say that the transformer is the most efficient of machines.

Transformers are of two types, the *core* or *ring* transformer (Fig. 204) and the *shell* transformer (Fig. 205). The shell transformer is the more frequently used but, for the sake of clearness, the following diagrams represent ring transformers.

In the actual construction of the shell transformer, the coils are usually wound in separate portions which are thoroughly insulated and then sandwiched together, after which they are placed in a form and the laminated iron punchings of which the shell is composed are built up around them. The completed coils are then put in an iron case which is usually filled with oil. This serves a double purpose; it aids the insulation of the coils, prevents the penetration of moisture into the wrappings and prevents excessive heating of the coils. In some of the larger transformers, the oil itself is cooled by water circulating in pipes which pass through the oil. In others, the oil is omitted and cooling is brought about by currents of air driven over the coils.

If a current be sent through the primary of a transformer, it will produce in the core a certain number of lines of force. These lines, as shown in Fig. 346, penetrate every turn of the coils in

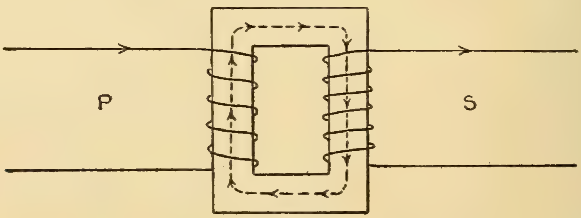


Fig. 346.

both primary and secondary. An equal E. M. F. is therefore induced in every turn. If this E. M. F. be e, and if there be N' turns in the primary and N'' in the secondary, the E. M. F. in the primary is $E' = N'e$, that in the secondary is $E'' = N''e$, whence

$$E' : E'' = N' : N''$$

or, as already shown (Par. 431), the E. M. F.s in the two coils are to each other as the number of turns in the respective coils.

650. Operation of Transformer.—In Par. 431 it was shown that the work done in the primary of a properly designed transformer is equal to that done in the secondary. It follows from this principle that the current in the primary varies with the

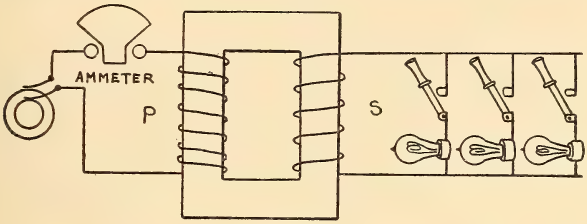


Fig. 347.

current in the secondary and that when the secondary circuit is open there should be no current in the primary. This can be shown experimentally by the arrangement shown in Fig. 347. With the switches in the secondary circuit open, the ammeter in the primary circuit indicates the merest trace of a current. Reflection will show that the primary, a coil of small resistance wrapped about a soft iron core, is nothing more nor less than a choke coil as described in Par. 621, and that the current is cut down by the choking effect. The small current which does get through, the "no load current," is just sufficient to maintain the magnetic flux.

If now one of the switches in the secondary be closed, the ammeter will indicate a current through the primary. If a second switch be closed, the current through the primary is doubled; if a third switch be closed, it is trebled, in other words, the current through the primary adjusts itself to conform to the current in the secondary, or, the primary acts as an automatic valve and permits only so much current to flow through it as is needed to supply the demands of the secondary.

This very remarkable property may be explained as follows. When an E. M. F. is impressed upon the primary, the secondary circuit being open (Fig. 346), a current flows and produces within the primary a magnetic flux. The lines of force, as shown by the arrowheads, travel around the magnetic circuit and enter the primary from below. This sets up an induced E. M. F. in the primary opposite to the actual E. M. F. (Par. 421) and conse-

quently cuts down the current in the primary. An E. M. F. is also set up in the secondary but produces no current since this circuit is open. When, however, the secondary circuit is closed, a current flows as indicated by the arrowhead. This current produces lines of force opposite in direction to those from the primary, that is, it diminishes the number of lines from the primary (Par. 418, *b*). This in turn diminishes the choking effect and allows a larger current to flow through the primary.

From the facts brought out above it will be seen that in any given transformer the voltage in the secondary varies directly with the voltage in the primary; on the other hand, the current in the primary varies directly with the current in the secondary; in other words, the primary determines the voltage; the secondary determines the current.

651. Connection of Transformers.—On account of the choking effect described in the preceding paragraph, transformers are not connected in series but in parallel. Fig. 348 represents an alter-

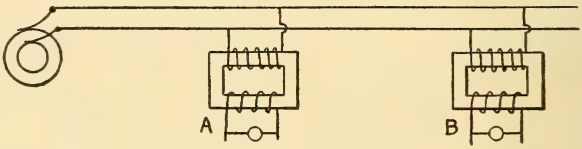


Fig. 348.

nator delivering high potential current to two mains and through transformers connected in parallel distributing energy from these mains to the stations *A* and *B*.

652. Auto-Transformers.—The transformers described in the preceding paragraphs are used when the voltage in the primary is to be very materially changed, as for example when it is to be increased or diminished tenfold, or, as a minimum, when it is to be doubled or halved. Smaller changes in voltage may be made by means of resistance, but this we have shown to be wasteful. A better method is to use the so-called *auto-transformer*, shown diagrammatically in Fig. 349. This is a transformer in which the primary and the secondary coils are combined in one. In principle it does not differ from the ordinary transformer. As explained in Par. 649, when a current is sent through the primary coil, an equal E. M. F. is developed in every turn. The E. M. F. in the

secondary therefore varies directly with the number of turns tapped by it. In the diagram, the secondary is used to step down the voltage in the primary. If the current were delivered to the

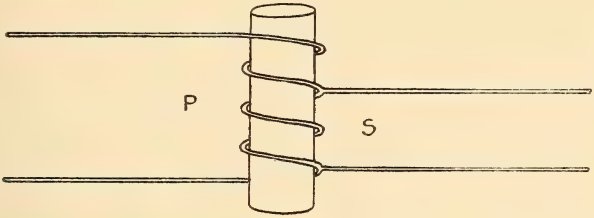


Fig. 349.

secondary and drawn from the primary, the voltage would be stepped up.

653. Rectification of Alternating Current.—An alternating current may be rectified in several ways. It has already been shown (Par. 556) how it may be rectified at the point of origin by means of a commutator. It is, however, frequently desirable to transmit the current to a distance as alternating and to rectify it at the receiving station. In this case it may be rectified by (a) mechanical means, or by (b) electro-chemical means.

An alternating current is rectified mechanically by means of a *synchronous converter*, also called a *rotary converter*. Briefly explained, this is a generator with both commutator and collector rings. The alternating current is delivered to the collector rings and the machine operates as a motor. While so operating, direct current is drawn from the commutator.

Alternating current may also be rectified mechanically by a *motor-generator* (Par. 605), the motor being driven by the alternating current and direct current being drawn from the commutator of the generator at the opposite end of the shaft.

The electro-chemical rectifiers are of several kinds. In one, the current is passed through a cell containing electrodes of aluminum and of lead or steel, the aluminum having the property of permitting the current to pass when it is the cathode but suppressing it when it is the anode. Allied to this is the mercury arc rectifier which will now be described.

654. The Mercury Arc Rectifier.—In the description of the mercury vapor lamp (Par. 527), it was shown that the resistance

to the passage of the current was confined mainly to the surface of the negative electrode and was so great that several thousand volts were required to break it down, but that once that it had been broken down, a current could be maintained by a small voltage provided that this current did not fall below a certain minimum. If it fell below this, the negative electrode resistance was re-established and the current was interrupted.

This principle is utilized in the mercury arc rectifier, an apparatus for the conversion of alternating currents into the relatively small direct currents such as are employed in charging the smaller storage batteries. It may be used with either single phase or polyphase currents. Its operation will be understood from the following. Fig. 350 represents diagrammatically one of these

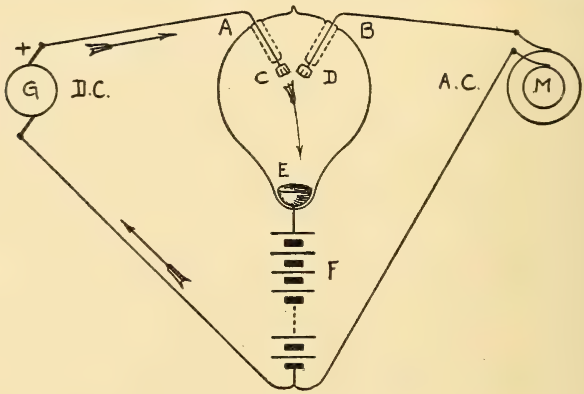


Fig. 350.

converters. It consists of a pear-shaped exhausted glass globe of about nine inches in diameter. Through its top extend the terminals *A* and *B* which connect on the interior with the iron electrodes *C* and *D*. A third terminal enters below and connects with the mercury electrode *E*. Suppose that desiring to charge the storage battery *F* by means of current from an alternator *M*, we should make connections as shown on the right of the diagram. No current can flow in either direction until the negative electrode resistance at either *D* or *E* be broken down. Suppose that as explained below this resistance be broken down at *E*. Current will now flow through the circuit in the direction *BDEF* but will continue to flow for only a small fraction of a second. As soon as the voltage between *D* and *E* drops to about ten volts, the

resistance at E is re-established and the current is interrupted. When the E. M. F. reverses, no current can flow, for the resistance at D has not yet been broken down. We see then that this arrangement could not be used. Now suppose a direct current generator G to be connected as shown on the left of the diagram. When the resistance at E has once been broken down, direct current from G will flow steadily in the direction $ACEF$. If now the alternator be turned on, the alternating E. M. F. in the direction BDE can send a current through the circuit because the direct current from G, by preventing the resistance at E from reasserting itself, keeps open the road through E, but the alternate impulses in the reverse direction can send no current since the resistance at D prevents. It is thus seen that by such an arrangement the alternating current from one-half of each cycle could be used to charge the battery.

The illustration above is purely hypothetical but is intended to bring out the fact that if in any manner the resistance at the negative electrode can be kept broken down, than the apparatus becomes selective in its operation and permits current to pass in one direction but not in the other, in other words, it becomes a rectifier.

655. Rectification of Single Phase Current.—The arrangement of the converter to rectify a single phase current is shown in Fig. 351. The leads from the alternator M terminate in the electrodes

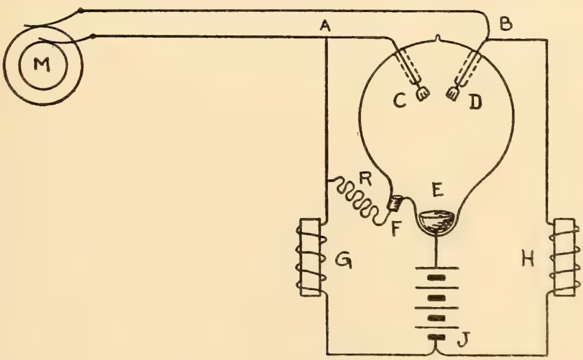


Fig. 351.

C and D, but at A and B branches are thrown off which include the inductance coils G and H and unite at J. To one side of the electrode E there is an auxiliary mercury electrode F which is

connected through a resistance R with the wire from A to G. The globe is mounted so that it may readily be tilted.

To charge a storage battery, the battery is connected between E and J as shown. The globe is then tilted until the mercury in E connects with that in F. At this instant the current passes through the path $MARFEJHBM$. The globe is now released and as the thread of mercury between E and F is broken, an arc is produced, some of the mercury is ionized and the vapor in the globe is thereby rendered a conductor. The path of the current is now $MACEJHBM$, but the E. M. F. acting in this direction soon dies down and then reverses, that is, acts in the direction MBD. The inductance of the coil H now comes into play and prolongs the current through H, a momentary current flowing around the circuit $JHBDEJ$. Before this delayed current has died down to the point where the resistance of E is re-established, it is picked up by the growing E. M. F. in the direction MBD, the circuit now being $MBDEJGAM$. At the next reversal, the inductance of the coil G comes into play, and so on, these induced delayed currents fulfilling the part of the direct current described in the preceding paragraph and keeping the path through E open.

656. Comparison of Alternating and Direct Currents.—Alternating current generators, since they require no commutator, are somewhat cheaper to construct than those for direct current, but this may be counterbalanced by the cost of the separate field exciter. The great advantage of alternating currents is the ease with which they may be transformed and the simplicity and the efficiency of the static transformers used for this purpose. On the other hand, they can not be used in electrolytic work nor in charging storage batteries and alternating current motors fall behind direct current motors both in efficiency and in speed regulation. While most incandescent lamps operate equally well with either kind of current, the arc lamp mechanism for alternating currents is not so satisfactory as that for direct currents. As a general statement therefore, alternating current is most suitable where power is to be transmitted to a distance; in all other cases direct current is to be preferred.

CHAPTER 45.

ALTERNATING CURRENT MOTORS.

657. Alternating Current Motors.—The electrical conditions encountered in motors designed for use with alternating currents are particularly complex. The interaction of the flux of the field coils and that of the armature coils, one or both of which may be shifting, the inductance, hysteresis and eddy currents necessarily developed in a machine in which alternating currents flow through coils embracing soft iron cores, render the mathematical treatment of the problem more intricate than is desirable in an elementary text book. In the following pages therefore, we can do no more than glance at the fundamental principles of a few of the simpler forms.

658. Classes of Alternating Current Motors.—Alternating current motors are usually classed under the following heads:

(a) Series motors.
(b) Synchronous motors.
(c) Repulsion motors.
(d) Induction motors.

The distinction between these will be brought out as we proceed.

659. Series Motors.—In Par. 604 it was shown that changing the direction of the current supplied to a shunt motor did not alter the direction of rotation. The same could have been shown for the series motor. At first sight, therefore, it would seem that whether supplied with direct or with alternating current, these motors would operate equally well. In the case of the shunt motor however, the inductance of the field coils is much greater than that of the armature coils. The current through the field coils therefore lags much more than that through the armature (Par. 617). The torque is a maximum when the armature current and the field flux reach their maxima simultaneously, but since the field and the armature currents differ in phase, but little power is developed. The shunt motor, therefore, is not used with alternating currents.

In the series motor, the field and armature coils being in series, there can be no phase difference and the above objections do not apply. When used with single phase alternating currents, series motors develop great starting torque and possess the advantages and disadvantages described in Pars. 602 and 603. They are therefore largely used as railway motors. The *A. C.* motors differ from the *D. C.* motors in certain minor arrangements by which the tendency of the *A. C.* machine to excessive sparking is reduced. Also, as in all other *A. C.* machines, the field cores must be laminated.

660. Synchronous Motors.—Suppose Fig. 352 to represent a rectified portion of the alternator shown in Fig. 337. *AB* represents the field which is excited by direct current and whose polarity

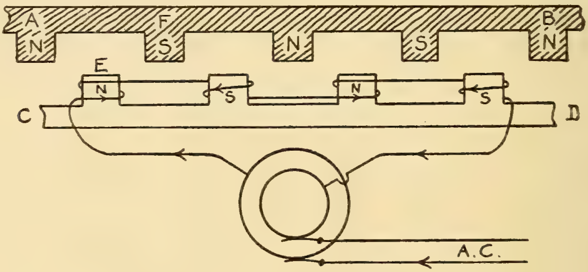


Fig. 352.

therefore does not vary. *CD* represents a portion of the revolving armature, the coils supplied with alternating current from a distant source. At the instant shown in the diagram, it will be seen that each pole of the armature experiences a force which tends to move *CD* from left to right. If *CD* does not move, it will at the next reversal of the current be urged in the opposite direction, or from right to left. Suppose it begins to move from left to right. If before the moving coil *E* arrives beneath *F*, the current through *E* reverses, the polarity of *E* also reverses and *E* will be driven back from *F*, in other words, the movement of *CD* will be checked. If *E* passes under *F* without reversing, it will be pulled back as soon as it begins to emerge on the other side. If it reverses as it passes under *F*, it will be pushed ahead.

The frequency of the alternating current supplied to the armature being constant, the relative positions of the fixed field poles and the rotating armature coils at the instant when the current

in these latter reverses depends upon the angular speed of the armature. The effect of variation in this speed can be shown graphically as follows. In Fig. 353 AB represents the fixed field, and a, b and c represent the successive positions of an armature coil moving at three different speeds. If the armature be rotated

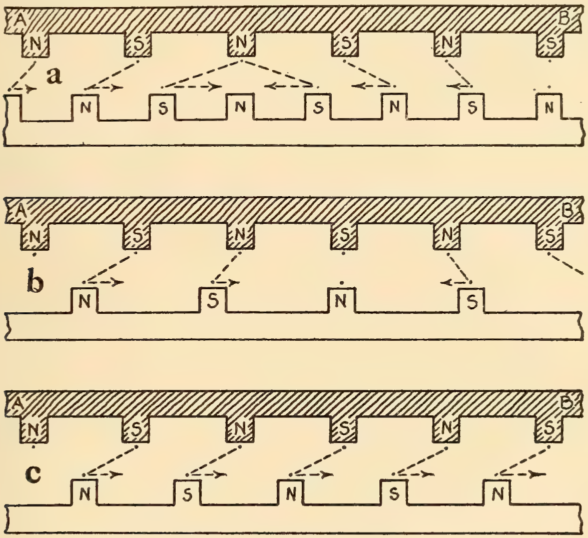


Fig. 353.

slowly, the angular distance between reversals is small; if it rotates rapidly, this angular distance is large. In a, the armature is turning slowly and the polarity of the coil reverses when the coil has travelled through less angular distance than that separating the field poles. In b, it is turning rapidly and the reversals occur at angular distances apart greater than that between the field poles. In c, the reversals occur at the same angular distance apart as that separating the poles. In this last case, the armature coil passes over the distance between two successive north poles of the field in the same time that a coil of the distant alternator supplying current to the armature passes over the distance between two successive north poles of *its* field, in other words, the armatures of the motor and of the alternator rotate in electrical synchronism.

For the sake of clearness only forces of attraction are represented in these diagrams. It is seen at a glance that only in the case of

the synchronous rotation is the torque the same in direction for the successive positions of the rotating coil. If, therefore, an alternator be brought up to synchronous speed and then supplied with alternating current, it will continue to rotate. Such machines are called *synchronous motors*. They differ in a few minor details from alternators. Either the field or the armature may revolve and they may be driven by either single phase or polyphase currents.

661. Operation of Synchronous Motors.—A serious objection to the single phase synchronous motor is that it can not of itself start from rest. An auxiliary motor is required to bring it up to synchronous speed before the current is turned on. The polyphase machines will start up of themselves, but even with these it is usual to employ an auxiliary starter.

Since these motors must maintain synchronous speed, it follows that their speed does not vary with variations in the load. The question then arises how is the supply of power varied to meet the different demands made upon it. The force on an inductor of the armature being $I.\ H.\ l$ (Par. 591) varies directly as the current. The current varies as the difference between the impressed E. M. F. and the back E. M. F. (Par. 593). The impressed E. M. F. is delivered by the alternator and is constant. The back E. M. F. varies with the speed of rotation of the armature, hence also is constant. If these two E. M. F.s reached their maxima and minima simultaneously, in other words, if they were in phase, the difference between them, and hence the current, would be a minimum. If, however, the armature coils should fall back a few degrees in angular position, still preserving synchronous rotation, the two E. M. F.s would no longer be in phase, their difference would increase and a greater current would flow. When, therefore, a load is thrown on a synchronous motor, the armature drops back a few degrees and thus exerts a greater torque. If the load be excessive, the machine is thrown out of synchronism and stops.

662. The Repulsion Motor.—The repulsion motor, shown diagrammatically in Fig. 354, consists of an ordinary D. C. armature placed in a field produced by a single phase alternating current. As the field alternates, an E. M. F. is induced in every coil in the armature except in the two at the opposite ends of the horizontal diameter. The direction of these E. M. F.s for an in-

creasing flux from N is indicated by the arrowheads in the diagram. No current is produced since the E. M. F.s in the two halves of the armature are equal and opposed. If a brush be applied to the commutator so as to touch two adjacent segments, a current will be produced in the coil thus short-circuited. If the

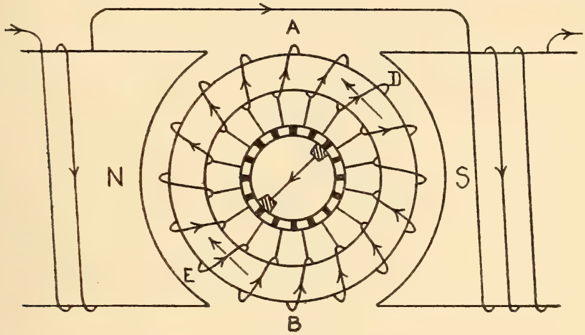


Fig. 354.

brushes be applied to the terminals of the coils A and B, the resulting flux in these coils will be opposite and parallel to the field and hence no torque will be developed. If, however, the coils D and E be short-circuited, the flux in these coils, as shown in the diagram, will be oblique to the field, D will be repelled from N and E will be repelled from S and clockwise rotation will ensue. When the field is reversed, the flux in the coils is also reversed and the rotation will continue in the same direction. As thus described, only the coils in the positions D and E contribute to the torque. If the brushes be enlarged so as to short-circuit a number of adjacent coils, all of these coils will contribute to the turning moment. Finally, if the brushes be connected as shown, currents will flow through the remaining coils and the torque will be correspondingly increased.

It will be noted that there is no direct electrical connection with the armature of this machine and that the currents are produced by induction. It is therefore a true *induction motor.*

663. Principle of Induction Motor.—The principle of the induction motor will be understood from the following.

SNS, Fig. 355, represents a series of magnetic poles, alternating in polarity and moving steadily from right to left as indicated by the arrow. Beneath these there is what may be compared to a

copper ladder with heavy copper rungs. Consider the opening $ABCD$ in this ladder. At the instant shown it is penetrated by the lines of force from N, but as N is moving off to the left, the number of lines embraced is decreasing and there is therefore induced a clockwise current in the direction $ABCD$ (Par. 421). In the adjacent opening $ABEF$, the number of lines embraced is increasing and there is therefore induced a counter-clockwise current in the direction $ABEF$, that is, the E. M. F. in the copper surrounding both of these openings produces a current from A to

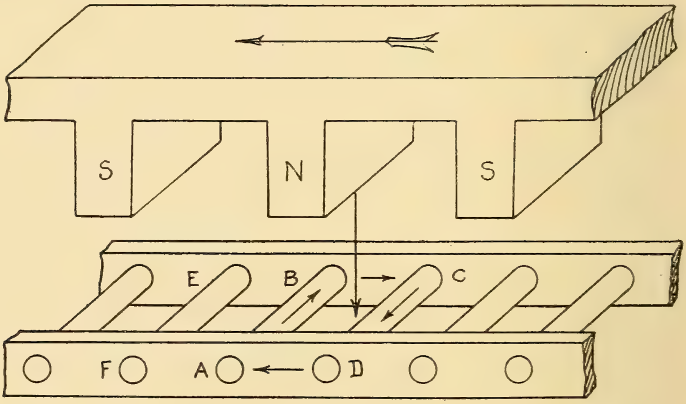


Fig. 355.

B. Since AB is a conductor carrying a current and placed in a magnetic field, it experiences a force urging it to follow along after the moving pole (Par. 352). In a similar manner it can be shown that the rungs under the south poles are also urged to the left. Reflection will show that this movement is also a consequence of Lenz's law (Par. 430).

Although the induced E. M. F. be small, the currents in the copper rungs are, on account of the low resistance, very large and the force $I. H. l$ on the rungs (Par. 356) is also large.

Suppose now the copper ladder to be bent into a cylindrical shape and fixed upon an axis like a squirrel cage (Fig. 356), and suppose the moving poles to be formed into a ring surrounding this cylinder, and their movement of translation to be converted into a movement of rotation. Corresponding rotation will be produced in the squirrel cage, which by a suitable pulley or by gearing could be made to do mechanical work. We have thus produced rotation

in the cage by rotating the magnetic field about it, but the thought arises at once that the energy expended in rotating the field might better have been applied to the cage direct. However, it will now be shown that it is possible to produce a rotating field without resorting to mechanical rotation.

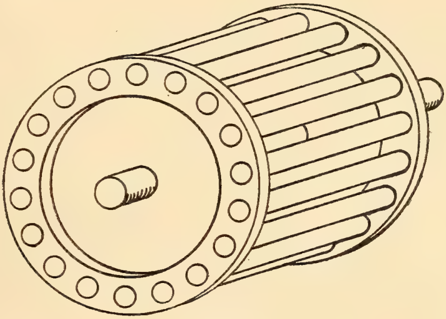


Fig. 356.

664. Production of Rotating Field.—Suppose Fig. 357 to represent a ring wound stationary iron frame and suppose there are connected to the winding at points 90° apart the leads CC' and DD' from a two phase alternator similar to the one shown in Fig. 341. Suppose we start with the armature in the position

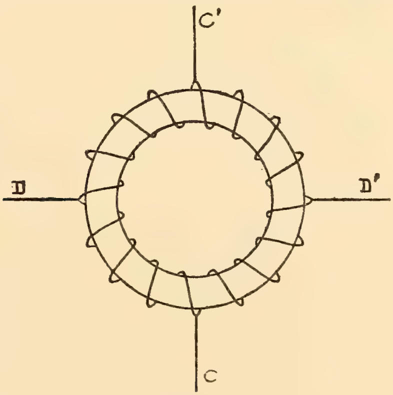


Fig. 357.

shown in that figure. At this instant the current in the leads CC' is a maximum and that in DD' is zero. Application of the right hand rule shows that the current entering at C and leaving at C' produces a south pole at C and a north pole at C'. The current

entering at C now begins to decrease, while an increasing current starts in at D. Currents leave by C' and D' and a north pole is produced between C' and D'.

When the current at C has dwindled to zero, the entire current enters at D and leaves by D'. A north pole is therefore produced at D'.

Without carrying this explanation farther, it is seen that during one complete cycle a north pole starts at C' and travels in a clockwise direction entirely around the iron frame, a corresponding south pole keeping pace at the opposite end of the diameter of the ring. The effect is therefore the same as if the frame work had held a pair of permanent magnetic poles and had been rotated through 360°. In the actual case, however, there has been no mechanical motion and no waste of energy in overcoming friction, such as would have occurred had the heavy iron frame been rotated. A squirrel cage placed inside of this ring would have been rotated by the rotating poles.

The rotating field described above was produced by a two-phase current. It may also be produced by a tri-phase current.

665. The Induction Motor.—The induction motor is based upon the foregoing principles. The rotating cage, although it resembles the armature of other motors, is not strictly an armature since it has no electrical connection with the power circuit. It is therefore called the *rotor*, the surrounding magnetic field being called the *stator*. This is the usual arrangement but it is quite possible to have the field the rotating member.

The inductors are of copper as described above, and in order to insure penetration by the lines of force of the field, the interior of the rotor is built up of laminated iron (Par. 565), in fact, the copper inductors are generally embedded in slots in this laminated core. The stator is likewise laminated and, instead of being ring wound as described in the preceding paragraph, it is wrapped somewhat as shown in Figs. 337 and 339.

If the rotor revolved synchronously with the rotating field, there would be no cutting of lines of force by the inductors, hence no induced current and no torque developed. In order then to develop torque the rotor must run below synchronism. It would therefore seem that the slower the rotor turned the greater would be the torque, but this is not correct. Examination of Fig. 355 will show that as the pole N moves over the interval $ABEF$, an

upward flux is produced, that is, a flux tending to demagnetize N. The force on an inductor is $I. H. l$ (Par. 356), but when the speed falls below a certain point, the field H is demagnetized more rapidly than I increases and the total force therefore falls off. If, therefore, an increasing load be applied to one of these motors, it will slow down until the maximum torque is developed, after which, if the load be further increased, it will come to a stop.

If one of these motors is to start from rest under load, as for example in operating an elevator, it is desirable that the maximum torque should be exerted at starting. This may be attained by constructing the rotor so that the resistance of the inductors may be varied. The resistance is introduced at starting and the currents through the inductors are thus kept down so that the demagnetizing effect described above will not be too great. As the motor gathers headway, this resistance may be cut out.

PART VI.

HIGH POTENTIAL.

CHAPTER 46.

DISCHARGE OF ELECTRICITY THROUGH GASES.

666. High Potential.—The two following chapters, with which we conclude this book, treat of the discharge of electricity through gases and of electrical oscillations. While these subjects stand somewhat apart from the divisions which we have hitherto considered, they can not be said to be very intimately interrelated, and they are here classed under one heading mainly because their most characteristic phenomena are usually produced by the use of high voltages. The title "high potential" must not therefore be regarded as descriptive but rather as used to avoid the comprehensive but still more indefinite designation "miscellaneous."

667. Conductivity of Gases.—Gases are ordinarily the most perfect of non-conductors. In the list of these bodies given in Par. 20, air was placed at the foot. However, under certain conditions described below (Par. 680) their conductivity can be greatly increased. Although some of these conditions have been known for upwards of fifty years, it is only within comparatively recent times that this subject has been systematically investigated, and as a result of these studies much light has been thrown both upon the mechanism of conduction and upon the ultimate nature of electricity itself.

668. Discharge Through Moderate Vacua.—Fig. 358 represents the arrangement already described in Par. 525. *AB* is a long glass tube into each end of which is sealed a platinum wire terminating on the inside in a small disc. The platinum wires are connected to the opposite sides of the spark gap *GH* of an induc-

tion coil. An air pump is attached to a small tube blown in one side of the larger tube and the air is gradually exhausted. At first, the sparks produced by the coil leap across the gap GH, but as the air is exhausted from the tube these sparks cease and a flickering light, like summer lightning, appears on the inside.

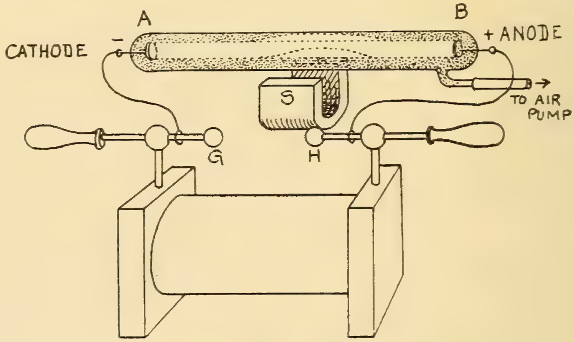


Fig. 358.

If the exhaustion be carried a little farther, or to a pressure corresponding to about an inch of mercury, a luminous column, the *positive column*, extends the entire length of the tube between the anode and the cathode. The spark gap may now be very materially decreased without a spark passing, thus showing that the conducting power of the gas within the tube has been greatly increased.

669. Effect of Magnetic Field on Positive Column.—If while the discharge is taking the form of the positive column the tube be placed in a crosswise magnetic field, the column is deflected for a portion of its length. Thus, if a horseshoe magnet be placed as shown in Fig. 358 so that the tube is penetrated at right angles by a magnetic field from rear to front, the portion of the column between the poles of the magnet will be bent upward as indicated by the dotted lines. Application of the left hand rule (Par. 352) will show that in this respect the column behaves as if it were a flexible conductor carrying a current.

670. Discharge Through High Vacua.—If the exhaustion of the tube described in Par. 668 be continued, when the pressure has been reduced to about that of two millimeters of mercury, the following changes are observed. The surface of the cathode is covered with a thin luminous layer. Adjoining this there is a

dark space C (Fig. 359), the *Crookes dark space,* which enlarges as the pressure diminishes, and adjoining this space there is a luminous region D, the *negative glow.* Beyond this there is a second dark space F, the *Faraday dark space,* followed by the positive column E which is now broken up into *striae,* transverse

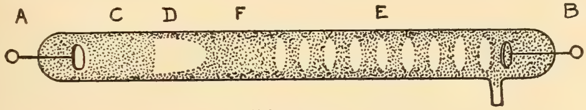


Fig. 359.

luminous discs. A potential sufficient to produce in air a spark one-eighth of an inch in length will now cause a discharge through a tube twenty inches long. Tubes exhausted to this extent are called *Geissler tubes.*

If the exhaustion be carried to about one-millionth of an atmosphere, the tube is called a *Crookes tube.* The luminous spaces entirely disappear, the Crookes dark space spreading throughout the tube, but the glass itself now begins to phosphoresce with a color which varies with its composition. Soda glass glows with a fine green color; lead glass with a pale blue. The resistance is now much greater and increases rapidly so that if the exhaustion be carried slightly farther it becomes no longer possible to send a discharge through the tube.

671. Cathode Rays.—It was discovered by Crookes that the phosphorescence of the glass tube described in the preceding paragraph is produced by certain invisible radiations proceeding

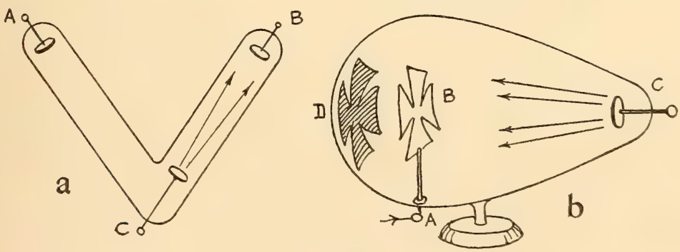


Fig. 360.

from the cathode, and these have accordingly been named *cathode rays.* It may be shown that they leave the cathode at right angles to the latter's surface. In the V-shaped Crookes tube shown in *a*, Fig. 360, whether A or B be used as the anode, only B, the arm up which the cathode points, will phosphoresce.

If the cathode be given a concave shape, a piece of platinum foil placed at the focus may be raised to a red heat by the rays. Many substances, even when not highly heated, fluoresce or emit brilliant light when placed in the path of these rays.

672. Nature of Cathode Rays.—Investigations lead to the belief that the cathode rays consist of minute material particles carrying electrical charges and moving with a velocity so great that they cause the bodies upon which they strike to emit light or fluoresce. These particles have been variously named *corpuscles, electrons* and *negative ions.*

In the Crookes tube shown in *b,* Fig. 360, a mica cross *B* is mounted upon the anode *A* and when the tube is in operation a distinct shadow of this cross appears upon the phosphorescent background at *D.* *B* therefore screens the glass from the rays from *C.* Since *B* is transparent, the cathode rays are not of the nature of ordinary light.

If, instead of the cross, a very delicate little paddle wheel be mounted at *B* and so placed that the rays from *C* strike upon the vanes of one side only, it will take up a motion of rotation as if it had been bombarded with small particles from *C.*

673. Effect of Magnetic Field on Cathode Rays.—The cathode rays are deflected by a magnetic field. In the Crookes tube

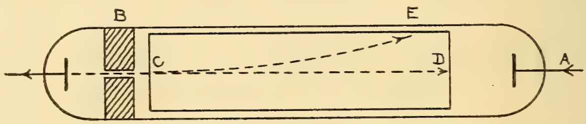


Fig. 361.

shown in Fig. 361, a diaphragm *B* with a narrow slit is placed in front of the cathode. Beyond this diaphragm and lying along the axis of the tube is a vertical sheet of mica coated with chalk. The narrow beam of rays through the slit causes in this chalk a bright line of fluorescence *CD.* If now a horseshoe magnet be placed as shown in Fig. 358, the field running from rear to front, the beam of rays will be deflected in the same direction as the positive column (Par. 669). There is, however, a great difference in the two cases. The positive column is simply deflected as it passes through the field and beyond this field returns to its original direction; the cathode rays, after passing beyond the field, continue

in their deflected direction and terminate upon the side of the tube at *E*.

674. Effect of Electric Field Upon Cathode Rays.—Cathode rays are also deflected by an electric field. Thus, if the tube shown in Fig. 361 be placed between two parallel metal plates, one above and the other below, and if the upper plate be charged positively, the rays will be deflected upward in the direction *CE*. The conclusion is that the corpuscles, or little particles of which the cathode rays are composed, carry negative charges and are consequently attracted by the positively charged and repelled by the negatively-charged plate.

The same conclusion might have been drawn from the deflection produced in the cathode rays by a magnetic field. Since these rays, although moving in opposite direction, were deflected in the same direction as the positive column, they must have constituted a current of *negative* electricity.

675. Nature of Charge Carried by Corpuscle.—The correctness of the above conclusion that the corpuscles carry negative charges

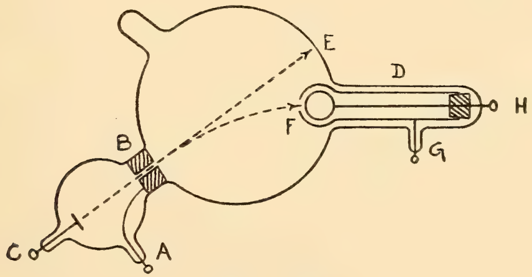


Fig. 362.

is experimentally confirmed as follows. In the two-chambered Crookes tube shown in Fig. 362, *B* is a metal diaphragm pierced with a narrow slit and together with *A* constituting the anode. *C* is the cathode. The side tube *D* contains a metal cylinder with a narrow opening at *F* and with a connection at *G* by which it may be grounded. Within this cylinder but insulated from it there is a second cylinder with a terminal at *H*. An electrometer is connected to this terminal. Normally, the cathode rays pass through the slit in the diaphragm and strike at *E* where they produce a luminous spot. By means of a magnet these rays are

deflected. The instant that they are bent enough to enter the opening at F, the electrometer indicates that the inner cylinder has received a negative charge.

676. Positive Rays.—If the cathode of a Crookes tube be pierced with small holes, luminous rays passing through these holes will be seen at the back of the cathode. These are found to consist of positively-charged ions and are accordingly called *positive rays,* sometimes also *canal rays.*

677. Lenard Rays.—While the cathode rays do not penetrate the Crookes tube in which they are produced, Lenard found that if a small window of aluminum foil be let into the side of the tube, the effect of the rays could be detected for a distance of several inches in the air on the outside. Since the fact that these rays apparently pass through metal appears contrary to the theory that they consist of small material particles, these exterior rays were at first considered to be something different and were called *Lenard rays.* It is now known that they are identical with cathode rays. It is thought that the cathode rays on the interior of the tube do not actually penetrate the aluminum but strike it with such energy that the percussion drives off ions from the outer surface.

678. X-Rays.—In addition to heating the objects upon which they fall, the cathode rays cause these objects to emit rays of a very remarkable penetrative power. Rontgen accidently discovered this fact in 1895. He noticed that a covered photographic plate in his laboratory became fogged by the rays from a Crookes tube with which he was working. It was known that the effect of the Lenard rays extended only a few inches beyond the tube and he realized that he was dealing with an unknown form. He therefore designated them as *X-rays,* though later, in his honor, they were called *Rontgen rays.*

They travel with the velocity of light, penetrate all bodies to some extent, are not reflected or refracted and are unaffected by electric or magnetic fields. Their penetration into the metals varies inversely as the atomic weights of these metals. Lead, whose atomic weight is 207, is therefore the metal most frequently used as a screen for these rays.

They excite powerful phosphorescence in many substances. Advantage is taken of this in the *fluoroscope.* This consists of a

light-proof frame shaped like the frustum of a pyramid. Over the larger end is spread a cardboard coated with barium platino-cyanide. The smaller end of the frustum is arranged so as to be applied to an observer's face as shown in Fig. 363. When exposed to the X-rays, the barium salt glows with a yellowish color and to the observer the effect is as if he were looking through a frosted glass window of that color. If the hand be interposed between the source of the X-rays and the fluoroscope and be applied to the coated cardboard, the X-rays penetrate the flesh more easily than they do the bones and the outline or shadow of the bones is clearly seen. This instrument is used in surgery in the examination of fractures, location of foreign bodies, etc.

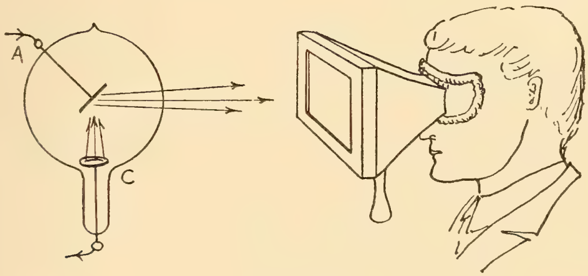


Fig. 363.

X-ray photographs or *sciagraphs* (shadow pictures) are made in a similar manner. The sensitive plate enclosed in its holder is usually placed on a table, the patient placing immediately above the plate the part of his body to be photographed. Exposure to the rays is then made and the plate is developed. In this way the greatest steadiness is secured.

In making these sciagraphs a special form of Crookes tube, a so-called *focusing tube*, is used. As shown in Fig. 363, the cathode is concave and the anode located at its focus is a flat plate of platinum inclined at an angle of 45° to the cathode rays. The X-rays thus emanate from a small area and more clear-cut shadows are produced.

These rays are particularly destructive to cells. They are therefore used in medicine for the treatment of superficial forms of cancer, tuberculosis and skin diseases, but they are not selective and destroy the healthy as well as the diseased, producing burns which are very difficult to heal.

679. Becquerel Rays.—In 1896 Becquerel in investigating the properties of phosphorescent bodies discovered that the compounds of the metal uranium emitted rays which partook of the nature of both the cathode rays and the X-rays. It was soon found that these *Becquerel rays* were not confined to uranium compounds but were produced by other substances. Those bodies which emit these rays are said to be *radio-active.*

The principal ore of uranium is the oxide, pitch blende. The Curies found that the residue left after extracting the uranium from this ore was more radio-active than the uranium itself, and in 1898 they succeeded in separating from this residue a compound of a new element, *radium,* whose radio-activity was over a million times greater than that of uranium. Radium has not yet been isolated but is known to be a metal chemically allied to barium. It exists in such minute quantities that from a ton of the ore only about two-tenths of a gram of the impure chloride or bromide is obtained. Associated with it are *polonium* and *actinium,* two still rarer metals possessing similar properties.

The Becquerel rays are complex but by passing them through a magnetic field they may be resolved into three types called the *alpha,* the *beta* and the *gamma* rays, respectively. The alpha and the beta rays are deflected, but in opposite directions; the alpha rays being positive rays, the beta rays being negative or cathode rays. The gamma rays are unaffected by the magnetic field and are allied to, if not identical with, the X-rays. They have almost incredible penetrative power, being able to penetrate upwards of a foot of solid iron.

680. Increase of Conductivity of Gases.—While, as already stated, the conductivity of a gas is normally very small, there are many widely different ways in which it may be greatly increased. Thus, a gas becomes a conductor if it be highly heated, or if it be mixed with gas drawn from the vicinity of glowing metals or of the electric arc, or if an electric spark be passed through it, or if it be exposed to any of the cathode, Lenard, Becquerel or X-rays described above, or if it be exposed to ultra-violet light, etc., etc. This increase in conductivity is best shown by means of a gold leaf electroscope. So long as the surrounding air remains in its normal state, the leaves, if charged, remain diverging, or if they fall together, do so very slowly. If, however, the air be rendered conductive, as for example by holding

within a foot or so of the leaves a minute quantity of a radium salt, the leaves collapse at once. The following experiment illustrates the production of conductivity by the X-rays. In Fig. 364, *A* is an X-ray tube enclosed in a thick box of lead with a small aperture in the top through which the rays may emerge. Immediately above this opening there is an inverted funnel *F*

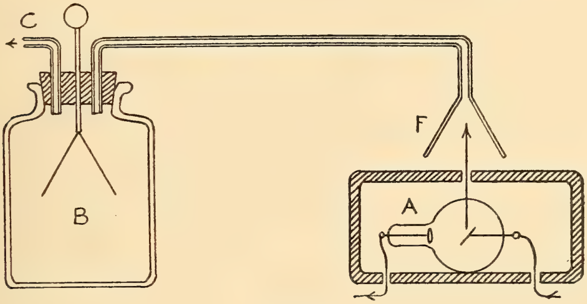


Fig. 364.

which communicates through a glass tube with the jar *B* in which the electroscope is suspended. The X-rays render the air through which they pass conductive, for if suction be applied to the tube *C* so as to draw the air within *F* over into the jar *B*, the leaves collapse as soon as this air enters the jar.

681. Ionization of Gases.—In the preceding paragraph we saw how the air within the funnel *F* (Fig. 364) was rendered conductive by the action of the X-rays and how it retained this conductivity after it had been drawn over into the jar *B*. If, however, there be placed in the tube between *F* and *B* a plug of glass wool, the air from *F* after passing through this plug will be found to have lost its conductivity. The same thing happens if this air be drawn through a metal tube of fine bore, or if it be caused to bubble through water. Since, therefore, the conductivity of the air may be thus removed by filtration, it is but natural to ascribe it to the presence of material particles. Furthermore, the conductivity is removed if the air be passed between two parallel metal plates between which a strong electric field is maintained. These material particles must therefore carry electric charges, and since the air as a whole shows no sign of a charge, there must be an equal amount of positive and negative charges present. The theory was therefore advanced by Thomson that the con-

ductivity of a gas is due to the presence of particles or part molecules, called *ions,* some positively and others negatively charged. These negatively-charged particles are found to be identical with the corpuscles of the cathode ray. The production of these ions, or *ionization,* is brought about by any of the agents mentioned in the preceding paragraph. The explanation advanced is that the ions or corpuscles associated with these various agencies are moving with such velocity that when they come into collision with the molecules of the gas through which they pass, they break these molecules up into other ions.

682. Investigation of Corpuscles.—From the time that the theory was advanced that the cathode rays consisted of charged corpuscles moving with high velocity, efforts were directed to determine the mass of these corpuscles, the charge which they carry and the velocity with which they move. In the solution of this problem the work of J. J. Thomson has been especially noteworthy. We can do no more than give a bare outline of his methods.

His first step was to determine the relation between these three quantities, and this he did as follows. In the neck of the two-chambered Crookes tube shown in Fig. 365, *B* and *D* are two

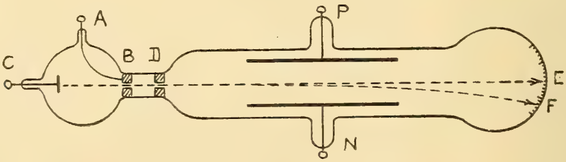


Fig. 365.

thick metal diaphragms pierced by an opening about a millimeter in width. The diaphragm *B* forms a part of the anode *A*. The rays from the cathode *C* pass in a narrow line through the slits in *B* and *D* and produce a small luminous spot at *E* on the far side of the other end of the tube.

Let the mass of each corpuscle be m grams, its velocity be v centimeters per second and its charge q electro-magnetic absolute units (Par. 536). Each moving corpuscle is equivalent to a current whose strength is vq absolute units. If the tube be placed in and at right angles to a uniform magnetic field of intensity H, each corpuscle will be acted upon by a force vqH at right angles

to its path (Par. 356). Now it is shown in mechanics that if a body moving with uniform velocity be acted upon by a constant force at right angles to its path, then the body will move upon the arc of a circle. The radius of this circle is given by the expression $r = \frac{mv^2}{f}$, f being the force at right angles to the path. The deflected corpuscles therefore move upon an arc whose radius is

$$r = \frac{mv^2}{vqH} = \frac{mv}{qH}$$

If the positive direction of the field be from front to rear, the rays DE will be curved downward to DF. DE, the tangent to the arc, and EF are measured, whence the radius of the circle is determined thus:

The triangles CDE and DFE (Fig. 366) being similar

$$EF : DE :: DE : CE$$

whence

$$EF : DE :: DE : 2r + EF$$

whence $2r = \frac{\overline{DE}^2}{EF} - EF$

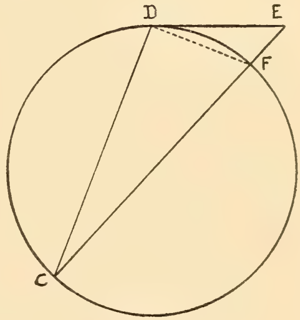


Fig. 366.

The intensity of the field being measured, we have H and r, and the value of $\frac{mv}{q}$ becomes known.

683. Velocity of Corpuscles.—The velocity v of the moving corpuscles was determined by Thomson as follows. Coils were placed in front and rear of the tube shown in Fig. 365 and a uniform transverse magnetic field H established. If the positive direction of this field was from front to rear, the rays were deflected downward by a force vqH.

The parallel metal plates P and N were connected to the terminals of the battery, thus establishing in the tube a vertical electric field F. If the plate P were positively charged, the rays would be deflected upward with a force Fq (Par. 674). By varying either field (generally the magnetic field) they could be so adjusted that the tendency of one to bend the rays down was

exactly balanced by the tendency of the other to bend the rays upward. At the instant

$$vqH = Fq$$

whence

$$v = \frac{F}{H}$$

and knowing F and H, the velocity v becomes known. This velocity, when the tube is highly exhausted, is about one-tenth of the velocity of light, and is independent of the nature of the gas within the tube.

By inserting this value of v in the expression deduced in the preceding paragraph we obtain the value of m/q, or the ratio of the mass of the corpuscle to the charge which it carries. More frequently, the reciprocal of this ratio, or the ratio of q/m is given. According to the latest determinations it is 1.7×10^7. It can be shown that in ordinary electrolysis the ratio of q/m for the hydrogen atom is about 10^4.

684. Mass of Corpuscle.—Having thus found the value of q/m, if either one of these quantities be separately obtained the value of the other follows at once. The charge q is the one usually determined directly. The actual process involves many steps into which we can not go in detail. It is based upon the following principles. If a volume of saturated vapor be suddenly expanded its temperature falls and there is a tendency for condensation to ensue. If in such supersaturated space microscopic particles of dust be introduced, a fog is produced at once, the particles of dust facilitating condensation by serving as nuclei upon which the drops form. Now, if a closed vessel with an aluminum cover be exposed to certain radiations, such as those from radium salts, or to X-rays, corpuscles are produced in the gas within the vessel. These corpuscles act just like the dust in that they serve as nuclei for drops and in a supersaturated space cause a fog to form at once. These drops of mist are very minute but may be seen through a glass and by suitable observations the velocity with which they slowly settle can be determined. Knowing this velocity and the density of the gas within the vessel, by the application of known formulae the size, and hence the mass, of the drops can be calculated. The vapor within the vessel contains both positive and negative ions but it has been found that if the expansion is not greater than one-quarter of the original volume

that only the negative ions serve as nuclei and are carried down. Each slowly-falling drop therefore has a corpuscle as a nucleus. The charge carried by this corpuscle has been determined in several ways. If the drop falls between two horizontal parallel metal plates, the lower plate can be given a negative charge so that its repulsion will counterbalance the force of gravity on the drop, or may drive it upward. By measuring the upward velocity, the force exerted upon it can be found, and hence the charge which it carries.

Within the limits of experimental error, this charge is found to be the same as the charge carried by the hydrogen atom in ordinary electrolysis. Since we saw in the preceding paragraph that the ratio of q to m for the corpuscle is 1.7×10^7, and for the hydrogen atom is 10^4, and since q is shown to be the same in the two cases, *the mass of the corpuscle is* 1700 *times less than that of the hydrogen atom*. On the other hand, the mass of the positive ions is found to agree with the mass of the corresponding atoms.

685. Nature of Corpuscles.—Upon the nature of the negative ions or corpuscles, scientists are not entirely agreed. From whatever substance produced, they appear to have the same mass. Some therefore maintain that they are the true atoms of a universal single matter and that the fact that the weights of the majority of the ordinary chemical atoms may be expressed in whole numbers is simply an expression of a law of multiples which would follow from these atoms being composed of definite numbers of corpuscles. It would seem therefore that we had confirmed the belief of the alchemists that all matter was composed of a single and ultimate element.

On the other hand, a few deny that they are matter and claim that they are portions of ether in rapid movement.

Whatever be the nature of the corpuscles themselves, it is quite certain that the charges which they carry are electric atoms in the sense that they are all the same and that no smaller charge has yet been obtained. The atomic character of these electrons has already been mentioned (Par. 280).

CHAPTER 47.

ELECTRIC OSCILLATIONS.

686. Henry's Theory of Oscillatory Discharge of Leyden Jar.—Seventy-five years ago the identity of static and voltaic electricity was not regarded as proven. It had just been discovered that a voltaic current sent through a coil wrapped about a steel bar converted the bar into a magnet. It occurred to investigators that this fact afforded a means of making the desired proof. A steel needle was placed in a coil, one end of which was connected to the outer coating of a Leyden jar (Fig. 367), the other end terminating in a knob near the knob which communicated with the inner lining. When a spark was caused to pass between the knobs, this charge passed through the coil and if it were of the same nature as voltaic electricity it should magnetize the needle. When this was done it was found, according to expectation, that the needle became magnetized. However, when these investigations were continued it was noted that although the charge was sent through the coil always in the same direction, the polarity of the resulting magnets varied in an anomalous manner so that it was not possible to predict which end of the needle would be the north pole. In seeking to explain this, Henry in 1842 advanced the theory that the discharge of a Leyden jar, although appearing to our senses as a single spark, was in reality an oscillation of a current back and forth, and the polarity of the needle depended therefore upon the direction of the last oscillation.

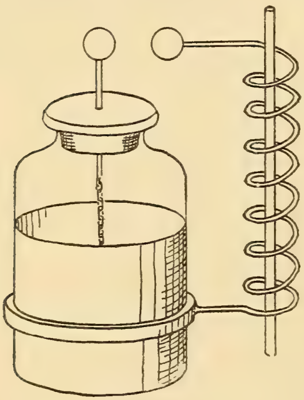


Fig. 367.

687. Thomson's Mathematical Proof of Oscillation.—Some ten years after Henry announced his theory, Sir William Thomson (Lord Kelvin) advanced a mathematical proof of its correctness. His deduction may be shown as follows:

Suppose that a Leyden jar of capacity K is discharged through a conductor of resistance R and of inductance L, and suppose that at any given instant it contains a charge q. The energy of the electro-static field of the jar at this instant is (Par. 97)

$$\frac{1}{2}(q^2/K)$$

This energy is being dissipated in two ways; (a) in establishing an electro-magnetic field about the conductor and (b) in heating this conductor.

If the instantaneous value of the current be I, the energy of the electro-magnetic field is (Par. 359) $\frac{1}{2}IN$, or since $N = LI$ (Par. 434)

$$\frac{1}{2}I^2L$$

Since $I = -\frac{dq}{dt}$, the rate at which the charge is diminishing, this may be written

$$\frac{1}{2}L\left(\frac{dq}{dt}\right)^2$$

The rate at which energy is being lost by the jar is equal to the rate at which energy is being gained by the field plus the rate at which it is being expended in heating the circuit.

The rate at which energy is being lost by the jar is

$$-\frac{q}{K}\cdot\frac{dq}{dt}$$

The rate at which it is being gained by the field is

$$L\cdot\frac{dq}{dt}\cdot\frac{d^2q}{dt^2}$$

The rate at which it is being dissipated in heat is

$$\frac{I^2Rt}{t} = I^2R = \left(\frac{dq}{dt}\right)^2 R$$

whence

$$-\frac{q}{K}\cdot\frac{dq}{dt} = L\frac{dq}{dt}\cdot\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2$$

whence

$$\frac{d^2q}{dt^2} + \frac{R}{L}\cdot\frac{dq}{dt} + \frac{q}{LK} = 0$$

A differential equation of this form may be solved by substituting ϵ^{mt} for q (Murray, Differential Equations, Par. 50). Making this substitution we have

$$m^2\epsilon^{mt} + \frac{R}{L}m\epsilon^{mt} + \frac{1}{LK}\epsilon^{mt} = 0$$

whence

$$\epsilon^{mt}\left(m^2 + \frac{R}{L}m + \frac{1}{LK}\right) = 0$$

which is satisfied when

$$m^2 + \frac{R}{L}m + \frac{1}{LK} = 0$$

or when

$$m = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LK}} \qquad \text{(I)}$$

The values of m are real when the quantity under the radical is positive, that is, when $R^2 > 4\,L/K$. Designating these values of m by $-m_1$ and $-m_2$ (since they are both negative), the corresponding value of q is

$$q = a\epsilon^{-m_1t} + b\epsilon^{-m_2t}$$

a and b being constants.

If t be made equal to zero, we have

$$q = a + b$$

q in this case being the value of the charge in the jar just before the discharge began. As t increases, corresponding to subsequent time, the value of q gets steadily smaller (since the exponents of ϵ are negative), that is, the charge dwindles away without fluctuations or change of sign. The discharge is therefore unidirectional.

If, however, R^2 is less than $4L/K$, the values of m in (I) above are imaginary. In this case we may proceed as follows:

The expression for m is written

$$m = -\frac{R}{L} \pm \sqrt{\frac{1}{LK} - \frac{R^2}{4L^2}}\,(\sqrt{-1})$$

and placing $\alpha = -\frac{R}{L}$, $\beta = \sqrt{\frac{1}{LK} - \frac{R^2}{4L^2}}$ and $i = \sqrt{-1}$, the corresponding roots may be written

$$m_1 = \alpha + \beta i$$
$$m_2 = \alpha - \beta i$$

whence as above

$$q = a\epsilon^{(\alpha+\beta i)t} + b\epsilon^{(\alpha-\beta i)t}$$

which can be put in the form (Murray, Par. 52)

$$q = \epsilon^{\alpha t}(A \cos \beta t + B \sin \beta t)$$

If the expression within the parentheses be multiplied and divided by $\sqrt{A^2 + B^2}$, we have

$$\sqrt{A^2 + B^2}\left(\frac{A}{\sqrt{A^2 + B^2}} \cdot \cos \beta t + \frac{B}{\sqrt{A^2 + B^2}} \cdot \sin \beta t\right)$$

which may be written

$$A_1 (\sin \phi \cos \beta t + \cos \phi \sin \beta t)$$

which is equal to

$$A_1 \sin (\beta t + \phi)$$

and substituting above

$$q = A_1 \epsilon^{\alpha t} \sin (\beta t + \phi)$$

In this expression, t being the only variable, we see that q varies harmonically, in other words, the discharge is oscillatory, although, since α is negative, the oscillations gradually die out.

Since βt is the variable angle and t is time, β is angular velocity and the periodic time of an oscillation is $2\pi/\beta$. Substituting the value of β from above

$$\tau = \frac{2\pi}{\sqrt{\dfrac{1}{LK} - \dfrac{R^2}{4L^2}}} = \frac{4\pi LK}{\sqrt{4LK - R^2K^2}}$$

If R be very small, which is usually the case, R^2K^2 may be neglected and this last expression reduces to

$$\tau = 2\pi \sqrt{LK}$$

whence, the periodic time increases with an increase in L, the inductance, or in K, the capacity.

688. Feddersen's Experiment with Revolving Mirror.—In 1859 Feddersen by a simple experiment proved the correctness of Henry's theory and of Thomson's deductions. The principle he employed will be understood from the following. In Fig. 368, G is the spark gap of a Leyden jar, M is a mirror mounted upon an axis about which it is capable of rapid rotation, and P is a photo-

graphic plate. If while the mirror is rotating a spark be passed across the gap G, the beam of light from this spark will fall upon the mirror and will be reflected. Owing to the movement of M, this reflected beam will sweep like a brush across P and when the plate is developed and printed the path will be revealed as a band of light. Examination will show along the edges of this band a series of bright beads, those on one side being opposite the gaps

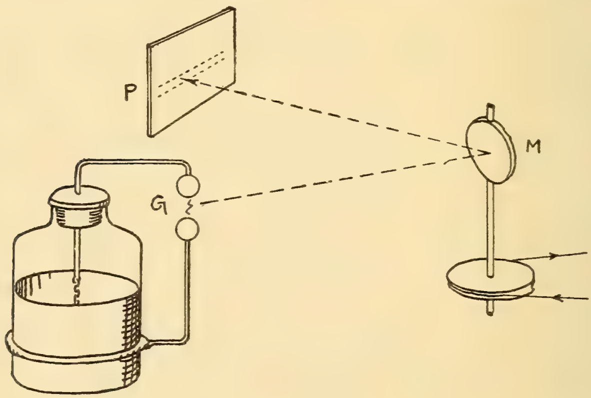


Fig. 368.

between those on the other and showing that the brightest point of the spark alternated between the knobs, in other words, the spark passed back and forth. Knowing the rate of rotation of the mirror, the time of oscillation of the spark may be determined with accuracy, even though this time is less than a millionth of a second.

689. Explanation of Oscillation.—An explanation of this oscillatory discharge is afforded by what we have already learned of inductance. Suppose a charge to be given to the jar and to be gradually increased until the discharge takes place. As the charge passes from the inner to the outer coating of the jar, it is a true current and the inductance of the circuit causes it to continue to flow beyond the point when the jar is completely discharged, in other words, the outer coating receives an excess charge. This then flows back in the opposite direction and, for the reason given above, the inner coating now acquires an excess charge, and so on. These oscillations do not continue indefinitely because at each one a portion of the energy is spent in heating the circuit and, as we

shall see shortly, another portion is radiated off into space. The total number therefore may not exceed ten or twelve.

Reflection will show that before the discharge takes place the energy of the field is electro-static, that is, the field is composed of tubes of force (Par. 62), but that during the discharge this energy is electro-magnetic, the conductor being surrounded by circular lines of force. At the end of the discharge the magnetic lines disappear and the tubes of force reappear, but reversed in direction. As the return oscillation begins, these tubes again give way to magnetic lines of force, which in turn are in reverse direction from the first set, and so on.

690. Maxwell's Electro-Magnetic Theory.—In 1865 Maxwell published a mathematical analysis of the effects produced in the surrounding medium by an oscillatory discharge. As bases for his discussion he took the facts (a) that a current flowing in a conductor produces about the conductor a magnetic field, (b) that if a magnetic field about a conductor be varied, an E. M. F. is induced in the conductor, (c) that the electric force exerted in the space about a charge varies inversely with the dielectric capacity and (d) that the magnetic field about a current varies with the permeability of the dielectric. To these he added the *displacement assumption* which is that when, for example, a charge flows into a condenser, an equal quantity of electricity moves in the dielectric between the plates, but that this movement takes place within the molecules of the dielectric and not from one molecule to another. The effect is as if in each molecule a positive charge had moved to one end, a negative charge to the other, and the positive charged ends all pointed in the same direction, that is, away from the positive plate of the condenser. The so-called displacement currents, just as any other current, produce about them a magnetic field.

As a result of his discussion he showed that these oscillations give rise to electric waves in the surrounding space, the wave front comprising electric displacements and magnetic forces at right angles to each other and both at right angles to the direction of propagation of the wave. He also showed that these waves moved with a velocity of thirty billion centimeters per second. Since light moves with the same velocity and is transmitted by the same agent, the ether, he concluded that light and electricity are identical and differ only in that the light waves are much the

shorter. As corroborating this last conclusion, he showed that since electric waves can not be transmitted through conductors, these bodies should not transmit light waves. As a fact, the metals are all opaque to light. The same reasoning would show that a transparent solid is a non-conductor and such substances are the best insulators. It does not follow however that all opaque bodies are conductors, for many, such as porcelain, marble, etc., owe their opacity to irregular crystallization or mechanically included impurities. The purest form of crystallized marble, Iceland spar, *is* transparent.

Maxwell's mathematical discussion can not be repeated here, but by following a similar line of reasoning we show in the three following paragraphs how he arrived at one of his conclusions.

691. Electric Elasticity.—The elasticity of a body is measured by the ratio of the stress exerted upon the body to the strain (elongation, compression, etc.) produced. Consider a sphere carrying a charge Q and surrounded by a concentric non-conducting shell of dielectric capacity K. Displacement will take place in the dielectric, a charge $-Q$ being induced on the inner surface of the shell and a charge $+Q$ being repelled to the outer surface. If the radius of the shell be r, the force per square centimeter exerted upon the inner surface is $\frac{1}{K} \cdot \frac{Q}{r^2}$ (Par. 90). This is the stress to which the shell is exposed. The strain per square centimeter consists in driving the positive charge to the outer surface of the shell and is therefore $\frac{Q}{4\pi r^2}$. The *electric elasticity*, the ratio of the stress to the strain, is $4\pi/K$.

692. Electric Density.—In Par. 435 there was deduced an expression for the inductance of a coil wrapped upon a circular core. If in this expression both I and L be absolute electro-magnetic units (instead of amperes and henrys), the expression becomes

$$L = \frac{4\pi^2 n^2 r^2}{l} \mu$$

in which n is the total number of turns, r is the radius of the coil and l is its length.

In Par. 687 it was shown that the energy of an electro-magnetic field is $\frac{1}{2}I^2L$. Substituting the above value of L and dividing

by $\pi r^2 l$, the volume of the core, we obtain for the energy per cubic centimeter

$$\frac{1}{2}4\pi\mu\left(\frac{In}{l}\right)^2$$

If for n/l, the number of turns per centimeter, we write N, this becomes

$$\frac{1}{2}4\pi\mu(IN)^2$$

In mechanics it is shown that the energy of a mass m moving with a velocity v is $\frac{1}{2}mv^2$. From analogy, therefore, $4\pi\mu$ is termed the electric mass per unit of volume. But mass per unit volume is density, therefore, $4\pi\mu$ is the *electric density* of the field.

693. Velocity of Propagation of Electric Wave.—If ϵ be the elasticity of a medium and δ be its density, the velocity with which waves are propagated through it is $v = \sqrt{\epsilon/\delta}$. Substituting the values of the electric elasticity and density given in the preceding paragraphs, we have for the velocity of propagation of electric waves

$$v = \frac{1}{\sqrt{K\mu}}$$

This is the expression which we have already obtained in Par. 548. This velocity, by many independent methods, has been shown to be thirty billion (3×10^{10}) centimeters per second, or as already stated, the same as the velocity of light.

694. Hertz's Confirmation of Maxwell's Theory.—During Maxwell's life time his theory made but moderate headway and he died before it had ever been experimentally proven. In 1887, twenty-two years after his theory had been announced, it received striking and abundant confirmation by a series of brilliant experiments performed by Hertz. The arrangement used in the first of these experiments is shown in Fig. 369 and consists of two parts which Hertz designated respectively as the *oscillator* and the *resonator.* The oscillator consisted of two sixteen-inch square zinc plates, A and B, placed two feet apart. A copper rod from each of these and upon their common axis terminated in polished knobs separated by a gap G of about one-quarter of an inch. These rods were connected as shown to the terminals of the secondary of an induction coil. When the coil was operated, series of sparks passed between the knobs. The resonator consisted of a copper wire bent into a circle with a narrow spark gap

between two knobs. It was found that to obtain the best results the dimensions of the resonator had to be adjusted, or it had to be "tuned" to suit the particular oscillator used. With the one described, the diameter of the resonator was about twenty-eight inches.

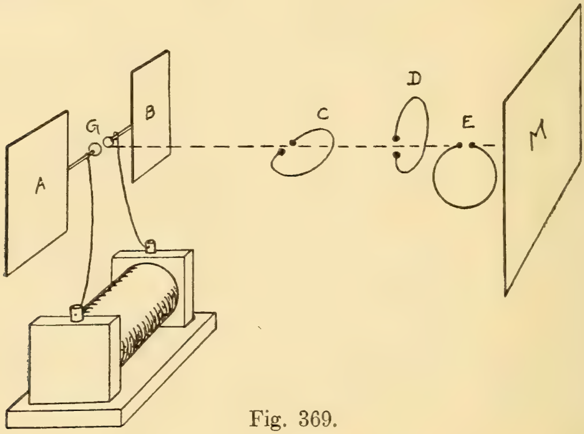


Fig. 369.

With the coil in operation and sparks passing between the knobs of the oscillator, the resonator was held in various near-by positions. When placed, as shown in the figure, along the line GM passing through and perpendicular to the spark gap G, it was found that sparks were produced in the gap of the resonator whenever the axis of this gap was parallel to the spark gap of the oscillator. Thus sparks were produced when the resonator was held as at C but not when held as at D or at E.

As thus carried out, this experiment does not conclusively show the existence of *waves* and might be considered a simple example of induction as explained in Par. 420. The demonstration of the existence of waves was made as follows: The oscillator was placed so that its axis was parallel to and at some distance from the opposite wall of the room in which the experiments were carried out. This wall was covered with large sheets of zinc M. Since according to Maxwell's theory the metals are opaque to these waves, this zinc sheet should act towards them as a mirror. Now it is well known that when waves strike a surface normally and are reflected back along the same path, the phenomenon of interference occurs. Beginning at the reflecting surface, at fixed points one-half of a wave length apart

the advancing and returning waves are in opposition and complete interference results, while at points midway between these nodes the waves are in phase and the resulting amplitude is twice that of the advancing wave. With the resonator close to the zinc sheet M, no sparks are obtained, but moving from M towards G, a point is found where the intensity of the sparks in the resonator is a maximum. Continuing to move towards G, the sparks in the resonator again die out, then again rise to a maximum. These electro-magnetic radiations are therefore waves, the distance between two successive points of maximum sparking or between two nodes of no sparking being one-half of the wave length. With the apparatus described above, the wave length was found to be about thirty-two feet.

By the method just outlined, the wave length can be determined. By photographing the spark seen in the revolving mirror, the periodic time of the oscillations can be measured. The reciprocal of this is the frequency, or number per second. The product of the wave length by the frequency gives the velocity with which the wave travels. The results entirely confirm the previous determinations of this velocity as 3×10^{10} centimeters per second.

695. Further Experiments by Hertz.—With slight modifications in his simple apparatus, Hertz succeeded in reproducing many of the characteristic experiments usually shown with light. Thus, by placing the spark gap of his oscillator at the focus of a reflector made by bending a sheet of zinc into a parabolic form, he was able to direct the waves so that they could be detected by a resonator placed at the focus of a corresponding reflector at a distance of over thirty yards. By using a huge prism of pitch, four feet on an edge, he was able to refract the beam from the parabolic reflector. Finally, he showed that these waves are polarized. By placing in the path of the beam from the reflector a screen made of a number of parallel wires strung on a wooden frame, he showed that the waves pass freely when the wires were perpendicular to the axis of the spark gap of the oscillator but were entirely cut off when these wires were turned so as to be parallel to this axis.

696. Length of Electro-Magnetic Waves.—The length of the longest light waves is a little over .00007 of a centimeter, while

we have seen above that that of the electro-magnetic waves produced by Hertz in his first experiment was thirty-two feet. Since the velocity of these waves is constant and is equal to the product of the wave length by the frequency, and since the frequency is the reciprocal of the periodic time, the wave length varies directly with the periodic time. In Par. 687 this was shown to be $\tau = 2\pi\sqrt{LK}$, therefore, by decreasing either the inductance or the capacity of the oscillator, the wave length may be shortened provided the condition for oscillatory discharge, $R^2 < 4L/K$ (Par. 687) be maintained. Since all conductors have both inductance and capacity, Hertz found that he could do away with the zinc plates of his oscillator and substitute for them simple straight wires. Other investigators, by reducing the dimensions of the oscillator, have produced electro-magnetic waves whose length has been about two-tenths of a centimeter.

697. Tuning of the Resonator.—By experiments based upon several different principles it has been shown that electric oscillations may also be transmitted along wires.

Consider a length of insulated wire and suppose an electric impulse to be applied to it. The wave produced will travel the length of the wire and having reached the far end will be reflected and return. If on its way back it encounters another wave traveling in the opposite direction, the two waves will combine. If they be in phase, the amplitude of the resultant wave is doubled and would be further increased by each successive wave. If they be in opposition, they mutually destroy one another and complete interference results. Between these extremes, recurring risings and fallings result and produce an effect analogous to the "beats" in sound waves. If the waves be of constant length, the time required for them to travel to the end of the wire and return varies with the length of the wire. It is therefore possible by lengthening or shortening the wire to adjust it so that the wave will return in exact time to receive the maximum increment from the succeeding impulse. When this adjustment, the *tuning* referred to in Par. 694, has been made, the waves in the wire are of maximum intensity and *resonance* is said to have been secured.

As an analogy, a pendulum has a natural period of vibration. If successive impulses be applied to the pendulum and be timed at its natural period, their effects are cumulative and, although

the individual impulses may be very feeble, they may finally produce motion through a wide arc. If, on the other hand, they be not timed at this period, little or no oscillation will be produced.

Reflection will show that resonance may be obtained in another way, that is, by varying the length of the wave instead of varying the length of the circuit. It was shown in the preceding paragraph how this may be done by varying the periodic time. There are, therefore, two ways of obtaining resonance in a resonator: (a) by shortening or lengthening the resonator and thus changing its natural period to suit the period of the waves and (b) by varying the period of the waves to suit the natural period of the resonator.

698. Principle of Wireless Telegraphy.—Hertz showed by his experiments that there could be produced at will electric waves which travel through space with the velocity of light. He also showed that by suitably-arranged apparatus these waves could be detected at a distance from their point of origin. It was quickly realized that these two observations comprised the fundamental principle of wireless telegraphy. Subsequent development has taken place along two lines: (a) the improving of the sending apparatus or oscillator so that a greater amount of energy could be thrown out in the form of waves, and (b) the perfecting of the receiving apparatus, increasing its sensitiveness so that the waves could be detected at greater and greater distances. Foremost among those engaged in these problems was Marconi who in 1895 took out his first patents on methods of wireless telegraphy.

In order to bring out clearly the object of the various parts of the modern apparatus, we shall describe the simpler forms and show why changes were found desirable.

699. The Aerial.—The primary form of apparatus for producing electric waves was the oscillator of Hertz. As stated above, it was soon discovered that the zinc plates could be replaced by straight wires. It was next found that if these wires be placed in a vertical position instead of horizontal, that the lower one could be dispensed with, the earth taking its place. In either case, the length of the waves produced remained the same, that is, a little over four times the length of the upper wire. To this vertical wire the names *aerial* or *antenna* are applied.

With other conditions constant, the distance to which signals can be sent varies as the square of the length of the aerial and for this reason Marconi at first proposed that it should be supported by kites or balloons. This proposition was found to be impracticable for permanent installations and antennae are now supported by towers or masts.

The signaling distance also increases directly with the amount of energy radiated. In Par. 97 it was shown that the energy of a condenser is $\frac{1}{2}V^2K$, K being its capacity. In this case the aerial and the grounded wire and the earth below the spark gap constitute the condenser. We may therefore increase its capacity by increasing the length of the aerial, but, as shown above, the practical limit of length is soon reached. The next solution therefore is to add to the capacity by using a number of wires in the aerial. The capacity of a single wire is considerable. Pierce states that a straight wire $\frac{1}{8}$ inch in diameter and 100 feet long has the same capacity as an isolated metallic disc 16 feet in diameter. If, however, more than one wire be used, owing to the mutual action of the like charges which they carry, the capacity is far from increasing in proportion to the number of wires, in fact, it increases more nearly as the square root of this number, that is, sixteen wires two feet apart have only four times the capacity of a single wire.

700. The Transmitter.—A simple form of transmitter is shown diagrammatically in Fig. 370. A is the aerial and D is the lower wire grounded at E. B is a battery in circuit with the primary of an induction coil C. The terminals of the secondary are connected to A and D on opposite sides of the spark gap G. When the key K is closed, the E. M. F. induced in the secondary drives a charge into A until a spark jumps across G, thereby producing the desired electrical oscillations.

We saw above that the signaling distance increases directly with the amount of energy radiated and we also saw that the energy of a condenser is $\frac{1}{2}V^2K$. The signaling distance therefore varies with two factors, K, the capacity, and V^2, the square of the potential in the aerial. The capacity may be increased by increasing the number of wires in the aerial, but, as shown above, this increase is far from being in proportion to the number of wires added. The most promising method is to increase the

difference of potential across G. This can be done by separating the knobs more widely, for, in order to throw a spark across this wider gap, the aerial must be charged to a higher potential. This solution, however, involves another difficulty for as we widen the gap G, we greatly increase the resistance and we have seen that in order that the oscillations may be produced by the spark, R^2 must be less than $4L/K$ (Par. 687). If, therefore, R be increased too much, the discharge is no longer oscillatory.

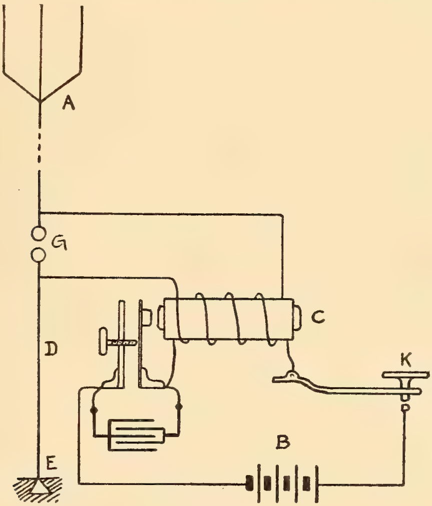


Fig. 370.

701. Coupled Circuits.—There still remains a way by which the potential in the aerial may be increased without increasing the resistance across the spark gap. This is by applying to the oscillatory circuit the principle of the step up transformer. One form of this arrangement is shown diagrammatically in Fig. 371. The battery, key and induction coil are just as described above but in addition there is shunted around the spark gap G a circuit containing a condenser D (usually a battery of Leyden jars), and a coil F. This coil is the primary of an air core, step up transformer, the secondary of which is the coil H in series with the aerial AE. In the actual apparatus, the coil H is within the coil F, although separated from it by considerable space. For the sake of clearness they are represented in the diagram as entirely separate. When the key K is closed, the high voltage of the secondary of C causes the condenser D to receive a large charge

and therefore when a spark occurs at *G*, a large current oscillates back and forth. As this current flows through *F*, it induces a corresponding oscillatory current in *H*, the voltage of this last being greater than that in *F* in proportion to the ratio of the number of turns in *H* to those in *F*. The voltage in the aerial is therefore stepped up and the waves which it sends forth have so much the more energy.

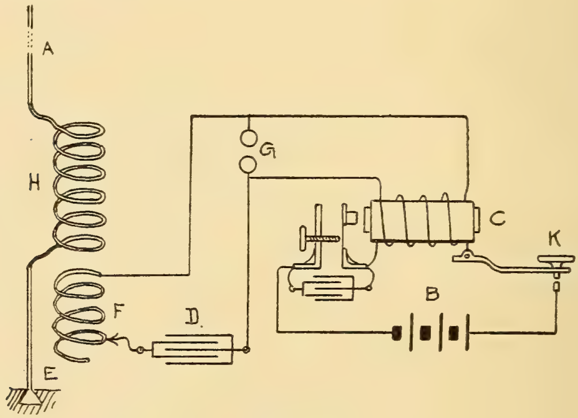


Fig. 371.

The arrangement just described is said to be *inductively-coupled*. Sometimes *F* and *H* are parts of one continuous coil, that is, they constitute an auto-transformer (Par. 652). In this case the apparatus is said to be *direct-coupled*. Owing to the very high frequency of the oscillations, hysteresis prevents the use of iron cores in these coils.

Commercial wireless telegraph plants no longer employ the battery and induction coil as described in the preceding paragraphs but use alternating current generators of from 2 to 5 kilowatts capacity and supplying 220 volts at 500 cycles. The current from these generators is stepped up by suitably-designed transformers.

702. Tuning of Coupled Circuits.—The aerial (Fig. 371) has a natural period, that is, there is an electric wave of a certain length which will produce in it resonance. If the waves in the circuit *FD* can be made of this length, resonance will be set up in the aerial and it will radiate a maximum amount of energy. This circuit contains capacity in the shape of the condenser *D*,

and inductance in the coil F. In Par. 696 it was shown that the length of a wave in an oscillatory circuit varies as $2\pi\sqrt{LK}$; we may therefore vary the wave length by varying either the inductance or the capacity or both. Condensers of variable capacity are of frequent use for this purpose. In the diagram, however, the inductance is represented as variable. The wire from D to F connects to F by means of a clip and may be slid up or down so as to embrace fewer or more turns of the coil. To determine when resonance is secured, a hot wire ammeter (Par. 463) is inserted in the aerial either above or below H and the inductance of F is varied until the ammeter shows a maximum current.

703. Branley's Coherer.—Having seen how electric waves may be produced, we shall now show how they may be detected at a distance.

The receiving circuit, like the transmitter already described, has an aerial, in fact, by means of a shifting switch uses the same aerial as the transmitter at the same station. The waves from a distance strike this aerial and produce in it electrical oscillations, but these are usually very feeble. The first efforts were therefore directed to produce a sensitive relay (Par. 412) which would operate under these feeble oscillations and close a delicate switch, thereby throwing in on some recording apparatus an auxiliary source of current.

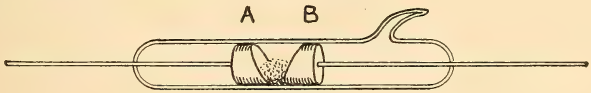


Fig. 372.

The first successful solution of this problem was made by a device due to Branley. In 1890 he found that metallic filings placed between two metal plugs in a glass tube, the plugs constituting the terminals of an electric circuit, were ordinarily non-conductive or at least of very high resistance, but were rendered conductive by electric oscillations in their vicinity. If, after being so rendered conductive, the filings were jarred, their original high resistance was restored. No entirely satisfactory explanation of this phenomenon has been given. His discovery resulted in a piece of apparatus, the *coherer*, which in the hands of Marconi took the form shown in Fig. 372. It consists of a slender glass tube in which are two metal plugs A and B separated by a nar-

row space in which are loosely piled rather coarse filings of a mixture of 95 parts nickel and 5 parts silver. The wires connecting with the plugs are sealed into the tube and the tube itself is then exhausted and sealed. The exhaustion of the tube is to prevent the filings from becoming tarnished.

704. Operation of Receiving Circuit.—The operation of the receiving circuit will be understood from the following: The incoming electric waves produce oscillations in the aerial *A* (Fig. 373) which render the coherer a conductor. This enables the

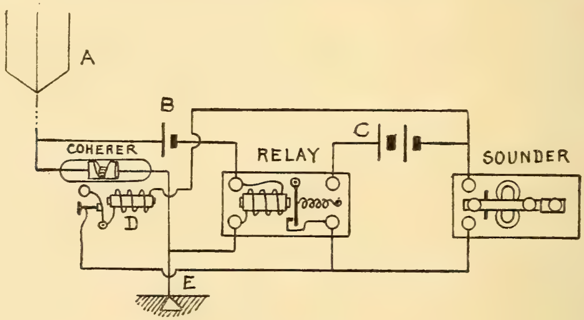


Fig. 373.

battery *B* to send a current through the coherer and the relay. This current is very feeble, being less than one-thousandth of an ampere, but is sufficient to cause the relay to operate and thus throw in the battery *C* on the Morse sounder, or on whatever recording apparatus may be used in its place. It also throws this battery in on the buzzer *D*, an apparatus identical in operation with the bell described in Par. 410. The small hammer of this buzzer beats against the coherer, jarring the filings sufficiently to cause them to decohere and restoring the resistance of the coherer so that it is in readiness to receive the next succeeding oscillations. These oscillations, however, in trains of ten or twelve (Par. 689), follow each other with such rapidity that the relay is kept closed and the sounder does not release its armature until the operator at the sending station opens his key. The sounder therefore repeats the dots and dashes made at the sending key.

705. Use of Telephone and Detectors.—The relay and coherer as described above have given way to much more sensitive forms

of receiving apparatus with corresponding increase in signaling distance.

In place of the relay, sensitive telephones are now employed. Pierce states that while it requires about 1/200 of a volt to operate a relay, a 540 cycle alternating E. M. F. of 8 *millionths* of a volt will produce an audible sound in such a telephone.

Instead of the coherer, many more sensitive forms of *detectors* have been devised. These are of a number of classes, only two of which we shall mention.

In 1896 General H. H. C. Dunwoody (Class of 1866, U. S. M. A.) discovered that a crystal of carborundum (Par. 488) inserted in an electric circuit served as a detector of electric oscillations. Many other crystalline substances, such as sulphide of molybdenum, oxide of zinc, silicon, etc., have since been found to possess the same property. Pierce by some beautifully conceived and brilliantly performed experiments has shown that the action of the crystals is to rectify the oscillatory currents in some way analogous to the operation of the mercury arc rectifier (Par. 654). He therefore designates them as "crystal rectifiers."

Their operation may be understood from the following: In the diagram (Fig. 374), *A* represents a metal point pressing upon a crystal *B*, and *T* is a telephone shunted around *AB*. Suppose that the oscillatory currents in the aerial may pass upward through *AB* but not downward (although this direction is immaterial). The successive oscillations follow at intervals which may be less than a millionth of a second, and the currents flowing upward but not downward, a charge accumulates in the antennae. When the oscillations cease, this charge flows down through the telephone. The extreme rapidity of the oscillations prevent them from causing vibrations in the diaphragm of the telephone, and even if they did cause such vibrations, their frequency would be far beyond anything that the ear can detect. The downward-flowing charges however follow in accordance with the intervals between the sparks at the sending station, and thus produce an audible note.

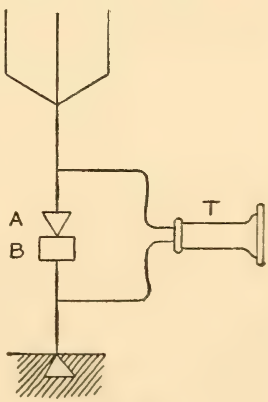


Fig. 374.

Fessenden has devised an *electrolytic detector* which also has been shown to be a rectifier. It differs from the arrangement shown in Fig. 374 in that B is a small platinum cup containing dilute acid, and A is a platinum wire, a thousandth of an inch or less in diameter and barely touching the acid in B. There is also inserted in the telephone circuit a single cell which sends a feeble but steady current through the telephone. The oscillatory current in the aerial causes the current through the telephone to vary and thus produce a sound.

706. Tuning of Receiving Circuits.—The receiving circuit is not quite so simple as represented in Fig. 374 but is usually a coupled circuit (Par. 701) and contains both a variable condenser and a variable inductance. By varying one or both of these, the circuit may be adjusted so as to be in resonance with the particular waves which are being received. If waves of different lengths are arriving, the circuit may be tuned to resonance with those of one length to the exclusion of those of other lengths. This naturally suggests that where two wireless stations are endeavoring to communicate and are being disturbed by signals from other stations, confusion may be avoided and perhaps privacy secured if by pre-arrangement the two stations concerned should use waves of different lengths from those used by the other stations. This involves the ability of the stations to ascertain the length of wave which they are emitting and to adjust their apparatus to emit waves of the desired length. This information is furnished by several forms of *wave meters*, instruments carrying a graduated scale from which, when they have been adjusted to resonance, may be read direct the length of the corresponding wave in the exciting circuit.

The standard wave length now used in communicating with vessels at sea is 425 meters.

707. Distance Attained by Wireless Telegraphy.—The distance to which wireless signals may be sent is constantly being increased and with more powerful sending apparatus, loftier antennae and more sensitive receiving instruments, there appears to be no reason why eventually they may not be exchanged between diametrically opposite points on our globe. Within the present year (1913) signals have been exchanged between the station at Arlington, Virginia, and Gibraltar, Panama and Alaska.

Several theories have been advanced to explain why it is possible to send these signals around considerable arcs of the earth's circumference. According to some, the radiations proceed in straight lines but at a height of about 50 miles in the atmosphere reach a point where the pressure is so reduced that the air is a conductor just as in the Geissler tubes (Par. 670). Since, as we have already seen (Par. 690), these waves can not penetrate a conductor, they are reflected from this tenuous stratum and thus by successive reflections pass around the arc.

According to others, the waves leave the grounded aerial and slide along over the surface of the earth like an immense inverted U. The better the conducting surface, the better the transmission of the waves. This is corroborated by the fact that signals can be exchanged at a much greater distance over salt water than over land.

Finally, it is a fact not yet explained that these signals can be sent nearly, if not quite, twice as far at night as they can during the day and that the conditions for attaining long distance are especially unfavorable at sun rise and at sun set.

INDEX.

References are to Paragraphs.